

MEASUREMENT OF THE DYNAMIC PROPERTIES OF DAMPING MATERIALS⁺

حساب الخواص الديناميكية للمواد المخمدة

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Abstract:

Most elastomers used in damping treatments for structures are visco - elastic in response to dynamic strains. The stress - strain relation can be described by two properties , the stress - strain modulus and a loss factor. The values of these properties are required for conditions tension or compression when the damping treatment consists of an unconstrained layer and in shear for the sandwich layer type of construction.

The different techniques of measurement using either a freely decaying vibration or a forced resonance motion are considered inadequate for materials with a high loss factor or with properties which are strain dependent.

A resonance technique for determining the properties in compression is described in which the exciting force is varied whilst keeping the strain amplitude constant . A second apparatus is described for testing elastomers in shear using a forced vibration technique. Typical results obtained with both techniques are given with particular attention to the dependence on strain amplitudes.

المستخلص :

معظم الباحثين في مرونة المواد يدرسون تأثير التخميد على مرونة الهياكل عند ديناميكية الإفعال. بحيث يمكن وصف العلاقة بين الإجهاد والإفعال بعاملين هما معامل المرونة (E) ومعامل التخميد (ζ) وللحصول على هذين العاملين لأي مادة تعرض لعمليات السحب أو الضغط. لما لهذين العاملين أهمية كبيرة في الدراسات الهندسية وخاصة في علوم ديناميكية الهواء، تطبق هذه الحالة على المواد من طبقة واحدة أما إذا تعددت الطبقات (غير محددة) فهنا يجب خضوع عامل القص أيضاً كأساس في بناء الهياكل. استخدمت تقنيات مختلفة لقياس الترددات الطبيعية للاهتزازات المخمدة بنوعها الحرة أو القسرية للمواد المختلفة وبمعامل تخميد عالي أو خواص الإفعال المعتمدة. تقنية قياس التردد الطبيعي (الرنين) لحساب خواص المواد عند ضغطها والتي يمكن وصفها لتحرير قوة متغيرة لجعل سعة الإفعال ثابتة. والثانية هو جهاز قياس القص للمواد المرنة باستخدام الإهتزاز القسري. ولكلا القياسيين التي استخدمت تم الحصول على نتائج مثالية تم اعتمادها لتحديد ساعات الإفعال.

Introduction:

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The design of a structure incorporating damping layers cannot be undertaken unless the dynamic properties of the damping material are known. The effects of variations in environmental conditions such as temperature and humidity should also be known, together with an accurate knowledge of the dependence upon frequency and strain amplitude.

The dynamic properties are required for conditions in tension or compression for materials used as unconstrained damping layers and as anti - vibration mountings under machinery and under foundation blocks conditions in shear apply for materials bonded between restraining layers forming a sandwich plate or beam and for the shear types of anti-vibration mountings.

Most elastomers are elastically imperfect, that is the applied stress leads the strain. An applied sinusoidal stress usually results in a sinusoidal strain, the phase angle, δ , between them remaining constant during a complete strain cycle, although it can vary with frequency of cycling and with the total strain amplitude.

A sinusoidal stress, σ , can be expressed as:

$$\begin{aligned} &= \hat{\sigma} \sin(\omega t + \delta) \\ &= \hat{\sigma} \cos(\omega t) \sin(\delta) + \hat{\sigma} \cos(\delta) \sin(\omega t) \end{aligned} \quad (1)$$

The resulting, ϵ , sinusoidal strain will be :

$$\epsilon = \hat{\epsilon} \sin(\omega t) \quad (2)$$

The "in-phase" elastic modulus will be the ratio of the coefficients of the stress and strain components of the, $\sin(\omega t)$ terms or :

$$\hat{E} = \frac{\hat{\sigma} \cos(\delta)}{\hat{\epsilon}}$$

and the "quadrature" component or loss modulus is:

$$\hat{E}'' = \frac{\hat{\sigma} \sin(\delta)}{\hat{\epsilon}}$$

The ratio of these two modulus give a measure of the amount of imperfect elasticity possessed by the material and a loss factor, ζ , is defined as

$$\zeta = \tan \delta = \frac{E''}{E'} \quad (3)$$

Eliminating the variables time, t , from equations (1 and (2) gives:

$$\sigma = \hat{E} \left\{ \epsilon \mp \sqrt{\hat{\epsilon}^2 - \epsilon^2} \right\} \quad (4)$$

which is a stress-strain law of elliptical form, a characteristic of all truly visco-elastic materials. Only two properties need be measured, for example the “in Phase” modulus, \bar{E} , and the loss factor, ζ .

Most visco-elastic materials exhibit a somewhat peculiar non-linearity in that a plot of stress versus strain will be a near perfect ellipse as predicted by equation (4), and discussed in detail by Fitzgeralds and Ferry (1) implying constant values for the properties, \bar{E} , and ζ , throughout a strain cycle, but the actual value of \bar{E} , and ζ will vary with frequency and strain amplitude. See for instance Gehmen (2) and Joseph W.T(3). Most experimental observations show a decrease in the elastic modulus with increase in strain amplitude where as the variation in loss factor is less marked. It is therefore necessary to control the strain amplitude and to be able to keep it constant at predetermined values for accurate measurements. This consideration is of great importance when choosing a method of testing.

