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CREATION A NEW MATHEMATICS RELATIONS USING ENGINEERING APPROACHES

Assist. Prof. Dr. Dhafer R. Zaghar
Department of Computer & Software Engineering/College of Engineering/University of Al-Mustansiriyah /Iraq,
Email-drz_raw@yahoo.com

Abstract

The mathematics laws, relations and functions are the guide for all the engineering designs and applications. This guide will simplified the work in the design phase but it capture the design in some times because the conditions of high accuracy in mathematics values. This accuracy not accepted any approximation in opposite side the values in the engineering fields that have small approximation in all times.

This paper will propose a novel approach with its proves to replacement some complex functions with a more simplicity functions that have approximate general shape, then modified the alternative functions to approximate them values for all times, i.e. reduce the error (maximum and accumulated) between the original function and the alternative function.

This paper discusses a new mathematics relation between some functions spatially the unintegralable functions such as arctan, arctanh, Gaussian and other functions. This relation will detect and prove using an engineering approach such as successive approximation in C++ programming also this paper will discuss the other derivatives for these relations if found. Finally it will give some important direct applications with some future expansions in this field.

الخـلاصــــــة

ان القوانين و العلاقات و الدوال الرياضية هي الموجه لكل التصاميم و الحسابات الهندسية. ان هذا الموجه بالرغم من تبسيطه للعمل في مرحلة االتصميم الا انه يقيد التصميم في بعض الاحيان بسبب الدقة العالية المطلوبة في العمليات الرياضية. هذه الدقة لاتقبل اي تقريب بخلاف المجالات الهندسية التي تتسم بالتقريب البسيط دائما.

ان هذا البحث يقترح طريقة جديدة مع الاثباتات اللازمة لابدال بعض الدوال المعقدة مع اخرى اكثر بساطة و لكنها تملك نفس الشكل العام ثم تطوير الدالة البديلة لتقليل الفرق بينها وبين الدالة الاصلية.

كما ان هذا البحث سيناقش علاقات جديده و خاصة الدوال الغير قابلة للتكامل مثل الدالة الظل المثلثيه (tan) ودالة الظل البيضويه المثلثيه المقلوبه (arctanh) و دالة (Gaussian). ان هذه العلاقات سوف تبين و تثبت باستخدام اساليب هندسيه مثل التقريب المستمر (successive approximation) وبرمجهه بلغة سي (C++) لايجاد هذه العلاقه المباشره و العلاقات المشتقه منها و كذلك سيركز على بعض التطبيقات الهندسيه المباشره لها مع بعض المقترحات المستقبلية.

Key Words

Mathematics relations, engineering approaches, trigonometric, hyperbolic, Gaussian, erf function, unintegrable functions, Taylor series.

1. Introduction

For a long time the engineering used the mathematics equations and relations to design and simplified the systems. But the mathematics conditions and laws capture the engineering works in some times, spatially the equality relation in mathematics. Hence, the relation a=b main a-b=0 for all times and for all cases while the engineering fields accept some errors in the manufacturing and results or in the design. For example the accept error in the value of resistance ±5% or in the values of the computer calculations that reach to about $3*10^{-20}$, so some one can noted this error (approximation) in the calculations of the value of π or in the value of exp(x), so the combination of them exp (π) reach to about $2.3*10^{-13}$ [1].

This paper will reverse this case by using the engineering to estimate new mathematics relations that will used in the future to design systems and simplified the previous systems. It will fox on the unintegralable functions such as arctan, arctanh, Gaussian functions and some classic functions such as cos(x).

This paper discusses in details a new mathematics relation between two functions first from trigonometric family (tan) and the other from inverse hyperbolic trigonometric family (inverse hyperbolic tan or arctanh). This relation will detected and proved using an engineering approach such as successive approximation under C++ programming also this paper will discuss the other images for this relation, then it will use same approach to find the relations of other functions and fox on the direct applications for it in the engineering world.

2. The Proposed Method

This section will discuss the proposed method using a clear example that represent the alternative between tan and arctanh functions. We will use here three symbols for the equality, first symbol is "=" to represent the mathematics equality, second symbol is "≅" to represent the engineering equality with very small error (typically maximum error less than 0.1% from the value of the function), while the third symbol is "≈" to represent the engineering equality with acceptable error (typically maximum error less than 3% from the value of the function),.

