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GENERELIZED CONHARMONIC CURVATURE TENSOR OF NEARLY KAHLER MANIFOLD

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Abstract

In this paper we study the relationship between tensor algebraic curvature tensor, and General conharmonic curvature tensor of Nearly Kahler manifold, i. e. it has a classical symmetry properties of the Riemann carvatur tensor. Relpenishing generalized Riemannian structur of certain classes of almost Hermitian manifold allows an additional symmetry properties of this tensor.

1- Introduction

Conformal transformations of Riemannien structures are the very significant object of study differential geometry, where this transformation which keeping the property of smooth harmonic function. It is known, that such transformations have tensor in variant so-called generalized conharmonic curvature tensor, The first practicts were in 1960 by Koto [1] when he constructed a manifold that carried the properties of kahler manifold. In 1975, Cray [2] found an accurate method for forming new examples on practical kinds of manifold, which were based on the remarks of Koto.

In 1975 a great change was made on these studies by the Russian researcher Kirichenko, when he studied the almost Hermitian manifold by adjoined G-structure space in particular, he defined two tensors which were the structure and virtual tensors [3]. These tensors helped him to find the structure group of almost Hermitian manifold. Later on, the studies started to represent this method which allowed to study the geometrical structure characteristics for each class of almost Hermitian manifold. Thes study encouraged the researchers who were working with Kirichenko. In 1993, Banaru [4] succed in re-classifying the sixteen classes of almost Hermitian manifold by using the structure and Virtual tensors, which were named Kirichenko's tensors [5]. Among the sixteen classes of almost Hermitian manifold there are eight which are invariant under the conformal transformation metric.

In this paper we investigated the generalized conharmonic curvature tensor of nearly kahler manifold.

2- Preliminaries

let M be a smooth manifold of dimension 2n, $C^{\infty}(M)$ is algebra of smooth function on M; X(M) is the module of smooth vector fields on manifold of M; g= <. , > is Riemanian metrics, ∇ is Riemannian connection of the metrics g on M; d is the operator of exterior differentiation. In the further all manifold, Tensor field, etc. objects are assumed smooth a class $C^{\infty}(M)$.

Definition 1.[6]: Almost Hermition structure on a manifold (AH) M is the pair (J, g), where J is an almost complex structure (J^2=-id) on M, g=<.,.> is a(pseudo)Riemannian metric on M. In this case $\langle JX,JY\rangle=\langle X,Y\rangle$, $\forall$ X,Y$\in$X(M).

Lemma 1.[7]: Every almost complex manifold there exist an almost Hermition (AH)-structure. The endomorphism J is called a structural endomorphism, Manifold which is fixed on it an almost Hermition structure is called an almost Hermition manifold (AH) -manifold and denoted by {M, J, g=<.,.>} or simply M.

Definition2.[2]: Almost Hermition structure (J, g) on manifold M is called;

1 -A Kahler (K) –structure, if
$\nabla_x(J)Y=0$; X,Y $\in$X(M).

2 -A nearly Kahler (NK)-structure, if M satisfies
$\nabla_x(J)Y+\nabla_Y(J)(X)=0$;X,Y$\in$X(M).

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Lemma 2.[8]: An almost hermition manifold successful :

1) $J^{\circ}\nabla_x(J)Y + \nabla_x(J)JY=0$
2) $(\nabla_x(J)Y,Z)+(Y,\nabla_x(J)Z)=0$

Theorem 3[8]: An Almost Hermition structure (J, g) on amanifold M is nearly Kahler if and only if the following identities are fair:

1) B(X, Y) = 0
2) C(X, Y) + C(Y, X)= 0,

this structure is called Kahler iff B=C=0.

Definition 3.[8] : The set of tensors of type (r,1) are linear or antilinear in its argument and the sum components in the tensor T is called the spectrum of the tensor T, and the tensors themselves are called elements of the spectrum.

