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Finding the Exact Solution of the Dynamic Model for the Spread of Computer Viruses Using the Homogeneous Balance Method

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Abstract

In this paper, we solved the SEIR system (a system that describes the spread of viruses in computers) using the homogeneous balance method, which is one of the methods concerned with finding the accurate solution for equations of various types, linear, non-linear, partial, and ordinary. SEIR is a system of nonlinear partial equations. Studying this system helps reduce the damage caused by viruses to computers and the problems they cause to devices by predicting the presence of viruses before they spread to computers. The system was solved and the exact solution of the SEIR system was found. The effect of the parameters $d1, d2\ and\ d3$ was studied, and it was found that the smaller these parameters were (close to zero), the slower the viruses leaked into computers. The impact of parameters and how much performance they can cause to devices is explained using 3D graphics. It has become clear that these viruses pose a great danger to devices, so studying them is a topic that deserves attention.

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1. Introduction:

Recently, interest has increased in research on exact solutions to nonlinear differential equations, whether partial or ordinary, and researchers have been interested in studying natural phenomena such as physics, chemistry, etc., and researchers have resorted to advanced methods to obtain accurate solutions[1], the inverse scattering method [2], and the function method Exponential [3]. Exact Solution of Partial Differential Equation Using Homo-Separation of Variables[4]. Exact solution of nonlinear partial differential equations describing PseudoSpherical Su [5]. Exact solutions of differential. equations using Tanh/Sech [6]. The true solution to types of partial and complex differential equations [7], True solutions of nonlinear differential equations with $\exp(-\varphi(\zeta))$-expansion method [8], The Adomian decomposition method to find the exact solution of some types of nonlinear differential equations, [9], The first integrative method to find the exact solution [10], Exact solutions of the mKdV equation[11], etc. This paragraph is an introduction to exact solutions methods and some of the methods used to solve various equations

Viruses cause significant damage to computers. Viruses are programs that are designed by some professional programmers to cause harm to other people's devices, for illegal purposes, such as disabling devices, stealing information, or other things. It can be said that computer viruses are similar to viruses that spread among humans in terms of the method of spread and not in terms of harm. The first appearance of computer viruses began in the early 1980s, and they had only a minor impact, so they did not receive much attention and many considered them a myth claimed by some of those interested in this field. Rather, the spread of viruses between devices expanded, and then became a threat to the information revolution. Viruses that infect computers include operating system viruses, parasitic viruses, general viruses, macro viruses, worms, Trojan horses, and others. The goal of any virus is to take control of control data on the computer. Among the worst viruses are viruses that infect programs. Microsoft Office because it manipulates user data and also affects the speed of the computer, and sometimes Some programs do not respond to user commands sometimes, As time goes by, hackers and virus developers develop programs containing viruses in more professional ways, to steal user information and data. At that time, researchers interested in the field were reminded of the necessity of studying the impact of viruses on computers. Mathematical models describing the spread of viruses have been studied. Researchers were interested in developing some mathematical formulas for checking and transmitting viruses over the Internet. The main source of viruses is mail and secondary storage devices. They were interested in improving the models. The spread of viruses in addition to other factors can be affected, as in the work mentioned in the following research[12],[13],[14], There are many types of viruses, including SEIR.[15]

The system was studied by many researchers in different ways, and each researcher studied the effect of a number of parameters. Because the system contains a large number of parameters, it is difficult to study the effect of all the parameters at the same time. Therefore, we chose to study the effect of the parameters d1, d2, and d3 to get acquainted with the previous studies. Take a look. On the following papers[16],[17],[18],[19],[20]

In this paper we will discuss. methods for solving the exact SEIR. virus system in detail. In the second section, the research includes the homogeneous equilibrium method, in the third section the mathematical system, in the fourth section the application. of the method, and in the final. section the references.

2. The Homogeneous Balance Method[21],[22],[23]

Assuming that the nonlinear partial differential equation has the following form:

$$\mu\left(u, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial t^2}, \dots\right) = 0 \tag{1}$$

Where u is the dependent variable in the equation to be found

The homogeneous balance method is based on converting the (PDE) into an (ODE)using the hypothesis

$$u(x,t) = u(\xi), \quad \xi = k(x + wt) + \xi_0 \tag{2}$$

Where k and w and ξ_0 are constant, eq (1) is transformed into the following nonlinear (ODE)

$$\psi\left(u, kwu', ku', k^2w^2u, k^2u'', \dots\right) = 0 \tag{3}$$

To solve eq (3) using the homogeneous balance method, Assuming that:

$$u = \sum_{i=0}^{m} q_i \phi^i \tag{4}$$

where q_i are constant and ϕ are satisfies the Riccati equation[24]

$$\phi' = a\phi^2 + b\phi + c \tag{5}$$

Where a,b,c are Constants,

Balancing the highest order derivative term with a nonlinear term in equation (3) gives leading order $m = n$,so we can choose:

$$u = q_0 + q_1\phi + q_2\phi^2 + \cdots \tag{6}$$

Where q_i , $i = 0,1,2,..n$ are constant, From (5) and (6) we get the value of , after that we substitute ϕ and the value of q_i in (4) to get exact solution of (1)