Two similar properties will be sufficient to specify the dynamic behavior of visco-elastic materials under conditions of shear strain as occurs in sandwich plates. Most elastomers exhibit less non-linearity under dynamic conditions of shear than under direct extension or compression use and the elliptic stress-strain law of equation (4) will be accurately followed. Experimental methods imposing a pure shear-strain are therefore to be preferred.

The damping property of a material is often expressed differently than by a loss factor, ζ . Approximate conversions are as follows for a system vibrating harmonically at a resonant frequency, ω :-

$$\text{logarithmic decrement} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

damping ratio $c/\bar{c} = \zeta/2$ where \bar{c} is the critical viscous damping coefficient.

Q factor = $1/\zeta$

Frequency band width At half power points = $\zeta \omega$

Different measuring techniques:

The dynamic properties of elastomers can be determined in several different ways, which can be divided as follows :

1. free vibration measurements of the frequency and the logarithmic decay;
2. forced resonance tests giving a response curve from which the required quantities can be calculated;
3. forced non-resonance oscillations with measurements of the force, strain and the phase angle between them.
4. wave propagation measurements of frequency and decay rates.

Reviews and detailed descriptions of most of these test techniques can be found in ref. [2, 4 and 5], using the free vibration techniques can be found in ref [2,4 and 5]. The free vibration techniques using either a cantilever specimen or the traditional pendulum apparatus is quite satisfactory for materials with low damping and without any strain dependence.

The development of high damping elastomers with logarithmic decrement values in excess of 10 makes this technique totally unsuitable. The effects of strain amplitude cannot be determined by this method.

The forced resonance test can be used with fairly high damping materials . Standard apparatus is available , where a cantilever specimen is forced into resonant flexural vibrations. Oberst [6] was used such an apparatus with success and has detected non-linear behavior by the distortion of the resonance curve but no accurate assessment of strain amplitude dependence can ever be made in a test where the amplitude is in fact the measured variable . A useful improvement to the torsional Sandwich beam has been introduced recently by Min Hao and Mohan. D.R. [7] where by Sandwich beam is maintained in a resonating state and the energy required is measured.

A number of different experiments have been described by payne [8] for either resonance or forced vibration tests. The forced non-resonance technique was first used extensively in the Fitzgerald apparatus [1 and 9]. Shear strains are induced into a tubular specimen bonded to a seismic mass in the form of a bar and on the outside to a vibrating tube. The exciting force and the motion of the tube are measured by electro - magnetic means. The suspended seismic mass has, unfortunately, six degrees of freedom and some resonant modes may be excited which would influence the measurements. This may be one of the reasons for the unaccountable “peaks” in the results obtained with the Fitzgreald apparatus at the higher frequencies. It is furthermore not always possible to provide a specimen in tubular form.

The wave propagation method is useful in studying some of the physical properties of elastomers but cannot yields engineering data. The frequency of propagation is usually must too high and the strains are less than 10-3%. [9] .

Dynamic properties in compression:

The main requirements are to load the visco-elastic material undergoing a test to the nominal operating pressure and to vibrate the material at a controlled frequency and at a constant amplitude. The nominal pressure is provided most easily by static weights; although a weight on a single pad of material does not produce a dynamically stable enough system. Three points of support for a larger weight with three equal size pads is fundamentally better. A simple resonance testing technique has been developed in which it is possible to keep the strain amplitude constant. This is achieved by defining resonance as occurring at that frequency at which the minimum value of exciting force has to be applied to maintain the constant amplitude. The exciting force has to be increased at frequencies lower and higher than the resonant frequency. Tests can be repeated at different values of strain

amplitude and the effects of amplitude on resonant frequency and thus the modulus of elasticity and on the damping ratio can be determined.

An outline of the apparatus is shown in figure 1. The three specimens are positioned equally under the circular main loading weights. An electro-magnetic vibrator is attached to these weights, there being a space below it for a vibration pick-up for measuring the amplitude of motion of the main loading weights. The exciting force is created by loading the table of the vibrator with some inertia weights and when the table is vibrated the inertia for reaction will act on the body of the vibrator and on the main loading weights. It is shown in Appendix 1 that the table suspension stiffness and damping forces are internal to the working of the vibrator and do not affect the exciting force; the acceleration of the table and the attached inertia weights will be measured.