Definition: 4: Suppose (M, J, g) - Nk manifold. We retard alert, that curvature tensor general conhatmonic was introduced by Ishii (1957) [9] as a tensor of type (4, 0) on n - dimensional Riemannian manifold, definite by the formulation:

$K(HR)_{ijkl} = (HR)_{ijkl} - \frac{1}{2(n-1)} \{g_{ik}\, r(HR)_{jl} + g_{jl}\, r(HR)_{ik} - g_{il}\, r(HR)_{jk} - g_{jk}\, r(HR)_{il}\}$........ (1)

Where (HR) is the general Riemann curvature tensor, r(HR) is general Ricci tensor. This tensor is invariant under general conharmonic transformation, i.e. with conformal transformations of space keeping a harmony of functions.

Consider properties tensor general conharmonic curvature *K(HR)*:

1- K(HR) (X, Y,Z,W) = (HR) (X,Y,Z,W) - 1/2(n-1) { g(x,z) r(HR) (Y,W) + g(Y,W) r(HR) (X,Z) –g(X,W) r(HR) (Y,Z) - g(Y,Z) r(HR) (X,W) }
= -(HR)(Y,X,Z,W) + 1/2(n-1) {g(X,Z) r(HR) (Y,W) + g(Y,W) r(HR)(X,Z) – g(X,W) r(HR)(Y,Z) – g(Y,Z) r(HR) (X,W) } = -K(HR) (Y,X,Z,W)

The next ownerships are similarity proved

2- K(HR) (X,Y,Z,W) = - K(HR) (X,Y,W,Z)
3- K(HR) (X,Y,Z,W) = K(HR) (Z,W,X,Y)
4- K(HR) (X,Y,Z,W) + K(HR)(X,Z,W,Y) + K(HR) (X,W,Y,Z) = 0

So general conharmonic curvature tensor *K(HR)* satisfies all the properties of algebraic curvature tensor :

1-K(HR)(X,Y,Z,W) = - K(HR)(Y,X,Z,W)
2- K(HR) (X,Y,Z,W) = - K(HR) (X,Y,W,Z)
3- K(HR)(X,Y,Z,W) = K(HR)(Z,W,X,Y)
4- K(HR)(X,Y,Z,W) + K(HR) (X,Z,W,Y) + K(HR) (X,W,Y,Z) = 0, X,Y,Z,W ∈ X(M) … .(2)

Tensor generalized conharmonic curvature *K(HR)* looks like :

K(HR)(X,Y)Z =(HR)(X,Y) Z - 1/2(n-1) {r(HR) (Y,Z)X + r(HR)(X,Z)Y – g(Y, Z)QX – g(X, Z)QY } …. (3)

Where g(QX,Y) = r(HR)(X,Y). By definition of aspectrum tensor, $K_3(HR)(X,Y)Z + K_4(HR)(X,Y)Z + K_5(HR)(X,Y)Z + K_6(HR)(X,Y)Z + K_7(HR)(X,Y)Z$; X,Y,Z ∈X(M)

