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Proca Electrodynamics Approach to The Massive Photon

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Abstract

The work presents a reflection on the importance of the existence of a massive photon. The discussion takes place based on the development of Maxwell's electrodynamics, introducing the massive term for the photon through Proca's electrodynamics. The presentation was based on theoretical development with the exposure of derivations by the divergent and rotational electromagnetic field, imposing the subsidiary condition with the terms of vector potentials. The work was intended to be simple, not going into details regarding applications and even Lagrangian terms, gauge transformations, and conservation of energy and momentum. The objective of this work is to discuss the possibility of the massive photon, through the derivation terms using divergent and rotational electromagnetic fields by extending the Proca model, presenting the dispersion relationship and consolidating with the Compton length and relating it to Planck's constant to understand the additional massive parameter of the theory. We made a comparison of some researchers in relation to experimental data, where they used an upper limit for the photon mass. We consolidate with a reflection on the study of the massive photon and its effects.

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1. Introduction

The photon is a spin 1 bosonic particle, with no charge and normally appears in various interactions, such as the electromagnetic one. A priori the photon is a massless particle and the objective of this work is a reflection of a theoretical discussion in the approach of Proca's electrodynamics to the existence of the massive photon.

There is a deep approach to this discussion, such as the exposition of Yukawa potential, gauge transformations, Lagrangian terms, and applications such as cosmological effects and superconductors.

The objective of this work is to discuss the possibility of the massive photon, through the derivation terms with the use of divergent and rotational electromagnetic field by the extension of the Proca model, presenting the dispersion relationship and consolidating with the Compton length and relating to Planck's constant to understand the additional massive parameter of the theory.

2. Method

Maxwell's electromagnetism considers the nullity of the photon's rest mass and several works for this discussion are being developed because the nullity of the photon mass in the introductory view of its construction is anchored in theory such as the inverse square law of the distance of Coulomb and Cavendish, experimentally proven [1]. To what extent is the masslessness of the photon interesting for physics? Should we have a more accurate experimental apparatus for high-energy measurements for such observation? Does physics need a massive photon for certain effects or phenomena?

For this discussion, we initially have a basis for the development of electromagnetism satisfactorily with Maxwell's equations, where he formulated equations, previously employed by Stokes in terms of vector operators (divergence and rotational), where the four equations that we know today appear. Maxwell's equations in a vacuum and with sources are presented in the form:

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0} \quad (1)$$

$$\vec{\nabla} \times \vec{E} = -\partial_t \vec{B} \quad (2)$$

$$\vec{\nabla}.\vec{B} = 0 \quad (3)$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \partial_t \vec{E} + \mu_0 \vec{J} \quad (4)$$

Where Gauss's law in the differential form appears in equation (1), obtained by Coulomb's law, with charge density ρ and electrical permittivity of the vacuum ε_0. In equation (2) we have Faraday's law of electromagnetic induction in the differential form, in equation (3), it indicates the non-observation of isolated magnetic monopoles, and in equation (4) it is expressed in an extended form of Ampere's law, with the part on the right, having the displacement current density in the first term and the second term $\mu_0 \vec{J}$ with the magnetic permeability of the vacuum and the current density.

The four equations above describe linear electromagnetism, but for certain specific effects and applied problems such as superconductivity and cosmological effects, it requires more detailed electrodynamics, as in the case of this work, the discussion of the mass of the photon. Experiments reach smaller and smaller distance limits, which can be even lower than the precision that the experiments provide [2].

According to Goldhaber and Nieto, all electromagnetism should be reformulated, taking into account the Gauge invariance, which considers the photon non-massive because in order to consider the photon massive, we must abandon this invariance [3].

For the discussion of this invariance, we have the development in equations (1-4) for the potentials, where the Gauge transformations are also known as gauge transformations.

Maxwell's equations for electromagnetism involve a relationship between the electric and magnetic fields in which a Lorentz force is presented, in the form:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad (5)$$

Let q be the charge and v be the velocity. Maxwell's equations can introduce the scalar potential Φ and the vector potential $\vec{A}$. Defining the fields according to the potentials in the form:

$$\vec{E} = -\vec{\nabla}\Phi - \partial_t \vec{A} \quad (6)$$

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad (7)$$

Where they satisfy Maxwell's two homogeneous equations. The point is that in classical electrodynamics, the fields are the directly observable quantities and their derivatives, although it is simpler to use the potentials to calculate the physical quantities expressed in these fields through equations (6, 7) [2].

To get into the discussion of Proca's model, according to Gonçales [1] Proca electrodynamics can be obtained using four hypotheses, such as the force law of electrostatics, the principle of superposition, the hypothesis that electric charge is a Lorentz scalar, and the requirement of invariance of form of the equations of electrostatics under Lorentz transformations. In this work, we will focus only on the development of the extensions of Maxwell's equations, with the use of divergence and rotational, in terms of potentials.