3. Mathematical. Model[15]

The SEIR system is a system that describes the spread of viruses in the computer and is as follows:

$$\frac{\partial S}{\partial t} = d_1 \frac{\partial^2 S}{\partial x^2} + \delta - \beta S(L+B) + \gamma_1 L + \gamma_2 B - \delta S$$

$$\frac{\partial L}{\partial t} = d_2 \frac{\partial^2 L}{\partial x^2} + \beta S(L+B) - \gamma_1 L - \alpha L - \delta L \tag{7}$$

$$\frac{\partial S}{\partial t} = d_3 \frac{\partial^2 B}{\partial x^2} + \alpha L - \gamma_2 B - \delta B$$

Where $\mathrm{B(x,t)}$ denotes computers vulnerable to viruses. While $\mathrm{S(x,t)}$ refers to healthy computers.$\mathrm{L(x,t)}$ denotes computers that are unknown whether they are infected or not,$S(\mathrm{L}+\mathrm{B})$ denotes the relationship between healthy and unhealthy in time period t,$d1, d2, d3, \gamma_1, \gamma_2$ It is the rate of spread of viruses.α It is the rate of computer virus penetration.δ and β are the infection rate between infection-free computers and latently affected computers,

4. **Application**

Now we use the homogeneous balance method to find some new exact solutions for the SEIR system

to solve the system (7) in the first step we will transform it into a system of (NODE) using the hypothesis (2), to become as follows:

$$kwS' = k^2 d_1 S'' + \delta - \beta S(L+B) + \Upsilon_1 L + \Upsilon_2 B - \delta S$$

$$kwL' = k^2 d_2 L'' + \beta S(L+B) - \Upsilon_1 L - \alpha L - \delta L \qquad (8)$$

$$kwB' = k^2 d_3 B'' + \alpha L - \Upsilon_2 B - \delta B$$

Now let the Riccati equation is as:

$$\phi' = a\phi^2 + b\phi + c \qquad (9)$$

and

$$S(\xi) = q_0 + q_1\phi(\xi) + q_2\phi(\xi)^2$$
$$L(\xi) = r_0 + r_1\phi(\xi) + r_2\phi(\xi)^2 \qquad (10)$$
$$B(\xi) = c_0 + c_1\phi(\xi) + c_2\phi(\xi)^2$$

Where q_0 , q_1 , q_2 and r_0 , r_1 , r_2 *and* c_0 , c_1 , c_2 are constant

Now by substituting (9) and (10) into equation (8) we get

$$kw(q_0 + q_1\phi(\xi) + q_2\phi(\xi)^2)' = k^2 d_1 (q_0 + q_1\phi(\xi) + q_2\phi(\xi)^2)'' + \delta - \beta(q_0 + q_1\phi(\xi) + q_2\phi(\xi)^2)(r_0 + r_1\phi(\xi) + r_2\phi(\xi)^2 + c_0 + c_1\phi(\xi) + c_2\phi(\xi)^2) + (r_0 + r_1\phi(\xi) + r_2\phi(\xi)^2) + \Upsilon_2(c_0 + c_1\phi(\xi) + c_2\phi(\xi)^2) - \delta(q_0 + q_1\phi(\xi) + q_2\phi(\xi)^2)$$

$$kw(r_0 + r_1\phi(\xi) + r_2\phi(\xi)^2)' = k^2 d_2 (r_0 + r_1\phi(\xi) + r_2\phi(\xi)^2)'' + \beta(q_0 + q_1\phi(\xi) + q_2\phi(\xi)^2)(r_0 + r_1\phi(\xi) + r_2\phi(\xi)^2 + c_0 + c_1\phi(\xi) + c_2\phi(\xi)^2) - \Upsilon_1(r_0 + r_1\phi(\xi) + r_2\phi(\xi)^2) - \alpha(r_0 + r_1\phi(\xi) + r_2\phi(\xi)^2) - \delta(r_0 + r_1\phi(\xi) + r_2\phi(\xi)^2) \qquad (11)$$