It agrees Lemma 2: tensor $K_0(HR)(X,Y)Z$ as nonzero the component can have only components of the form $\{K_0(HR)^a{}_{bcd},\ K_0(HR)^{\hat{a}}_{\hat{b}\hat{c}\hat{d}}\} = \{K(HR)^a_{bcd}, K(HR)^{\hat{a}}_{\hat{b}\hat{c}\hat{d}}\}$ components of the from $\{K_1(HR)^a_{bc\hat{d}} K_1{}^{\hat{a}}_{\hat{b}\hat{c}d}\} = \{K(HR)^a_{bc\hat{d}}, K(HR)^{\hat{a}}_{\hat{b}\hat{c}d}\}$;tensor K_2 (HR)(X,Y)Z- ingredients of the formulation $\{K_2(HR)^a_{b\hat{c}d}, K_2(HR)^{\hat{a}}_{\hat{b}c\hat{d}}\} = \{K(HR)^a{}_{b\hat{c}d}, K(HR)^{\hat{a}}_{\hat{b}c\hat{d}}\}$; tensor $K_3(HR)(X,Y)Z$– ingredients of the form $\{K_3(HR)^a_{\hat{b}\hat{c}d}, K_3(HR)^{\hat{a}}_{\hat{b}c\hat{d}}\} = \{K(HR)^a{}_{\hat{b}\hat{c}d}, K(HR)^{\hat{a}}_{\hat{b}c\hat{d}}\}$; tensor $K_4(HR)$ (X,Y)Z – ingredients of the from $\{K_4(HR)^a_{\hat{b}cd}, K_4{}^{\hat{a}}_{b\hat{c}\hat{d}}\} = \{K(HR)^a_{\hat{b}cd}, K(HR)^{\hat{a}}_{b\hat{c}\hat{d}}\}$; tensor $K_5(HR)(X,Y)Z$–ingredients of the form$\{K_5(HR)^a_{\hat{b}c\hat{d}} K_5(HR)^{\hat{a}}_{b\hat{c}d}\} = \{K(HR)^a_{\hat{b}c\hat{d}}, K(HR)^{\hat{a}}_{b\hat{c}d}\}$; tensor $K_6(HR)(X,Y)Z$ – ingredients of the form $\{K_6(HR)^a_{\hat{b}\hat{c}d}, K_6(HR)^{\hat{a}}_{bc\hat{d}}\} = \{K(HR)^a_{\hat{b}\hat{c}d}, K(HR)^{\hat{a}}_{bc\hat{c}}\}$; tensor $K_7(HR)(X,Y)Z$ – ingredients of form $\{K_7(HR)^a_{\hat{b}\hat{c}\hat{d}}, K_7(HR)^{\hat{a}}_{bcd}\} = \{K(HR)^a_{\hat{b}\hat{c}\hat{d}}, K(HR)^{\hat{a}}_{bcd}\}$

Tensors $K_0(HR)=K_0(HR)(X,Y)Z, K_1(HR)=K_1(HR)(X,Y)Z,..,K_7(HR)=K_7(HR)(X,Y)Z$ we shall name the basic invariants conharmonic AH-manifold.

Now we introduces anew definition:-

Definition 5: AH-manifold for each K_i(HR)=0, We shall name AH – manifold of class K_i (HR), i=0, 1,...,7.

Theorem2:

1) AH-manifold of class K_0((HR) are characterized by identity
K(HR)(X,Y)Z-K(HR)(X,JY)JZ-K(HR)(JX,Y)JZ-K(HR)(JX,JY)Z-JK(HR)(X,Y)JZ-JK(HR)(JX,Y)Z-JK(HR)(JX,Y)Z+JK(HR)(JX,JY)JZ=0;X,Y,Z∈ X(M).

2) AH-manifold of class K_1(HR) are characterized by identity
K(HR)(X,Y)Z+K(HR)X,JY)JZ-K(HR)(JX,Y)JZ+K(HR)(JX,JY)Z+JK(HR)(X,Y)JZ-JK(HR)(X,JY)Z-JK(HR)(JX,Y)Z-JK(HR)(JX,JY)JZ ; X,Y,Z∈ X(M).

3)AH-manifold of class K_2(HR) are characterized by identity
K(HR)(X,Y)Z-K(HR)(X,JY)JZ+K(HR)(JX,Y)JZ+K(HR)(JX,JY)Z-JK(HR)(X,Y)JZ-JK(HR)(X,JY)Z+JK(HR)(JX,Y)Z-K(HR)(JX,JY)JZ=0; X,Y,Z∈X(M).

4)AH-manifold of class K_3(HR) are characterized by identity
K(HR)(X,Y)Z+K(HR)(X,JY)JZ+K(HR)(JX,Y)JZ-K(HR)(JX,JY)Z-JK(HR)(X,Y)JZ+JK(HR)(X,JY)Z+JK(HR)(JX,Y)Z+JK(HR)(JX,JY)JZ; X,Y,ZX(M)

5)AH-manifold of class K_4(HR) are characterized by identity
K(HR)(X,Y)Z+K(HR)(X,JY)JZ+K(HR)(JX,Y)JZ-K(HR)(JX,JY)Z+JK(HR)(X,Y)JZ-JK(HR)(X,JY)Z-JK(HR)(JX,Y)Z-JK(JX,JY)JZ=0 ; X,Y,Z∈ X(M).