The Proca equations are developed in Maxwell's equations, but adding the term massive for the photon, in the form:

$$\vec{\nabla}.\vec{E} + \xi\Phi = \frac{\rho}{\varepsilon_0} \quad (8)$$

$$\vec{\nabla} \times \vec{E} = -\partial_t \vec{B} \quad (9)$$

$$\vec{\nabla}.\vec{B} = 0 \quad (10)$$

$$\vec{\nabla} \times \vec{B} + \xi\vec{A} = \mu_0 \varepsilon_0 \partial_t \vec{E} + \mu_0 \vec{J} \quad (11)$$

The addition of these potentials makes no transformations that change the intensities of the fields. Because of this presence, the electric field and the magnetic field will not function with the inverse square of the distance. There will be a loss of linearity, and non-linearity will arise in the dynamics of this new electromagnetism. The quantity ξ is a quantity that we will discuss later [4].

The equations (6, 7) that relate the fields and potentials are still valid, and by deriving equation (8) concerning time and applying the divergent to equation (11), we get:

$$\vec{\nabla}.\partial_t \vec{E} = \partial_t \left(\frac{\rho}{\varepsilon_0} - \xi\Phi\right) \quad (12)$$

$$\nabla.\left(\vec{\nabla} \times \vec{B}\right) + \xi\nabla\vec{A} = \mu_0 \varepsilon_0 \nabla \partial_t \vec{E} + \mu_0 \nabla\vec{J} \quad (13)$$

And by relating it to equations (12) and (13), we get the form:

$$\xi\left(\nabla\vec{A} + \mu_0 \varepsilon_0 \partial_t \Phi\right) = \mu_0\left(\nabla\vec{J} + \partial_t \rho\right) \quad (14)$$

Equation (14) corresponds to the right-hand term for the conservation of charge, where zero is the ratio $\nabla\vec{J} + \partial_t\rho$, as a result, also zeroing out the term on the left-hand side $\nabla\vec{A} + \mu_0\varepsilon_0\partial_t\Phi$ [5].

It is interesting to discuss the speed of light, to clarify our discussion of the search for the massive photon in the Proca approach. We also use Maxwell's equations without the source terms, i.e., in the absence of charges and currents, i.e., $\rho = 0$ e $\vec{J} = 0$, Getting in form [6,7]:

$$\vec{\nabla}.\vec{E} = 0 \tag{15}$$

$$\vec{\nabla} \times \vec{E} = -\partial_t\vec{B} \tag{16}$$

$$\vec{\nabla}.\vec{B} = 0 \tag{17}$$

$$\vec{\nabla} \times \vec{B} = \mu_0\varepsilon_0\partial_t\vec{E} \tag{18}$$

Because to get to the speed of light, we need to know the speed of propagation of the electromagnetic wave and for that, we must take the rotational from equations (16) and (18) which are also rotational of the electric and magnetic fields. Taking the rotational from the rotational of the equations mentioned, we will have in the form:

$$\nabla \times (\nabla \times E) = -\partial_t(\nabla \times B) \tag{19}$$

$$\nabla \times (\nabla \times B) = \mu_0\varepsilon_0\partial_t(\nabla \times E) \tag{20}$$

By relating equations (16) and (18) to equations (19) and (20), we obtain

$$\nabla \times (\nabla \times E) = -\mu_0\varepsilon_0{\partial_t}^2 E \tag{21}$$

$$\nabla \times (\nabla \times B) = -\mu_0\varepsilon_0{\partial_t}^2 B \tag{22}$$

Well, as for every field A = (x,t) The identity of the vector calculus is valid $\nabla \times (\nabla \times A) = \nabla(\nabla.A) - \nabla^2 A$, then equations (21) and (22) can be rewritten, in the form:

$$\nabla(\nabla.E) - \nabla^2 E = -\mu_0\varepsilon_0{\partial_t}^2 E \tag{23}$$

$$\nabla(\nabla.B) - \nabla^2 B = -\mu_0\varepsilon_0{\partial_t}^2 B \tag{24}$$

Using equations (15) and (17), we cancel the first terms of equations (23) and (24), arriving at the following results:

$$\mu_0\varepsilon_0{\partial_t}^2 E - \nabla^2 E = 0 \tag{25}$$

$$\mu_0\varepsilon_0{\partial_t}^2 B - \nabla^2 B = 0 \tag{26}$$

Where we decouple first-order differential equations in terms of second-order. These equations satisfy the wave equation for any given function $\psi = \psi(x,t)$, in the form:

$$\frac{1}{v^2}{\partial_t}^2\psi - \nabla^2\psi = 0 \tag{27}$$

Let the velocity of propagation v in each medium, in the case of light and in a vacuum, $v = c$. When we compare these last three equations, the electromagnetic field is interpreted as an electromagnetic wave propagating with speed $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$. Com $\varepsilon_0 = 8{,}85 \times 10^{-12} C^2/N.m^2$ e $\mu_0 = 4\pi \times 10^{-7} N/A^2$, Getting to the speed of light in a vacuum $c = 2{,}99 \times 10^8 m/s.$