2-1 The Functions tan and arctanh

Fig (1) shows the plot of the two functions *tan(x)* and *arctanh(x)*, this figure shows how these two functions are identical in them general shapes. However, some one can be asked if the two functions are equal? The answer is no but the difference between them is very small value of error e(x) where defined in equation (1)

$$\mathrm{e(x)} = \mathrm{arctanh}(x) - \tan(\pi x / 2) \qquad (1)$$

Hence, the e(x) is a function of x and the two functions in right hand side, so the modification one from these two functions will reduce the error to limited as zero or very small values, this paper will to trade on this similarity to find a new relation through modification the second function (arctanh) as in the equation (2).

$$\tan(x) \cong \operatorname{arctanh}(\alpha(x)) \qquad (2)$$

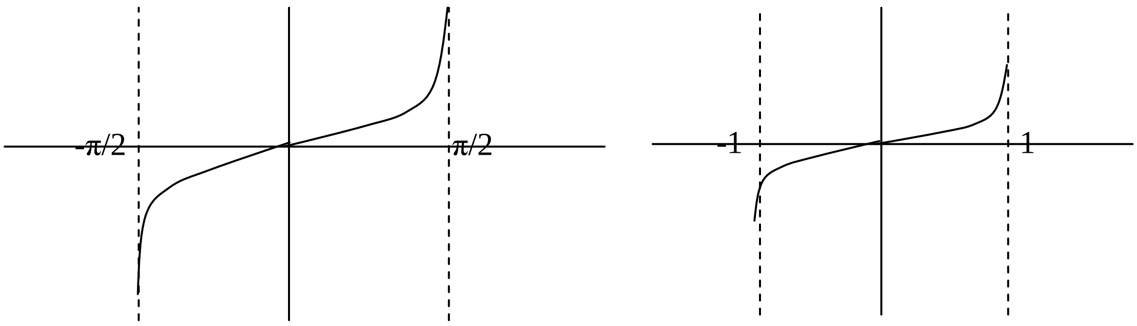

Fig (1): The plot of tan(x) function and arctanh(x) function.

2-2 Properties of α(x) Function

The function α(x) in equation (2) is the key to reduce the e(x), but it has some properties to satisfy the equation (2) these properties are:

1- The function α(x) has odd symmetry.

2- The domain of α(x) is (-π/2, +π/2) and the range is (-1, +1).

3- α(0) = 0 and α(π/2) = 1.

4- Linear combination of x powers. This condition is not compulsory but it is necessary for simple relation.

3. Calculation of α(x) Function

The direct value of α(x) can be calculated from equation (3), but this value is very complex and can be simplified using more than one method.

$$\alpha(x) = \tanh(\tan(x)) \qquad (3)$$

The classic methods to simplify the value of α(x) is the Taylor series [2] of equation (3) satisfied using equation (4), this method is very complex and gives an infinite term as in equation (5). Equation (5) shows that all the even derivatives at x equal to 0, the x coefficient equal 1 and the x^3 coefficient equal 0, while the higher coefficients are complex calculations. However, the finite practical solution can be found using the results of equation (5) as initial values with a successive approximation under C++ program to find an approximation term of α(x) as in equation (6).

$$f(x) = f(0) + \frac{x}{1!} f^{\backslash}(0) + \frac{x^2}{2!} f^{\backslash\backslash}(0) + \frac{x^3}{3!} f^{\backslash\backslash\backslash}(0) + \ldots\ldots + \frac{x^n}{n!} f^{(n)}(0) \qquad (4)$$

$$f(x) = 0 + \frac{x}{1}1 + \frac{x^2}{2}0 + \frac{x^3}{6}0 + \frac{x^4}{24}0 + \frac{x^5}{15}a3 + \frac{x^6}{6!}0 + \frac{x^7}{15}a4 + \ldots\ldots + \frac{x^n}{n!} f^{(n)}(0) \qquad (5)$$

$$\alpha(x) \cong x - 0.1x^3 - 0.014x^5 - 0.017x^7 + 0.00606x^9 \qquad (6)$$

However, the result in equation (6) is complex with very low error while most engineering applications remains reliable with higher error, therefore this relation can be rewrite as in equation (7) that has higher error but it very simple expression and the final result as in equation (8).