6)AH-manifold of class K_5(HR) are characterized by identity
K(HR)(X,Y)Z-K(HR)(X,JY)JZ+K(HR)(JX,Y)JZ+K(HR)(JX,JY)Z+J

K(HR)(X,Y)JZ+JK(HR)(X,JY)Z-JK(HR)(JX,Y)Z+JK(HR)(JX,JY)JZ=0;X,Y,Z∈X(M).

7) AH-manifold of class $k_6(HR)$ are characterized by identity K(HR)(X,Y)Z+K(HR)(X,JY)JZ-K(HR)(JX,Y)JZ+K(HR)(JZ,JY)Z+JK(HR)(X,Y)JZ-JK(HR)(X,JY)Z+JK(HR)(JX,Y)Z+JK(HR)(JX,JY)JZ =0; X,Y,Z∈ X(M).

8)AH-manifold of class *K7 (HR)* are characterized by identity K(HR)(X,Y)Z-K(HR)(X,JY)JZ-K(HR)(JX,Y)JZ-K(HR)(JX,JY)Z+JK(HR)(X,Y)JZ +JK(HR)(X,JY)Z+JK(HR)(JX,Y)Z-JK(HR)(JX,JY)JZ=0; X,Y,Z∈ X(M).

Proof:

The manifold of class K_0 *(HR)* is characterized by acondition $K_0(HR)^a{}_{bcd}=0$, or $K^a{}_{bcd}=0$, i.e. $[k(HR)(\varepsilon_c,\varepsilon_d)\varepsilon_b]^a\varepsilon_a=0$. As σ – aprojector on $D_J^{\sqrt{-1}}$,that $\sigma\circ\{K(HR)(\sigma X,\sigma Y)\sigma Z\}=0$, i.e. $(id-\sqrt{-1}J)\{K(HR)(X-\sqrt{-1}JX,Y-\sqrt{-1}JY)(Z-\sqrt{-1}JZ)\}=0$. Rrmoving the brackets we shall receive:
K(HR)(X,Y)Z-K(HR)(X,JY)JZ-K(HR)(JX,Y)JZ-K(HR)(JX,JY)Z-JK(HR)(X,Y)JZ-JK(HR)(X,JY)Z-JK(HR)(JX,Y)Z+JK(JX,JY)JZ-
$\sqrt{-1}\{K(HR)(X,Y)JZ+K(HR)(X,JY)Z+K(HR)(JX,Y)Z-K(HR)(JX,JY)JZ\}-\{JK(HR)(X,Y)Z-JK(HR)(X,JY)JZ-JK(HR)(JX,Y)JZ-JK(HR)(JX,JY)Z\}=0$,
i.e.

1)K(HR)(X,Y)Z-K(HR)(X,JY)JZ-K(HR)(JX,Y)JZ-K(HR)(JX,JY)Z-JK(HR)(X,Y)JZ-JK(HR)(X,JY)Z-JK(HR)(JX,Y)Z+JK(HR)(JX,JY)JZ=0

Thus AH-manifold of class $K_0(HR)$ are characterized by identity

2)K(HR)(X,Y)JZ+K(HR)(X,JY)Z+K(HR)(JX,Y)Z-K(HR)(JX,JY)JZ+JK(HR)(X,Y)Z-JK(X,JY)JZ-JK(HR)(JX,Y)JZ-JK(HR)(JX,JY)Z=0, X,Y,Z∈ X(M).

These equality are equivalent, the second equality turns out from the first replacement Z on JZ.
K(HR)(X,Y)Z-K(HR)(X,JY)JZ-K(HR)(JX,Y)JZ-K(HR)(JX,JY)Z-JK(HR)(X,Y)JZ-JK(HR)(X,JY)Z-JK(HR)(JX,Y)Z+JK(HR)(JX,JY)JZ=0, $X,Y,Z\in X(M)$. Similarly, considering AH-manifold of class $K_1(HR)$ - $K_7(HR)$,we shall receive the following theorem.