$$kw(c_0 + c_1\phi(\xi) + c_2\phi(\xi)^2)' = k^2 d_3 (c_0 + c_1\phi(\xi) + c_2\phi(\xi)^2)'' + \alpha(r_0 + r_1\phi(\xi) + r_2\phi(\xi)^2) - \Upsilon_2(c_0 + c_1\phi(\xi) + c_2\phi(\xi)^2) - \delta(c_0 + c_1\phi(\xi) + c_2\phi(\xi)^2)$$

we derive equation (11) for ξ we get

$$(\beta q_2 r_2 + \beta q_2 c_2 - 6k^2 d1 q_2 a^2)\phi^4 + (\beta q_2 r_1 + \beta q_1 c_2 - 10k^2 d1 q_2 ab + 2kwaq_2 + \beta q_1 r_2 + \beta q_2 c_1 - 2k^2 d1 q_1 a^2)\phi^3 + (\beta q_2 c_0 + kwaq_1 - 3k^2 d1 q_1 ab - 4k^2 d1 q_2 b^2 - \gamma_2 c_2 - \gamma_1 r_2 + 2kbq_2 + \beta q_1 r_1 + \beta q_0 r_2 - 8k^2 d1 q_2 ac + \beta q_1 c_1 + \delta q_2 + \beta q_2 r_0 + \beta q_0 c_2)\phi^2 + (-\gamma_2 c_1 + \beta q_0 r_1 - 2k^2 d1 q_1 ac + \beta q_0 c_1 + \beta q_1 r_0 - 6k^2 d1 q_2 bc + 2kwcq_2 + kwbq_1 + \delta q_1 - k^2 d1 q_2 bc - \gamma_1 r_1 + \beta q_1 c_0)\phi - 2k^2 q_2 c^2 - \delta - \gamma_1 r_0 + \beta q_0 r_0 + kwcq_1 + \delta q_0 - \gamma_1 c_0 + \beta q_0 c_0 - k^2 \mathrm{d}1 q_1 bc = 0$$

$$(-6k^2 d2 r_2 a^2 - \beta q_2 c_2 - \beta q_2 r_2)\phi^4 + (-\beta q_2 c_1 - \beta q_2 r_1 - 2k^2 d2 r_1 a^2 - \beta q_1 r_2 + 2kwar_2 - \beta q_1 c_2 - 10k^2 d2 r_2 ab)\phi^3 + (-4k^2 d2 r_2 - \beta q_2 r_0 - \beta q_2 c_0 - \beta q_0 r_2 - 8k^2 d2 r_2 ac + \alpha r_2 - \beta q_1 c_1 - \beta q_1 r_1 + kwar_1 + \delta r_2 - \beta q_0 c_2 - 3k^2 d2 r_1 ab + \gamma_1 ab + \gamma_1 r_2 + 2kwbr_2)\phi^2 + (\delta r_1 - 2k^2 d2 r_1 ac + \delta r_1 + kwbr_1 - 6k^2 d2 r_2 bc - \beta q_0 r_1 + 2kwcr_2 - \beta q_0 r_0 - \beta q_0 c_1 + \gamma_1 r_1 - k^2 d2 r_1 b^2 - \beta q_1 c_0)\phi + kwcr_1 - 2k^2 d2 r_2 c^2 + \alpha r_0 - \beta q_0 r_0 - k^2 d2 r_1 bc - \beta q_0 c_0 + \gamma_1 r_0 + \delta r_0 = 0$$

$$-6k^2 d3 c_2 a^2 \phi^4 + (-10k^2 d3 c_2 ab + 2kwac_2 - 2k^2 d3 c_1 a^2)\phi^3 + (-4k^2 d3 c_2 b^2 + 2kwbc_2 + \gamma_2 c_2 - 8k^2 d3 c_2 ac + \delta c_2 - \alpha r_2 - 3k^2 d3 c_1 ab + kwac_1)\phi^2 + (2kwcc_2 + \gamma_2 c_1 + kwbc_1 - 2k^2 d3 c_1 ac + \delta c_1 - k^2 d3 c_1 b^2 - 6k^2 d3 c_2 bc - \alpha r_1)\phi + \gamma_2 c_0 + kwcc_1 - 2k^2 d3 c_2 c^2 + \delta c_0 - k^2 d3 c_1 bc - \alpha r_0 = 0$$

Setting the coefficients $\phi^i (i = 0,1,2,..4)$ to zero we get the following equations:

$\beta q_2 r_2 + \beta q_2 c_2 - 6k^2 d1 q_2 a^2 = 0$

$\beta q_2 r_1 + \beta q_1 c_2 - 10k^2 d1 q_2 ab + 2kwaq_2 + \beta q_1 r_2 + \beta q_2 c_1 - 2k^2 d1 q_1 a^2 = 0$