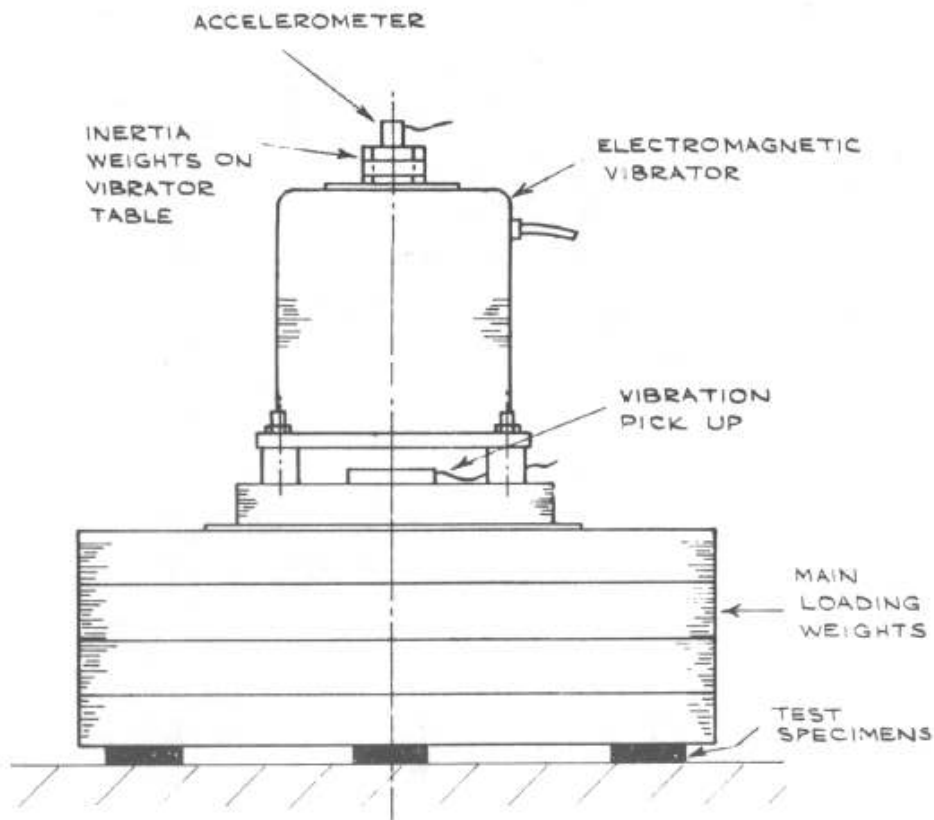


FIG.1. THREE POINT COMPRESSION APPARATUS WITH AN ELECTRO-MAGNETIC VIBRATOR FOR RESONANCE TESTING.

figure 1: outline of the apparatus

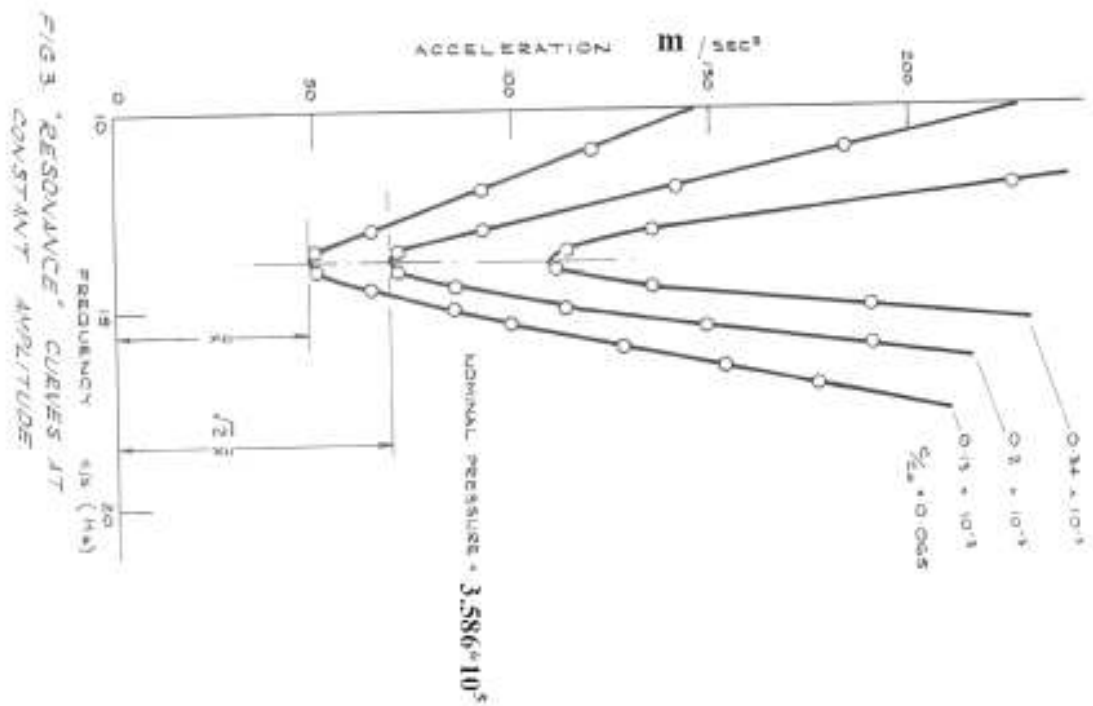
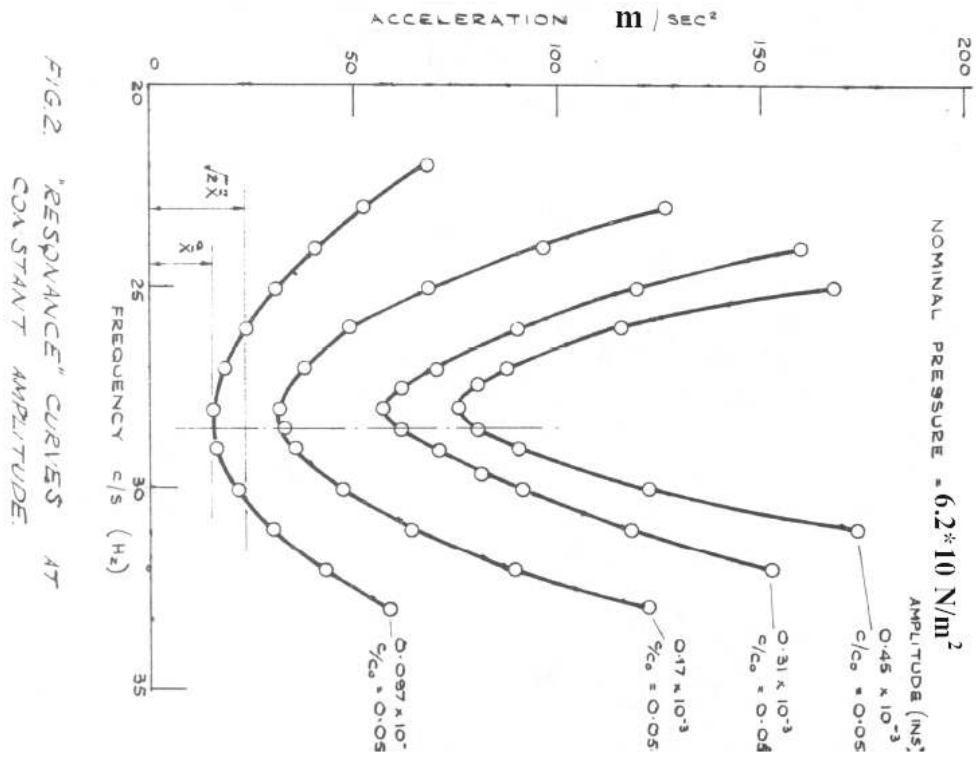
A typical test would consist in loading three specimens until the static creep had diminished to an insignificant rate compared with the time a dynamic test would