Theorem3.[3]: Components of Riemannian curvature tensor of NK-manifold in the adjoint G-structurespace are award as follows :

$$R^a_{bcd}=R^a_{\hat b\hat c\hat d}=R^{\hat a}_{\hat bcd}=R^{\hat a}_{\hat b\hat c\hat d}=R^{\hat a}_{bcd}=R^{\hat a}_{bc\hat d}=R^{\hat a}_{b\hat cd}=R^a_{\hat b\hat c\hat d}=R^a_{\hat bc\hat d}=R^a_{\hat b\hat c\hat d}=0$$

1) $R^a_{bc\hat d}=B^{adh}B_{hbc}+A^{ad}_{bc}$

2) $R^a_{b\hat cd}=B^{ahc}B_{bdh}-A^{ac}_{bd}$

3) $R^{\hat a}_{\hat bc\hat d}=B^{bdh}B_{ahc}-A^{bd}_{ac}$

4) $R^{\hat a}_{\hat b\hat cd}=B^{bch}B_{adh}+A^{bc}_{ad}$

5) $R^{\hat a}_{b\hat c\hat d}=2B^{dch}B_{abh}$

6) $R^a_{\hat bcd}=2B^{abh}B_{dch}$

Definition 6.[10]: Ageneralized Riemannian curvature tensor on AH-manifold M is a tensor of type (4,0) which is defined as following formulation:

(HR)(X,Y,Z,W)=$\frac{1}{16}$ {3[R(X,Y,Z,W)+R(JX,JY,Z,W)+R(X,Y,JZ.JW)+R(JX,JY,JZ,JW]-R(X,Z,JW,JY)-R(JX,JZ,W,Y)-R(X,W,JY,JZ)-R(JX,JW,Y,Z)+R(JX,Z,JW,Y) +R(X,JZ,W,JY)+R(JX,W,Y,JZ)+R(X,JW,JY,Z)}

Where *R(X,Y,Z,W)* is Riemannian curvature tensor, $X,Y,Z,W\in T_p(M)$ and satisfies the following properties:

1) (HR)(X,Y,Z,W)=-(HR)(Y,X,Z,W)=-(HR)(X,Y,W,Z);

2) (HR)(X,Y,Z,W)=(HR)(Z,W,X,Y);

3) (HR)(X,Y,Z,W)+(HR)(X,Z,W,Y)+(HR)(X,W,Y,Z)=0;

4) (HR)(X,JX,JX,X)=R(X,JX,JX,X)

The components of the generalizied Riemannian curvature tensor (HR) of NK-manifold are given by the next theorem.

Theorem 4.[6]: The components of the generalized Riemannian curvature by the next formulation :

1) $(HR)_{\hat ab\hat cd}=-A^{ac}_{bd}$

2) $(HR)_{\hat abc\hat d}=A^{ad}_{bc}$

And the others are conjugate to the above components or equal to zero.

Definition 7.[11]: A tensor of type (2,0) which is defined as $r(HR)_{ij}=(HR)^k_{ijk}$ is called ageneralized Ricci tensor.

Theorem 5.[12]: The components of generalized Ricci tensor of NK-manifold in the adjoint G-structure space are given as the following form:

$R(HR)_{\hat ab}=-A^{ac}_{bc}$

And the others are conjugate to the above component or equal to zero.

Definition 8.[12]: A generalized scalar curvature tensor is denoted by *K(HR)* and defined as :

$K(HR)=g_{ij}\,r(HR)_{ij}$

Theorem 6.[12]: The component of generalized scalar curvature tensor of NK-manifold in the adjoint G-structure space is given as the following form :

$K(HR)=\ -2A^{ac}_{ac}$

Theorem 7: The components of the generalized conharmonic curvature tensor of NK-manifolds in the adjointG-structure are given as the following form:

$K(HR)_{ijkl}=R(HR)_{ijkl}-\frac{1}{2(n-1)}\{g_{ik}\,r(HR)_{jl}+g_{jl}\,r(HR)_{ik}-g_{il}\,r(HR)_{jk}-g_{jk}\,r(HR)_{il}\}$