$\beta q_2 c_0 + kwaq_1 - 3k^2 d1 q_1 ab - 4k^2 d1 q_2 b^2 - \gamma_2 c_2 - \gamma_1 r_2 + 2kbq_2 + \beta q_1 r_1 + \beta q_0 r_2 - 8k^2 d1 q_2 ac + \beta q_1 c_1 + \delta q_2 + \beta q_2 r_0 + \beta q_0 c_2 = 0$

$-\gamma_2 c_1 + \beta q_0 r_1 - 2k^2 d1 q_1 ac + \beta q_0 c_1 + \beta q_1 r_0 - 6k^2 d1 q_2 bc + 2kwcq_2 + kwbq_1 + \delta q_1 - k^2 d1 q_2 bc - \gamma_1 r_1 + \beta q_1 c_0 = 0$

$-2k^2 q_2 c^2 - \delta - \gamma_1 r_0 + \beta q_0 r_0 + kwcq_1 + \delta q_0 - \gamma_1 c_0 + \beta q_0 c_0 - k^2 \mathrm{d}1 q_1 bc = 0$

$-6k^2 d2 r_2 a^2 - \beta q_2 c_2 - \beta q_2 r_2 = 0$

$-\beta q_2 c_1 - \beta q_2 r_1 - 2k^2 d2 r_1 a^2 - \beta q_1 r_2 + 2kwar_2 - \beta q_1 c_2 - 10k^2 d2 r_2 ab = 0$

$-4k^2 d2 r_2 - \beta q_2 r_0 - \beta q_2 c_0 - \beta q_0 r_2 - 8k^2 d2 r_2 ac + \alpha r_2 - \beta q_1 c_1 - \beta q_1 r_1 + kwar_1 + \delta r_2 - \beta q_0 c_2 - 3k^2 d2 r_1 ab + \gamma_1 ab + \gamma_1 r_2 + 2kwbr_2 = 0$

$\delta r_1 - 2k^2 d2 r_1 ac + \delta r_1 + kwbr_1 - 6k^2 d2 r_2 bc - \beta q_0 r_1 + 2kwcr_2 - \beta q_0 r_0 - \beta q_0 c_1 + \gamma_1 r_1 - k^2 d2 r_1 b^2 - \beta q_1 c_0 = 0$

$kwcr_1 - 2k^2 d2 r_2 c^2 + \alpha r_0 - \beta q_0 r_0 - k^2 d2 r_1 bc - \beta q_0 c_0 + \gamma_1 r_0 + \delta r_0 = 0$

$-6k^2 d3 c_2 a^2 = 0$

$-10k^2 d3 c_2 ab + 2kwac_2 - 2k^2 d3 c_1 a^2 = 0$

$-4k^2 d3 c_2 b^2 + 2kwbc_2 + \gamma_2 c_2 - 8k^2 d3 c_2 ac + \delta c_2 - \alpha r_2 - 3k^2 d3 c_1 ab + kwac_1 = 0$

$2kwcc_2 + \gamma_2 c_1 + kwbc_1 - 2k^2 d3 c_1 ac + \delta c_1 - k^2 d3 c_1 b^2 - 6k^2 d3 c_2 bc - \alpha r_1 = 0$

$\gamma_2 c_0 + kwcc_1 - 2k^2 d3 c_2 c^2 + \delta c_0 - k^2 d3 c_1 bc - \alpha r_0 = 0$

With the help of the Maple program, by taking part of the above equation (only the first five equations), because if all the equations are taken, it is difficult to find the solution, so only the first five equations were taken, we get several cases, including:

Case: I .we have

$q_0 = q_0 \quad , q_1 = q_1 \quad q_2 = -\frac{1}{2}\frac{\delta+\gamma_1 r_0-\beta q_0 r_0-kwcq_1-\delta q_0+\gamma_2 c_0-\beta q_0 c_0+k^2 d1bc}{k^2 d1 c^2} \quad ,$

$r_0 = r_0 \quad , r_1 = \frac{3\beta d1b\delta^2-\beta w\delta^2,\cdots,-10\gamma_2 k^3 d1^2 abr_0 c\gamma_1+2\gamma_2 k^2 war_0 cd1\gamma_1}{\beta kcd1(-\delta_2+\delta_1)(\delta+\gamma_1 r_0-\beta q_0 r_0-kwcq_1-\delta q_0+\gamma_2 c_0-\beta q_0 c_0+k^2 d1 q_1 bc)}$

$r_2 = \frac{1}{2}\frac{-\beta\delta^3+8\beta\delta^2 k^2 d1ac,..,16\gamma_1 k^2 d1acr_0\beta\gamma_2 c_0-16\gamma_1 k^2 d1ac\beta^2 q_0 r_0 c_0}{\beta c^2 d1k^2(-\delta_2+\delta_1)(\delta+\gamma_1 r_0-\beta q_0 r_0-kwcq_1-\delta q_0+\gamma_2 c_0-\beta q_0 c_0+k^2 d1 q_1 bc)}$