take. The vibrator would then be operated at a frequency below the resonance for vertical oscillations of the main system and the level of power supplied to the vibrator would be adjusted to give the required strain amplitude as measured by the center vibration pick-up. The frequency of vibration would then be increased by small increments and the power input to the vibrator adjusted so as to maintain the amplitude constant. Some typical results for rubber anti-vibration pad material are given in figures 2 and 3. The measured acceleration of the inertia weights is plotted against the excitation frequency. The graphs are essentially inverted resonance curves and the same technique of measuring the frequency band width at a half power point will hold for estimating the damping ratio, except that the minimum acceleration has to be multiplied by $\sqrt{2}$. The damping ratio is given by

$$\frac{c}{\dot{c}} = \frac{\text{frequency band width}}{2 \omega}$$

Where ω is the resonant frequency. The results in figure 2 are at higher frequencies than for figure 3 and show much fuller curves although the actual damping ratio is in fact slightly smaller. A definite strain amplitude effect can be observed in the results of figure 2; it indicates a reduction in the in-phase modulus of elasticity of some 10%. No such effect is shown in the results of figure 3 for the softer material.

It would be quite feasible to measure the phase angle between the acceleration and the amplitude as measured by the center pick-up and to construct a frequency locus plot. This will give more accurate values for the resonance frequency and band width but the taking of such measurement is somewhat tedious and was not thought to be worthwhile in this case. It would not alter the principle introduced into technique of carrying out a resonance at constant amplitude by varying the excitation force.