1- $K(HR)_{\hat ab\hat cd}=-A^{ac}_{bd}-1/2(n-1)\{\delta^a_d\,r(HR)_{\hat cb}+\delta^a_b\,r(HR)_{\hat ad}\}$

2- $K(HR)_{\hat abc\hat d}=A^{ad}_{bc}+1/2(n-1)\{\delta^a_c\,r(HR)_{b\hat d}+\delta^d_b\,r(HR)_{\hat ac}\}$

3- $K(HR)_{\hat a\hat bcd}=1/2(n-1)\{\delta^a_c\,r(HR)_{\hat bc}+\delta^b_d\,r(HR)_{\hat ac}-\delta^a_d\,r(HR)_{\hat bc}-\delta^b_c\,r(HR)_{\hat ad}\}$

And other components Generalized conharmonic curvature tensor for NK are equal to zero.

Proof:

1- Let i=a, j= b, k=c and l=d

$K(HR)_{abcd}=(HR)_{abcd}-\frac{1}{2(n-1)}\{g_{ac}\,r(HR)_{bd}+g_{bd}\,r(HR)_{ac}-g_{ad}\,r(HR)_{bc}-g_{bc}\,r(HR)_{ad}\}$.

$k_{abdc}=0-\frac{1}{2(n-1)}\{(0)(0)+(0)(0)-(0)(0)-(0)(0)\}=0$

$k(HR)_{abdc}=0$

2-Let i=â, j= b, k =c and l=d

$K(HR)_{\hat{a}bcd} = (HR)_{\hat{a}bcd} - \frac{1}{2(n-1)}\{ g_{\hat{a}c}\, r(HR)_{bd} + g_{bd}\, r(HR)_{\hat{a}c} - g_{\hat{a}d}\, r(HR)_{bc} - g_{bc}\, r(HR)_{\hat{a}d}\}$

$k(HR)_{\hat{a}bdc} = 0 - \frac{1}{2(n-1)}\{ \delta_c^a(0) + (0)(-A_{ck}^{ak}) - \delta_d^a(0) - (0) - A_{dk}^{ak} - \frac{1}{2(n-1)}\{0\}$

$k(HR)_{\hat{a}bdc} = 0$

3-Let i=a, j =$\hat{b}$, k =c and l=d

$K(HR)_{a\hat{b}cd} = (HR)_{a\hat{b}cd} - \frac{1}{2(n-1)}\{ g_{\hat{a}c}\, r(HR)_{\hat{b}d} + g_{\hat{b}d}\, r(HR)_{\hat{a}c} - g_{\hat{a}d} r(HR)_{\hat{b}c} - g_{\hat{b}c} r(HR)_{\hat{a}d}\}$

$k(HR)_{a\hat{b}cd} = 0 - \frac{1}{2(n-1)}\{\delta_c^a(0) + (0)(-A_{ck}^{ak}) - \delta_d^a(0) - (0)(-A_{dk}^{ak})\}$

$= -\frac{1}{2(n-1)}\{0\}$

$K(HR)_{a\hat{b}cd} = 0$

4-Let i=a, j= b, k=ĉ and l=d

$K(HR)_{ab\hat{c}d} = (HR)_{ab\hat{c}d} - \frac{1}{2(n-1)}\{ g_{a\hat{c}}\, r(HR)_{bd} + g_{bd}\, r(HR)_{a\hat{c}} - g_{ad}\, r(HR)_{b\hat{c}} - g_{b\hat{c}}\, r(HR)_{ad}\}$

$k(HR)_{ab\hat{c}d} = 0 - \frac{1}{2(n-1)}\{\delta_a^c(0) + (0)(-A_{ak}^{ck}) - (0)(-A_{bk}^{ck}) - \delta_b^c(0)\}$

$= -\frac{1}{2(n-1)}\{0\}$

$K(HR)_{ab\hat{c}d} = 0$

5-Let i=a, j= b, k=c and l=$\hat{d}$

$K(HR)_{abc\hat{d}} = (HR)_{abc\hat{d}} - \frac{1}{2(n-1)}\{ g_{ac}\, r(HR)_{b\hat{d}} + g_{b\hat{d}}\, r(HR)_{ac} - g_{a\hat{d}}\, r(HR)_{bc} - g_{bc} r(HR)_{a\hat{d}}\}$