$c_0 = c_0 \quad , c_1 = -\frac{(-10k^3 d1^2 ab\delta c\gamma_1,..,2k^2 wa\gamma_2 c_0 cd1\gamma_1)}{\beta kcd1(-\delta_2+\delta_1)(\delta+\gamma_1 r_0-\beta q_0 r_0-kwcq_1-\delta q_0+\gamma_2 c_0-\beta q_0 c_0+k^2 d1 q_1 bc)}$

$c_2 = -\frac{1}{2}\frac{(-\beta\delta^3+8\beta\delta^2 k^2 d1ac,..,12\gamma_1 k^4 d1^2 c^2 a^2\beta q_0 c_0)}{\beta c^2 d1k^2(-\delta_2+\delta_1)(\delta+\gamma_1 r_0-\beta q_0 r_0-kwcq_1-\delta q_0+\gamma_2 c_0-\beta q_0 c_0+k^2 d1 q_1 bc)}$

Now where $q_0 = 1, q_1 = 1, r_0 = 1, c_0 = 1, d_1 = 1, \gamma_1 = 1, \gamma_2 = 2, \beta = 1, \delta = 1, d_1 = 1$, w=1,k=1, we get:

$$
\begin{aligned}
q_2 &= -\frac{1}{2c^2} \\
r_1 &= -\frac{2+3c-8ca-8a^2c^3-2ac^2}{c} \\
r_2 &= -\frac{-1+8ca-12a^2c^2+12ac^2+16c^4a^2}{2c^2} \\
c_1 &= \frac{2+3c-2ac^2}{c} \\
c_2 &= \frac{-1+8ca+12ac^2+16c^4a^2}{2c^2}
\end{aligned} \qquad (12)
$$

The Riccati equation(9) we will solve it by the Homogeneous balance method as follows:

$\phi = \sum_{i=0}^{m} e_i tanh^i \Sigma$ where m=1 we get

$\phi = e_0 + e_1 tanh\Sigma$ (13)

Substituting (13) into (9) we have the

$\Phi = -\frac{1}{2a}(b + 2tanh\Sigma), \quad ac = \frac{b^2}{4} - 1$ (14)

From (12)and(13) and(10) we have exact of (7) where:

δ=1,β=1,α=1, a=1, b=1, c=1,

$d1 = \frac{1}{16}\frac{\partial}{\chi}$

where

$\partial = -11232tanh(k(x + wt) + \Sigma_0)^2 - 168\tanh(k(x + wt) + \Sigma_0)$
$+48tanh(k(x + wt) + \Sigma_0)^3 + 48tanh(k(xwt) + \Sigma_0)^4$

$\chi = -3\tanh(k(x + wt) + \Sigma_0) + 3tanh(k(x + wt) + \Sigma_0)^3 + 1 - 4tanh(k(x + wt) + \Sigma_0)^2 +$
$3tanh(k(x + wt) + \Sigma_0)^4$

$d2 = \frac{1}{16}\frac{\mathrm{E}}{\mathrm{H}}$

where

$\mathrm{E} = -392\cosh(k(x + wt) + \Sigma_0) - 368\sinh(k(x + wt) + \Sigma_0), \ldots,$
$-184\tanh(k(x + wt) + \Sigma_0)^2\cosh(k(x + wt) + \Sigma_0)^3$

$\mathrm{H} = \cosh(k(x + wt) + \Sigma_0)^3(-49\tanh(k(x + wt) + \Sigma_0) + 49\tanh(k(x + wt) + \Sigma_0)^3 + 23 - 92\tanh(k(x + wt) + \Sigma_0)^2 + 69\tanh(k(x + wt) + \Sigma_0)^4$

$d3 = \frac{1}{8}\frac{\mathrm{Z}}{\Gamma}$

where

$\mathrm{Z} = 116\cosh(k(x + wt) + \Sigma_0), \ldots, 280\tanh(k(x + wt) + \Sigma_0)^2\cosh(k(x + wt) + \Sigma_0)^3$

$\Gamma = \cosh(k(x + wt) + \Sigma_0)^3(-29\tanh(k(x + wt) + \Sigma_0) + 29\tanh(k(x + wt) + \Sigma_0)^3 + 35 - 140\tanh(k(x + wt) + \Sigma_0)^2 + 105\tanh(k(x + wt) + \Sigma_0)^4)$