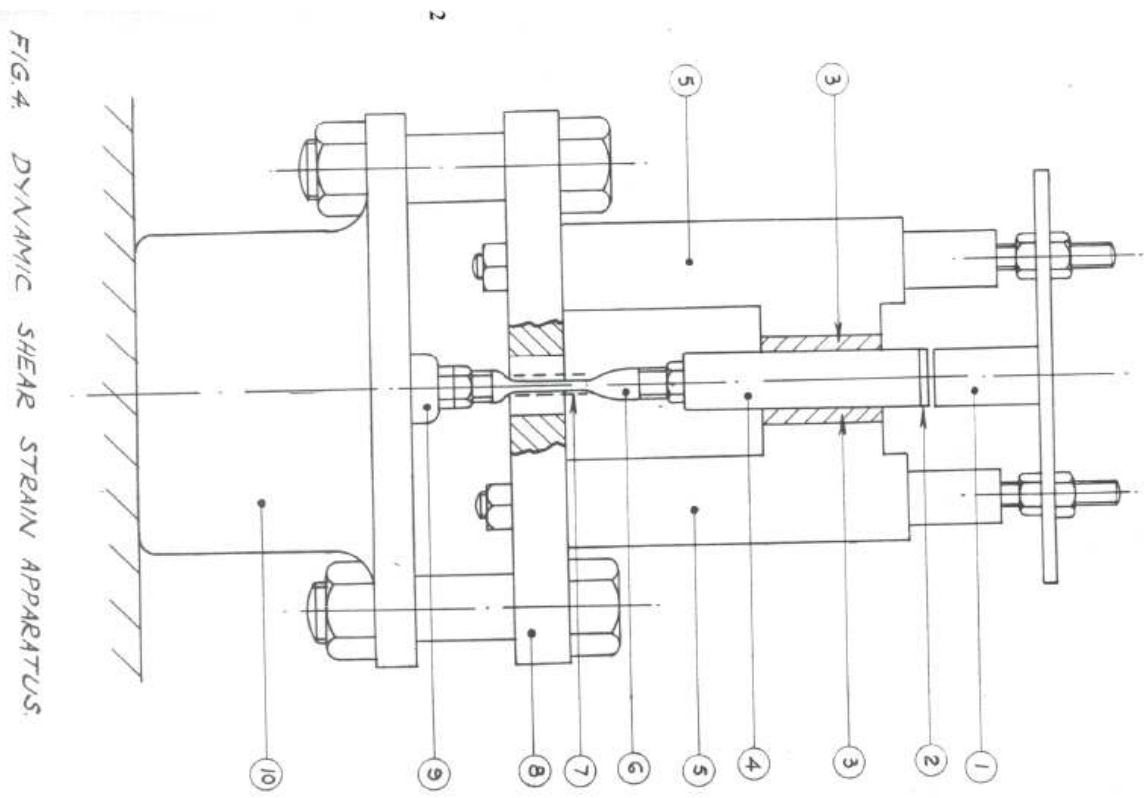


Dynatic properties in shear :

A direct way to obtain the properties in shear is to measure the mechanical force required to induce a shear strain. A forced non-resonance technique has been used to cover a wide frequency range and the apparatus developed for this is shown figure 4.

Two identical specimens of material, 3, were bonded to a moving center piece, 4, and two stationary supports, 5. The force and motion were provided by an electromagnetic vibrator, 10, and transmitted from the output connection, 9, through the drive rod, 6, to the center piece, 4. The drive rod, 6, was a 6.4 mm diameter aluminum rod machined over its middle half length to form two flat surfaces and reduced to a thickness of 1.26 mm. Two identical resistance strain gauges, 7, were cemented to these surfaces to measure the strain in the drive rod so that the applied force could be calculated. The strain gauges were connected in series to cancel out any strain effects due to bending of the drive rod. The strain gauge and measuring bridge could be calibrated statically before the test specimens were bonded in place by turning the apparatus up-side-down and by attaching known weight to the free end of the center drive piece. This calibration was carried out before and after a test.

The displacement of the center piece was measured with an inductance transducer, 1, mounted in an adjustable cross beam which was supported on the stationary frame of the apparatus. A disc, 2, of brass was attached to the end of the aluminum center piece to increase the sensitivity. The inductance transducer was part of a frequency modulated bridge circuit and could be calibrated statically against a known displacement. The center piece, 4, was 12.7 mm square cross-section and 6 mm long and made from aluminum to keep the inertia force low. The stationary supports, 5, and the base plate, 8, were made from steel with ground mating faces to ensure accurate alignment. The base plate, 8, had slots in it for the bolts darning the supports, 5, so that specimens of different thicknesses could be accommodated. A heater could be placed around the apparatus for tests at elevated temperatures and thermocouples were embedded in the specimens. A gradual heating of the strain gauges on the drive rod did not affect the alternating strain measurements as it produced a d.c. voltage change only.



The measured force applied to the drive rod was not the shear force acting on the specimens owing to the inertia of the center piece, but this could be accounted for as follows. Let the displacement of the center piece be

$$x = \bar{x} \sin(P t)$$

Where, p, is the forcing frequency in rad/sec. The corresponding shear strain in each specimen, assuming pure shear is:

$$\phi = \bar{\phi} \sin(P t) = \frac{\bar{x}}{b} \sin(P t) \quad (5)$$

Where b is the thickness of a specimen. The shear stress will lead the strain by the angle, δ , such that

$$\tau = \hat{\tau} \sin (pt + \delta) \quad (6)$$

the shear force is:

$$A\hat{\tau} \sin (pt + \delta) \quad (7)$$

Where A is the total area of bonding of both specimens to the center piece. The inertia force due to the center piece and part of the drive rod, of combined mass, M, is:

$$M\ddot{x} = -M\bar{x} p^2 \sin pt \quad (8)$$

The measured force, P, will be out-of-phase with the displacement by an angle which can be measured by comparing the voltages obtained from the strain bridge and from the inductance transducer. Combining equations (7) and (8) gives the measured force as:

$$\begin{aligned} P &= \hat{P} \sin (pt + \delta) \\ &= A\hat{\tau} \sin (pt + \delta) - M\bar{x} p^2 \sin pt \end{aligned} \quad (9)$$

Expanding equation (9) and can by comparing similar terms gives:

$$P \sin \delta = \tau \sin \delta$$

$$\hat{P} \cos \delta = \hat{\tau} \cos \delta - M\bar{x} p^2$$

(10)

Hence by using the definitions of the properties gives in the introduction we obtain for the in-phase shear modulus:

$$G' = \frac{\hat{\tau} \cos \delta}{\hat{\phi}} = \frac{\hat{F} \cos \delta + M \hat{x} \omega^2}{A} \left(\frac{b}{\hat{x}} \right) \quad (11)$$