$k(HR)_{abc\hat{d}} = 0 - \frac{1}{2(n-1)}\{(0)(-A_{bk}^{dk}) + \delta_b^d(0) - \delta_a^d(0) - (0)(-A_a^d)\}$

$= -\frac{1}{2(n-1)}\{0\}$

$K(HR)_{abc\hat{d}} = 0$

6-Let i=â, j=$\hat{b}$, k=c and l=d

$K(HR)_{\hat{a}\hat{b}cd} = (HR)_{\hat{a}\hat{b}cd} - \frac{1}{2(n-1)}\{ (HR)_{\hat{a}c} r(HR)_{\hat{b}d} + g_{\hat{b}c}\, r(HR)_{\hat{a}c} - g_{\hat{a}d}\, r(HR)_{\hat{b}c} - g_{\hat{b}c}\, r(HR)_{\hat{a}d}\} = 0 - \frac{1}{2(n-1)}\{ \delta_c^a(-A_{dk}^{bk}) + \delta_d^b(-A_{ck}^{ak}) - \delta_d^a(-A_{ck}^{bk}) - \delta_c^b(-A_{dk}^{ak})\}$

$K(HR)_{\hat{a}\hat{b}cd} = \frac{1}{2(n-1)}\{\delta_c^a A_{dk}^{bk} + \delta_d^b A_{ck}^{ak} - \delta_d^a A_{ck}^{bk} - \delta_c^b A_{dk}^{ak}\}$

7-Let i=â, j=b, k=ĉ and l=d

$K(HR)_{\hat{a}b\hat{c}d} = (HR)_{\hat{a}b\hat{c}d} - \frac{1}{2(n-1)}\{g_{\hat{a}\hat{c}}\, r(HR)_{bd} + g_{bd}\, r(HR)_{\hat{a}\hat{c}} - g_{\hat{a}d} r(HR)_{b\hat{c}}\} =$

$-A_{bd}^{ac} - \frac{1}{2(n-1)}\{0 + 0 - \delta_d^a(-A_{bk}^{ck}) - \delta_b^c(-A_{dk}^{ak})\}$

$K(HR)_{\hat{a}b\hat{c}d} = -A_{bd}^{ac} - \frac{1}{2(n-1)}\{\delta_d^a A_{bk}^{ck} + \delta_b^c A_{dk}^{ak}\}$

8-Let i=â, j=b, k=c and l=$\hat{d}$

$K(HR)_{\hat{a}bc\hat{d}} = (HR)_{\hat{a}bc\hat{d}} - \frac{1}{2(n-1)}\{ g_{\hat{a}c}\, r(HR)_{b\hat{d}} + g_{\hat{a}c}\, r(HR)_{\hat{a}c} - g_{\hat{a}\hat{d}}\, r(HR)_{bc} - g_{bc} r(HR)_{\hat{a}\hat{d}}\} = A_{bc}^{ad} - \frac{1}{2(n-1)}\{\delta_c^a(-A_{bk}^{dk}) + \delta_b^d(-A_{ck}^{ak}) - 0 - 0\}$

$K(HR)_{\hat{a}bc\hat{d}} = A_{bc}^{ad} + \frac{1}{2(n-1)}\{\delta_c^a A_{bk}^{dk} + \delta_b^d A_{ck}^{ak}\}$ ■

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تنزر الانحناء الكونهورمني المعمم لمنطوي كوهلر التقريبي

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الملخص

في هذه البحث تم دراسة العلاقة بين تنزر الانحناء الجبري، وتنزر الانحناء الكونهورمني المعمم لمنطوي كوهلر التقريبي، حيث ان هذا التنزر يمتلك خصائص التناظر الكلاسيكي لتنزر الانحناء الريماني، مع احتساب مركبات تنزر الانحناء الكونهورمني المعمم في بعض أصناف المنطوي الهرميتي التقريبي إضافة الى دراسة خصائص التناظر لهذا التنزر.