Then, the exact solutions for system (7) are:

$$S_{Exact}(x,t) = \frac{1}{2} - tanh(k(x + wt) + \Sigma_0) - \frac{1}{8}(1 + 2\,tanh(k(x + wt) + \Sigma_0))^2$$
$$L_{Exact}(x,t) = -\frac{11}{2} - 13\,tanh(k(x + wt) + \Sigma_0) - \frac{23}{8}(1 + 2\,tanh(k(x + wt) + \Sigma_0))^2 \quad (15)$$
$$B_{Exact}(x,t) = -\frac{1}{2} - 3\,tanh(k(x + wt) + \Sigma_0) + \frac{35}{8}(1 + 2\,tanh(k(x + wt) + \Sigma_0))^2$$

This precise solution shows the extent of the spread of the virus in computers. The farther the parameters $d1 = \frac{1}{2}, d2 = \frac{1}{6}, d3 = \frac{1}{2}$ are from zero, the faster the rate of virus spread in devices, where $d1, d2, d3$ are default values.

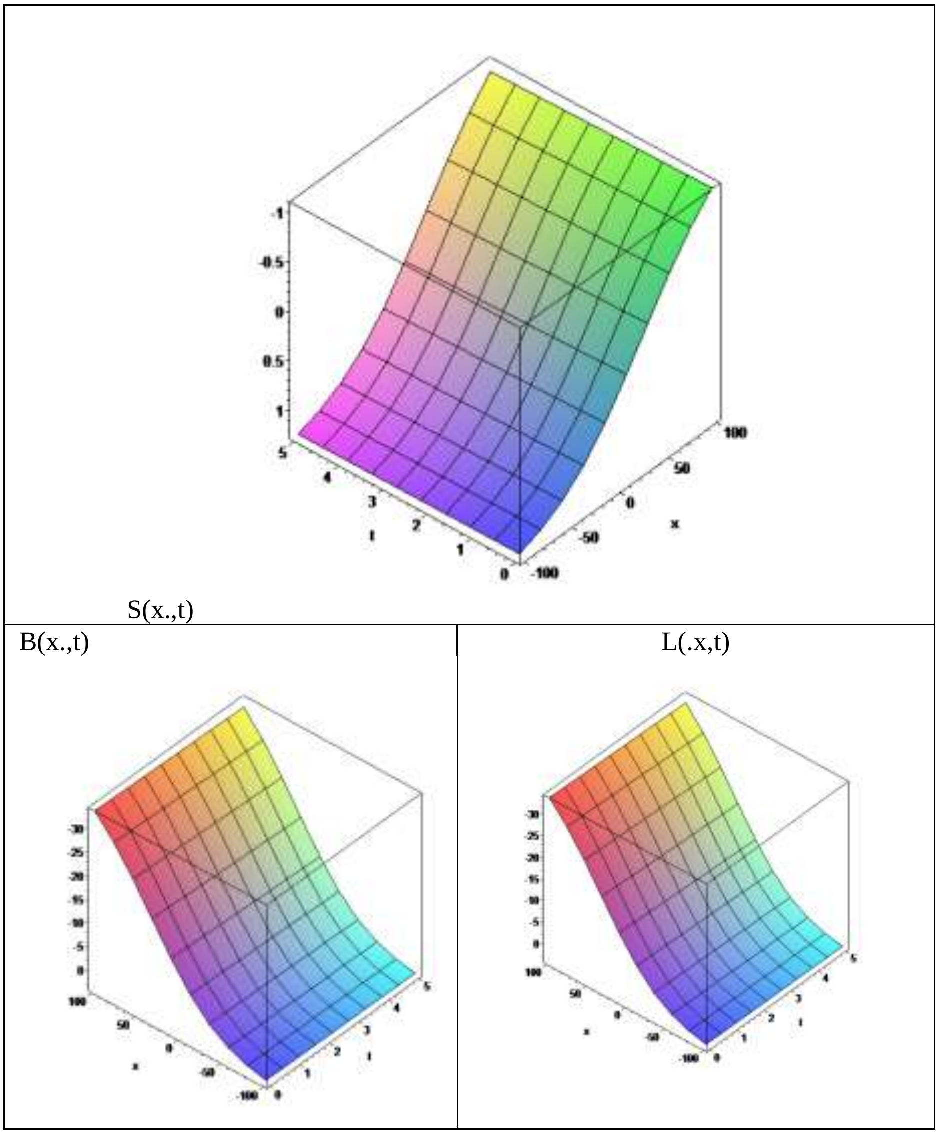

Figure 1.shows the solution path for computer types S(.x,t),L(x,,t),B(x,,t) sequentially.