And for the loss factor

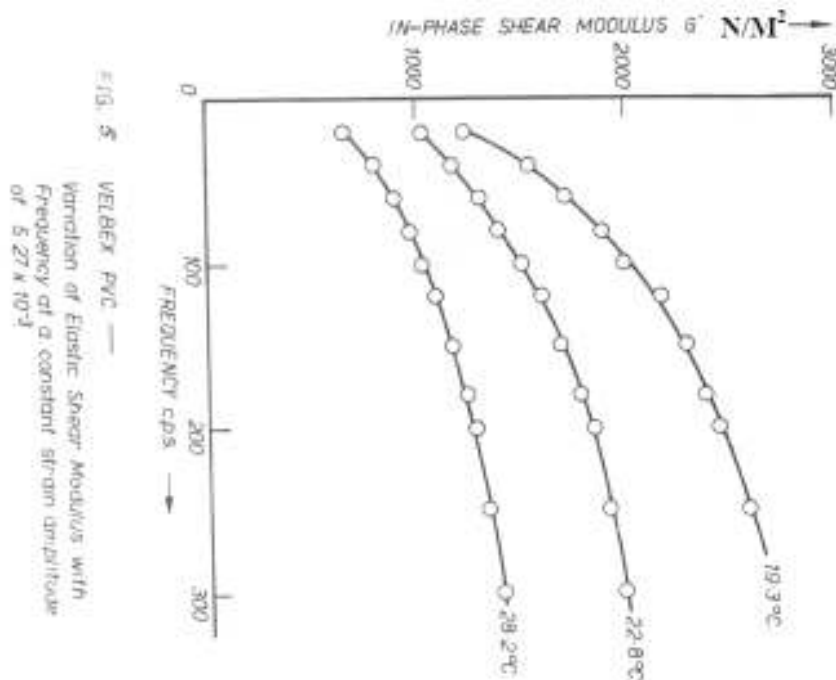
$$\zeta = \tan \delta = \frac{\hat{F} \sin \delta}{\hat{F} \cos \delta + M \hat{x} \omega^2} \quad (12)$$

The measurements which have to be taken for a sinusoidal excitation at a frequency $2\pi\omega$, are the maximum, \hat{p} , of the force signal measured by the strain gauges, the maximum displacement, \hat{x} , measured by the inductance transducer and the phase angle, δ , between these two signals, and in addition, of course, the constant quantities relating to the test is A , b , M , and the temperature.

It can be shown easily that with pure shear allowance for the inertia effects of mass of the material to the mass, M , of the center piece. This is worth while only with thick or dense specimens. Experiments at vary large strain amplitudes will not induce a pure shear strain in the material because initially horizontal element will be stretched when the centerpiece is displaced. The force required to cause this extension is in addition to the shear force applied to the center

piece but it has been shown by Agbasiere [7] that the additional force required is only about $1 \frac{1}{2}$ % of the shear force for a strain as large as 10%. A similar error would be present in the Fitzgerald apparatus.

The dynamic properties of an elastomer material suitable for sandwich plate constructions are given in figures 5,6 and 7. The material was a soft polyvinyl choride (p.v.c) with 35-40% of puthalate plasticiser (known as velbex). The frequency dependence at a constant strain amplitude and at three different temperatures of the shear modulus is shown in figure 5 and of the loss factor in figure 6.



The effect of strain amplitude at constant frequency and temperature upon both properties is shown in figure 1 and confirms the effect usually observed of a reduction in the modulus and thus the rigidity with an increase in strain. These results do not confirm, however, the rather strange “resonance” features found by Cook et. al. [1] for a similar p.v.c. and other materials. They used a Fitzgerald apparatus and it has already been pointed out in the Introduction that this type of apparatus must suffer a number of rigid body resonances for which no allowances were made by Cook et. al.

A damping material suitable for use in concrete structures, see for example Grootenhuis [11] is a bitumen-rubber latex mixture known as Eroseau. This has a high loss factor of about 1.5 and the strain amplitude dependence is shown in figure 8. The modulus again decreases with an increase in strain but the loss factor is increased. The properties are obviously very non-linear but the material is nevertheless visco-elastic during a strain cycle because the force-displacement loop is still an ellipse. A trace taken from the oscilloscope of the force and displacement loop is shown in figure 9 and is compared with a number of points calculated for a true ellipse of similar size. The agreement is quite remarkable.