For the discussion of this propagation in terms of the potentials Φ e $\vec{A}$ in Proca electrodynamics, let's start using equation (6), with the absence of a source of equation (8), we will have:

$$-\nabla^2\Phi - \partial_t\nabla\vec{A} + \xi\Phi = 0 \tag{28}$$

With $\nabla\vec{A} = -\mu_0\varepsilon_0\partial_t\Phi$ e $\mu_0\varepsilon_0 = c^{-2}$, we have for the presentation of only Φ, in the form:

$$\frac{1}{c^2}{\partial_t}^2\Phi - \nabla^2\Phi + \xi\Phi = 0 \tag{29}$$

And when using the d'Lambertian operator $\frac{1}{c^2}{\partial_t}^2 - \nabla^2 = \square$, get:

$$(\square + \xi)\Phi = 0 \tag{30}$$

And analogously for potential $\vec{A}$, Substituting the relations (6) and (7) into equation (11) without a source, we have:

$$(\square + \xi)\vec{A} = 0 \tag{31}$$

It is observed that the potentials Φ e $\vec{A}$ satisfy the one-dimensional wave equation, similar to the Klein-Gordon equation for the photon [5].

3. Results And Discussion

The experiments carried out to determine the mass of the photon provided upper limits. The inability to determine the mass of the photon does not mean that it has zero mass, but it has a mass that is less than the precision limit of the experiment. In fact, this discussion is sensitive, because the finite mass photon is not described in Mawell's electrodynamics, hence the belief that it is zero .

But what is the need to have a massive photon? Certain effects require the existence of the massive photon, for example in cosmology and superconductors. Proca's electrodynamics comes in search of this implementation and response to the various effects.

We have seen that the Proca equations incorporate the mass parameters by changing the terms with the source. In Maxwell's equations, the fields were invariant under the gauge transformations, but in the Proca equations, there is a break in gauge symmetry, and there can no longer be this transformation of the potentials without modifying the equations, so with Proca, we no longer have this gauge invariance.

We also saw that we arrived at the Klein-Gordon equation for the photon in the absence of sources, because the Klein-Gordon field is a relativistic one that describes a massive, neutral scalar boson [2].

What about wave propagation with the terms of potentials? In a vacuum, potentials, and fields can be obtained by analyzing plane waves, e.g. monochromatic waves. In the case of the potential Φ For example, we have a solution for wave equation in the form:

$$\Phi = \Phi_0 e^{i\vec{k}.\vec{x} - i\omega t} \tag{32}$$

Analogous to potential $\vec{A}$ and the fields $\vec{E}$ and $\vec{B}$. Relating equation (29) to equation (32), we have:

$$\left(\frac{1}{c^2}\partial_t{}^2 - \nabla^2 + \xi\right)\Phi_0 e^{i\vec{k}.\vec{x} - i\omega t} = \left(-\frac{\omega^2}{c^2} + \overrightarrow{k^2} + \xi\right)\Phi_0 e^{i\vec{k}.\vec{x} - i\omega t} \tag{33}$$

The term $\Phi_0 e^{i\vec{k}.\vec{x} - i\omega t}$ It's non-zero what it takes $-\frac{\omega^2}{c^2} + \overrightarrow{k^2} + \xi$ to be zero, where we get the dispersion equation:

$$\omega^2 = c^2\overrightarrow{k^2} + c^2\xi \tag{34}$$

If we relate the magnetic field divergence to the magnetic field wave equation, we have $\vec{B}.\vec{k} = 0$, where that field is perpendicular to the propagation of the wave.

By relating equation (15) to the wave equation of the electric field, we will have the form:

$$\nabla\left(\vec{E}_0 e^{i\vec{k}.\vec{x} - i\omega t}\right) + \xi\left(\Phi_0 e^{i\vec{k}.\vec{x} - i\omega t}\right) = 0$$

$$\vec{E}.\vec{k} = -\xi\Phi_0 \tag{35}$$

This shows the non-perpendicularity of the electric field to the direction of propagation of the wave, being different from Maxwell's electrodynamics. The massive photon deflects the direction of the electric field, generating a state of longitudinal polarization.