Case: II . we have:

$$q_0=q_0, q_1=\frac{\beta q_0 r_0+\beta q_0 c_0-\gamma_1 r_0-\delta+\delta q_0-\gamma_2 c_0}{kc(-w+kd1b)}, q_2=0, r_0=r_0, r_1=\frac{-2k^2ac\delta+\delta^2,\cdots,2k^2d1ac\beta q_0 r_0-\beta q_1 c_1 k^2 cd1b}{kc(\gamma_1+\beta q_0)(-w+kd1b)}, r_2=\frac{-4k^5d1^2a^2c^2\gamma_2 wb\gamma_1+2\beta^2\delta^2k^2d1ac,\cdots,-2\gamma_2k^4d1a^2c^2w^2\beta q_0-2\gamma_2k^6d1^3a^2c^2b^2\beta q_0+4\gamma_2k^5d1^2a^2c^2b\beta q_0}{c^2k^2(-w+kd1b)^2(-\gamma_2+\gamma_1)(-\gamma_1+\beta q_0)},$$

$$c_0=c_0, c_1=c_1, c_2=-\frac{2\beta^2\delta^2k^2d1ac,\cdots.-4k^5d1^2a^2c^2w\beta^2{q_0}^2b}{c^2k^2\beta(-w+kd1b)^2(-\gamma_2+\gamma_1)(-\gamma_1+\beta q_0)} \qquad (16)$$

Now where: $c_0=1, c_1=1, r_0=1, d1=1, \gamma_1=4, \gamma_2=2, \beta=1, w=1, \delta=1, k=1, b=3, q_2=0, q_1=1$ *we get*

$$q_1=-\frac{2}{c}, r_1=-\frac{-12-6c}{6c}, r_2=-\frac{48+24c^2-144c}{24c^2}, c_2=\frac{48+72c^2-144c}{24c^2} \qquad (17)$$

The Riccati equation(9) we will solve it by the Homogeneous balance Method as follows:

$\phi=\sum_{i=0}^{m} e_i tanh^i\Sigma$ where m=1 we get

$$\phi=e_0+e_1 tanh\Sigma \qquad (18)$$

$$\phi=-\frac{1}{2a}(b+2tanh\Sigma), \quad ac=\frac{b^2}{4}-1 \qquad (19)$$

From (17)and(18) and(19) we have exact of (7) where:

δ=1,β=1,α=1, a=1, b=1, c=1,

$$d1=\frac{1}{8}\frac{\cosh(x+t+\xi_0)^3-10\sinh(x+t+\xi_0)\cosh(x+t+\xi_0)^2+8\sinh(x+t+\xi_0)-8\cosh(x+t+\xi_0)}{\sinh(x+t+\xi_0)}$$

$$d2=-\frac{1}{24}\frac{\cosh(x+t+\xi_0)\left(40\sinh(x+t+\xi_0)-4\sinh(x+t+\xi_0)\cosh(x+t+\xi_0)^3+35\cosh(x+t+\xi_0)^2-36\cosh(x+t+\xi_0)\right)}{2\cosh(x+t+\xi_0)^2-3}$$

$$d3=-\frac{1}{8}\frac{\cosh(x+t+\xi_0)\left(-12\cosh(x+t+\xi_0)+8\sinh(x+t+\xi_0)+19\cosh(x+t+\xi_0)^3+16\sinh(x+t+\xi_0)\cosh(x+t+\xi_0)^2\right)}{2\sinh(x+t+\xi_0)\cosh(x+t+\xi_0)+2\cosh(x+t+\xi_0)^2-3}$$

The exact solution of system (7) is:

$$S_{Exact}(x,t)=2+2\tanh(k(x+wt)+\xi_0) \qquad (20)$$

$$L_{Exact}(x,t)=-\frac{1}{2}-3\,tanh(k(x+wt)+\Sigma_0)+\frac{3}{4}(1+2\,tanhh(k(x+wt)+\Sigma_0))^2$$

$$B_{Exact}(x,t)=\frac{1}{2}-tanh(k(\,x+wt)+\Sigma_0)-\frac{1}{4}(1+2\,tanh(k(x+wt)+\Sigma_0))^2$$

This precise solution shows the extent of the spread of the virus in computers. The closer the parameters $d1=\frac{1}{10}, d2=\frac{1}{24}, d3=\frac{7}{8}$ are to zero, the lower the rate of virus spread in devices, where $d1, d2, d3$ are default values.