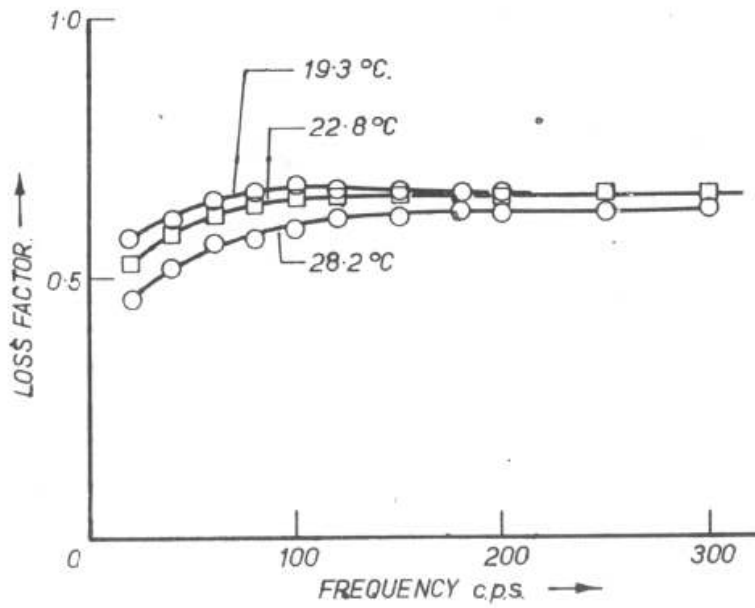
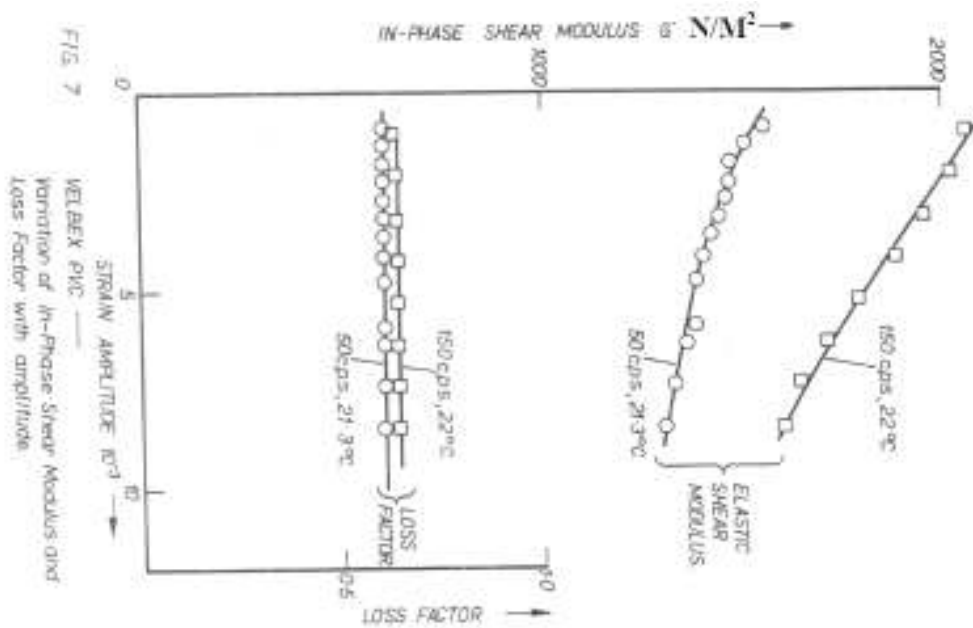


FIG. 6. VELBEX PVC. —
 Variation of Loss Factor with Frequency
 at a constant strain amplitude of 5.27×10^{-3}



Conclusions :

1. The first apparatus described uses a resonance technique and the compression modulus of elasticity and loss factor can be obtained.
2. The second apparatus is based on a forced non-resonant technique for testing in shear.

3. The properties obtained by either technique can be related, in that it is usually assumed that the loss factors for compression and shear are equal and that $E' = 3 G'$. A correction to this comparison has been developed by Mark [12] which should be used in particular when the conditions of frequency or temperature are such that the material is not in the transition region.
4. The modulus of elasticity decreases with an increase in strain but the damping factor is increased.
5. It would be quite feasible to measure the phase angle between the acceleration and the amplitude as measured. This gave more accurate results for the natural frequency and the band width.
6. This a simple technique of resonance testing has been developed in which it is possible to keep the strain amplitude constant.

Appendix:

Attaching the electro-magnetic vibrator to the main loading weight has the effect of introducing a second degree-of-freedom into the system. The moving table in the vibrator is supported by a spring suspension system which will inevitably introduce some damping.

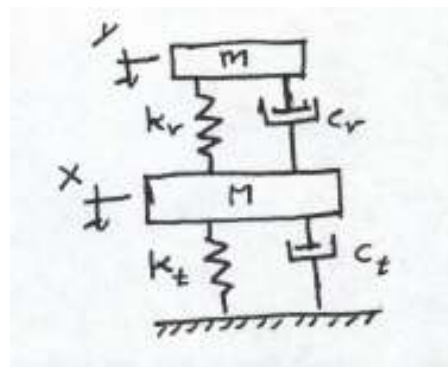
The rigid body mode of vibration of the table resonating on its spring suspension lies usually in the frequency range of 20-40 c.p.s (HZ) and this can be adjusted by either adding a mass to the table or by changing the suspension stiffness.

The non rigid body modes of the table-moving coil system can be made to be in the kilo c.p.s. range as shown by Randall, R.B. [13], who discussed the design of electro magnetic vibrators in more detail. The equations of motion for the two degree-of-freedom.