$$\alpha(x) \approx (2/\pi)x \tag{7}$$

$$\tan(x) \approx \operatorname{arctanh}(\frac{2}{\pi}x) \tag{8}$$

4. The Generalization of the Proposed Method

The proposed method is not spatial for single or few cases but it's general method so it can be generate new relations in different ways as example first using the families of the original and alternative functions to generate additional relations at same type of these functions as in 4.1. The second way to generalize this method depends on the more searches to find other useful relations as in later. The new relations may be an absolute relations i.e. satisfied for all times and all cases as the relation in section 3 or limited under specific condition as in sub-section 4.3. This section proposes some additional relations with some proposed future challenges to find more relations as below:

4-1 The Additional Family Relations

The proposed relation in section 3 is not unique relation but it is a connection between two function families. This relation will generate set of relations as in equations (9) when β(x) is the inverse function of α(x) and can be found in same method in section 3.

$$\begin{aligned} &\arctan(\mathrm{x}) \cong \tanh(\alpha(\mathrm{x})) \\ &\tan(\mathrm{x}) \cong \operatorname{arctanh}(\alpha(\mathrm{x})) \\ &\arctan(\beta(\mathrm{x})) \cong \tanh(\mathrm{x}) \\ &\tan(\beta(\mathrm{x})) \cong \operatorname{arctanh}(\mathrm{x}) \end{aligned} \tag{9}$$

4-2 The New Absolute Relations

The proposed method can be used to find new relations to solve the unitegrable functions such as Gaussian function (e^{-x^2}) with defend integral function as in equations (10 &11).

$$e^{-x^2} \cong (\sec h(\alpha(x)))^2 \tag{10}$$

$$e^{-x^2} \approx (\sec h(x))^2 \tag{11}$$

These equations will satisfy an alternating expression for *erf(x)* function as in equation (12) that has a wide range of applications [3], this approximation will has error less than 2.7% in maximum but it will give a very high simplicity for the calculations of probability and channel design.

$$\int e^{-x^2}dx \approx \int (\sec h(x))^2 dx \quad \Rightarrow \quad erf(x) \approx \tanh(x) \tag{12}$$

4-3 The New Conditional Relations

This method can be used to find a new relation under limited time or limited conditions, these relations used to simplify the calculations under pacific cases, as example the relation in equation (13).

$$\cos(x) \approx \sqrt{(2/\pi)^2\left((\pi/2)^2 - x^2\right)} \qquad \text{for } -\frac{\pi}{2} \le x < \frac{\pi}{2} \tag{13}$$

4-4 The Challenges Relations

The additional advantages for this method are the non-limited relations and the wide range of ideas, but some thinks require to deep analysis and hard work as challenges. One from the most important and most difficulty challenges is the finding an alternative function using the proposed method for the factorial function (n!) or to the gamma function ($\Gamma(x)$) in general. Other important and unitegrable functions (challenges) are $(\ln x)^2$ and $e^{\sqrt{x}}$.

5. Applications of the proposed relations

The most useful direct application for the proposed relations fill in the field of unintegrable functions that used in the engineering calculations such as erf function, the nonlinear differential equations that have arctan or arctanh. While the indirect engineering applications fields fill in the applications that use the direct applications above and that have large number of functions such as the solutions of the matrix function of the differential equations [4], the complex circuit analysis such as [5] and the analysis of the transmutation line analysis [6].

6. Conclusions

One from the ingrained facts is that "no engineering without mathematics relations" but at the same time these relations capture the calculations in some times spatially when it has complex and unintegrable functions such as Gaussian function and its integration (erf function).
The deep analysis and careful comparator for the complex and unintegrable functions -that used in the different engineering calculations and designs- will leads to find an alternative functions have general symmetric shape with different values.
The new relations are not acceptable in the mathematics definitions but they accept in some engineering applications because the engineering fields accept some approximations and some errors in opposite of the mathematic.
This paper proposed a novel approach to simplify some complex engineering calculations by detect an expression for the alternative function to reduce the complexity and the error ratio at

the same time. This process will give acceptable, simple and integrate-able functions, so it in result will give a simple design and reduce the complexity and the calculations cost.

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