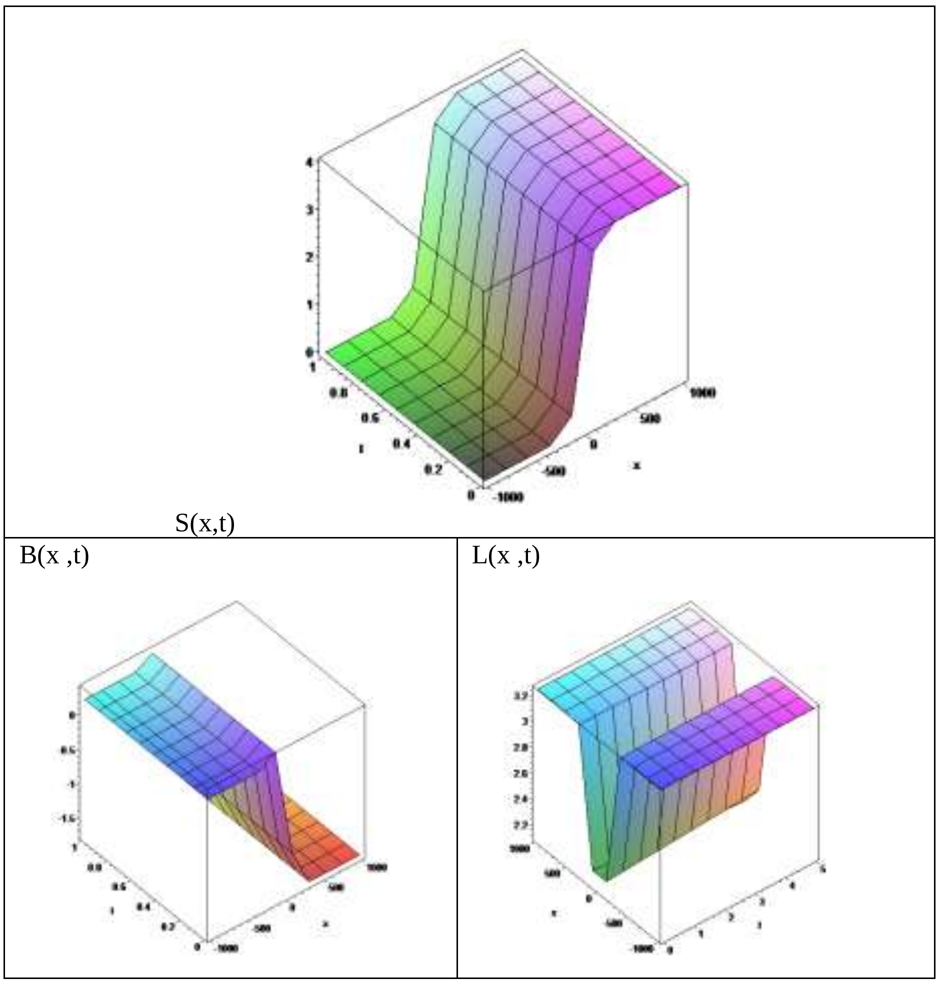

Figure 2. shows the solution path for computer types S(.x,t),L(x,,t),B(x,,t) sequentially

5. Conclusions

The SEIR model, a system of nonlinear differential equations that describes the transmission of viruses between computers, was solved. The real (exact) solutions of the nonlinear SEIR system were obtained using the homogeneous balance method. The effect of variables (parameters) d1, d2, d3 on virus transmission was studied in virus-free computers S(x, t), computers unaware of the infection site L(x, t) and control computers B (x, t) in the system (7) and it turns out that the smaller these parameters are, the slower the diffusion. This is the goal of the paper to reduce the spread, as shown in Figure (1,2). Maple software was used to obtain results and graphs.

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7. References

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إيجاد الحل الدقيق للنموذج الديناميكي لانتشار فيروسات الكمبيوتر باستخدام طريقة التوازن المتجانس

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المستخلص:

في هذا البحث قمنا بحل نظام SEIR (نظام يصف انتشار الفيروسات في أجهزة الكمبيوتر) باستخدام طريقة التوازن المتجانس، وهي إحدى الطرق المعنية بإيجاد الحل الدقيق للمعدلات بمختلف أنواعها، الخطية، غير الخطية وجزئية واعتيادية. SEIR هو نظام من المعادلات الجزئية غير الخطية. دراسة هذا النظام تساعد في تقليل الأضرار التي تسببها الفيروسات لأجهزة الكمبيوتر والمشاكل التي تسببها للأجهزة من خلال التنبؤ بوجود الفيروسات قبل انتشارها في أجهزة الكمبيوتر. تم حل النظام وتم العثور على الحل الدقيق لنظام SEIR. تمت دراسة تأثير المعلمات d1,d2, d3، وتبين أنه كلما كانت هذه المعلمات أصغر (قريبة من الصفر)، كلما كان تسرب الفيروسات إلى أجهزة الكمبيوتر أبطأ. تم شرح تأثير المعلمات ومدى الأداء الذي يمكن أن تسببه للأجهزة باستخدام الرسومات ثلاثية الأبعاد. وبات واضحاً أن هذه الفيروسات تشكل خطراً كبيراً على الأجهزة، لذا فإن دراستها موضوع يستحق الاهتمام.