The presence of three polarization states can be perceived, one longitudinal and two transverse in the discussion of the massive photon. The decrease in the longitudinal polarization state will occur when we tend the mass of the photon to zero, recovering Maxwell's equations [5].

To work with the parameter ξ, we can work with equation (33) multiplying with the square of Planck's constant in each term, we have the equation in the form:

$$\hbar^2\omega^2 = \hbar^2 c^2\overrightarrow{k^2} + \hbar^2 c^2\xi \tag{36}$$

Where we get the De Broglie correspondence relation for wave-particle and being $\hbar\vec{k} = \vec{p}$ and $\hbar\omega = E$, So equation (35), is in the form:

$$E = \overrightarrow{p^2}c^2 + \hbar^2 c^2\xi \tag{37}$$

Getting $\hbar^2 c^2\xi = m_\gamma^2 c^4$. As the Compton wavelength of the photon is $\lambda_c = \frac{\hbar}{mc}$, where the parameter is of the form:

$$\xi = (\lambda_c)^{-2} \tag{38}$$

Or

$$\xi = \left(\frac{m_\lambda c}{\hbar}\right)^2 \tag{39}$$

Where it shows that the parameter ξ relates to the Compton wavelength of the massive photon, showing it to be a mass parameter, being interpreted as the rest mass of the photon [2].

By analyzing the Proca equation (11) and substituting with the vector potential $\vec{B} = \nabla \times \vec{A}$, We will have the following form:

$$\nabla \times \nabla \times \vec{A} + \xi\vec{A} = \mu_0\varepsilon_0\partial_t\vec{E} + \mu_0\vec{J} \tag{40}$$

Getting:

$$\nabla^2\vec{A} = \xi\vec{A} \tag{41}$$

Shows that the parameter ξ carries the mass information of the radiation from the equation (39) [5].

The experiments that are carried out, set upper limits for the mass of the photon, and currently the most restrictive is $m_\lambda \leq 1{,}78 \times 10^{-54} kg$ [8].

In an analysis of the descriptions used here, we observe that the continuity equation for electric charge does not arise from the automatic consequence of the equations of motion, but from the imposition of the subsidiary condition on the potentials Φ and $\vec{A}$ [9].

4. Conclusion

The object of study of this work was the discussion of the massive photon in the Proca electrodynamics approach. We do not try to go into detail about applications such as superconductivity and cosmological effects, but the idea is to reflect on the extent to which physics needs a term with mass, in this case for the photon, for the study of certain effects.

For example, if the presence of a massive photon would modify the Casimir effect and in potential barriers, the study of optics, because we have a component of the electric field in the direction of propagation of electromagnetic radiation [5].

Various experiments on the superconductor [10] were made for the massive photon, with a length of 39 to 64 *nm*, Getting to 10^{-35} *kg*. Other researchers for a penetration length of 129 nm, found a mass of $1{,}7.10^{-34}$ *kg*. [11 − 15].

Proca electrodynamics is a versatile theory, as it can be applied in several situations, because the charge, energy, and momentum are conserved, and the Proca field is more energetic than Maxwell's.

Through this theory, we can study quantum effects still in development, such as a possible unification, of matter and dark energy.

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نهج Proca الديناميكي الكهربائي للفوتون الضخم

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الخلاصة

يقدم العمل تسليط الضوء على أهمية وجود فوتون ضخم. تجري المناقشة بناءً على تطور الديناميكا الكهربائية لماكسويل، حيث يقدم مصطلح الكتلة للفوتون من خلال الديناميكا الكهربائية لبروكا. اعتمد العرض على التطوير النظري مع التعرض لاشتقاقات المجال الكهرومغناطيسي المتباعد والدوراني، مما يفرض الشرط الفرعي مع شروط الإمكانات المتجهة. كان المقصود من العمل أن يكون بسيطًا، دون الخوض في التفاصيل المتعلقة بالتطبيقات وحتى مصطلحات لاغرانج، وقياس التحولات، والحفاظ على الطاقة والزخم. الهدف من هذا العمل هو مناقشة إمكانية وجود الفوتون الهائل من خلال مصطلحات الاشتقاق باستخدام المجالات الكهرومغناطيسية المتباعدة والدورانية من خلال توسيع نموذج بروكا وعرض علاقة التشتت ودمجها مع طول كومبتون وربطها بثابت بلانك لفهم معلمة ضخمة إضافية للنظرية. وقمنا بإجراء مقارنة بين بعض الباحثين فيما يتعلق بالبيانات التجريبية، حيث استخدموا حداً أعلى لكتلة الفوتون. ندمج مع التفكير في دراسة الفوتون الهائل وتأثيراته.