System are:

$$M \ddot{x} + C_t \dot{x} + K_t x + C_v (\dot{x} - \dot{y}) + K_v (x - y) = F \quad (A1)$$

$$m \ddot{y} + C_v (\dot{y} - \dot{x}) + K_v (y - x) = -F \quad (A2)$$



Where: M=mass of main system
 m = mass of vibrator table and added masses
 K_t = stiffness of test specimen

C_t = damping coefficient of test specimen
 K_v = stiffness of table suspension
 C_v = damping coefficient of the suspension
 F = force produced by the vibrator

The Equation A1 can be re-written as:

$$M \ddot{x} + C_t \dot{x} + K_t x = F - C_v(\dot{x} - \dot{y}) - K_v(x - y)$$

On inspection of equation A2, it is seen that the right hand side is equal to- my that is the inertia force of the moving table. the forces internal to the vibrator have cancelled out, all what we needs to measure is the acceleration of the table motion.

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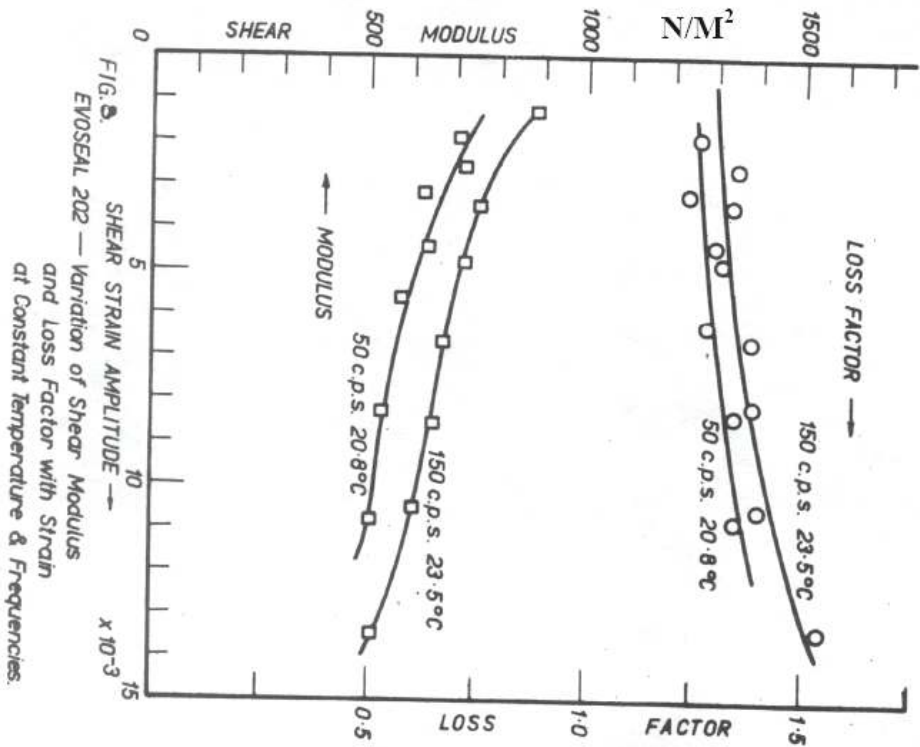


FIG. 8. SHEAR STRAIN AMPLITUDE →
 EVOSEAL 202 — Variation of Shear Modulus
 and Loss Factor with Strain
 at Constant Temperature & Frequencies.

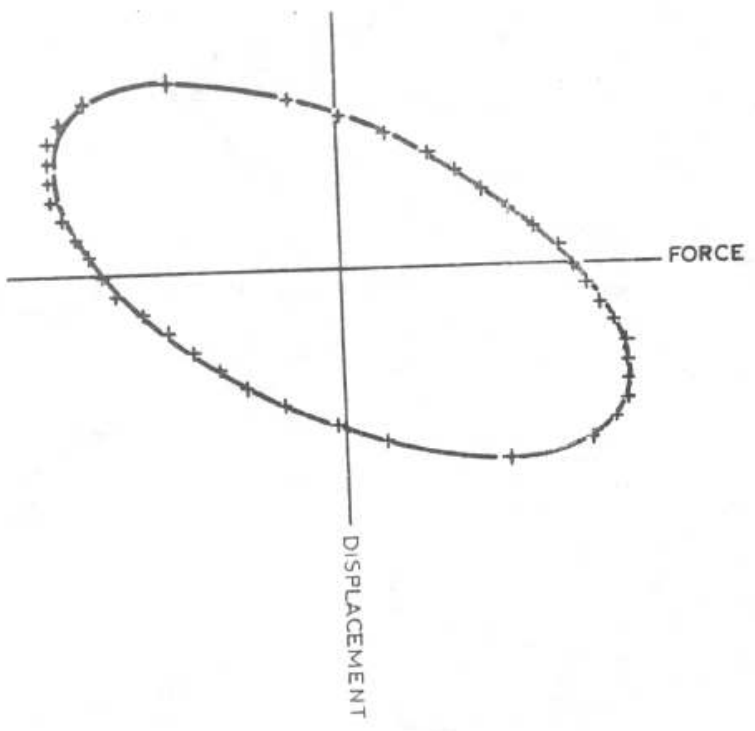


FIG. 9
 Force-Displacement trace
 for EVOSEAL specimen