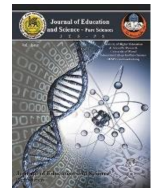
Journal of Education and Science (ISSN 1812-125X)

www.edusj.mosuljournals.com



Enhancing Conjugate Gradient Method Through a Novel Investigation Parameter For Unconstrained Minimization

A. A. Hassan [(1)], **H. Th. Saeed** [(2)]

[1*,2] *Department of Mathematics, College of Computer Sciences and Mathematics, University of Mosul, Mosul, Iraq*

Article information

Article history:
Received: March 30, 2024
Accepted: May 27, 2024
Available online: June 01, 2024

Keywords:
Conjugate Gradient Technique
Unconstrained Optimization
Numerical Studies
Preconditioning
Sufficient Descent Condition
Global Convergence.

Correspondence:
Abd Ahmad Hassan
abd.23csp76@student.uomosul.edu.iq

Abstract

Conjugate gradient algorithms come in a wide range of flavors. The conjugate gradient technique primarily concentrates on the spectral parameter. It follows the standard method proposed by Hastens and Stiefel, In this study, we have devised an innovative approach to spectral conjugate gradient methods we get a new direction conjugate gradient method to solve unconstrained optimization problems, which is based on non-linear function using an inexact line searching introduced a novel direction. In specific scenarios, this groundbreaking direction not only guarantees global convergence but also ensures a downward trajectory. Our numerical experiments unequivocally demonstrate that when compared to traditional CG techniques, depending on the number of functions (NOF), the number of iterations (NOI), and time (CPU), and evaluated using the Dolan-More performance profile, our novel method consistently exhibits superior performance across a diverse set of unconstrained function minimization test. and the convergence condition under some Hypotheses by using a strong -Wolfe line search.

DOI: 10.33899/edusj.2024.148330.1439,

1. Introduction

The conjugate gradient method is one of the best methods for solving large-scale unconstrained optimization problems due to the iterative formula.CG methods were studied for two reasons:

First, solving systems of linear equations in high dimensions using these methods, is one of mathematics's oldest and most well-known areas. Second nonlinear optimizing issues are amenable to these techniques[1]. The main motivation behind deriving the spectral conjugate parameter in the conjugate gradient method lies in improving the algorithm's performance concerning with rapid convergence towards the optimal solution. The conjugate gradient method is an effective technique for solving large-dimensional optimization problems, especially those involving quadratic functions. An important family of unconstrained optimization algorithms is known as CG methods. The main benefits of CG methods are their small memory required, fast convergence, and quadratic termination property, which allows them to quickly find the minimum of
a quadratic function within a fixed number of iterations[2],[3]

The CG methods represent significant iterative approaches, serving as efficient and well-structured tools for addressing unconstrained optimization problems, which include but are not limited to:

$$\underset{x \in R^n}{\text{minimize}} \quad f(x) \tag{1}$$

Where $f : R^n \to R$ is a real-valued CG method generated $\{ x_k \}$ as:

$$"x_{k+1} = x_k + \alpha_k d_k" \tag{2}$$

The step length, denoted as α_k, can be computed through a line search procedure.

The direction, represented as d_k , can be defined as follows:

$$d_{k+1} = \begin{cases} -g_{k+1} & \text{for } k = 0 \\ -g_{k+1} + \beta_k d_k & \text{for } k \geq 1 \end{cases} \quad (3)$$

Where β_k define in [4][5][6][7][8][9][10][11]

There is another type of direction of CG method, which is defined as

$$d_{k+1} = -(1+\theta)g_{k+1} + \beta_k d_k \quad (4)$$

Zhang [12] and others have introduced a modified FR method, referred to as the MFR method, the standard conjugate gradient method is defined as follows :

$$d_{k+1} = -(1+\theta)g_{k+1} + \beta d_k \quad (5)$$

Where $\beta_k = \beta_k^{FR} = \frac{\|g_{k+1}\|^2}{\|g_k\|^2}$

And θ defined as $\beta_k^{FR} \frac{g_{k+1}^T d_k}{\|g_{k+1}\|^2}$

Mamat and Mohd in [13]defined a direction that a modification of QN direction and defining as

$$d_{k+1} = -\nabla f_{k+1}^{-1} g_{k+1} - \lambda g_{k+1}, \ 0 < \lambda < 1 \quad (6)$$

Where $-\nabla f_{k+1}^{-1}$ is the symmetric positive definite matrix. To ensure the convergence analysis of the Conjugate Gradient CG method, the weak Wolfe conditions are typically employed:

$$f(x_k + \alpha_k d_k) - f(x_k) \leq \delta \alpha_k \nabla f(x_k)^T d_k \quad (7)$$

$$\nabla f(x_k + \alpha_k d_k)^T d_k \geq \sigma \nabla f(x_k)^T d_k \quad (8)$$

Used the strong Wolfe conditions consisting of (6) and

$$\left| g(x_k + \alpha_k d_k)^T d_k \right| \leq -\sigma \ g_k^T d_k \text{ "} \quad (9)$$

Where $0 < \delta < \sigma < 1$,[4]. "

1. A modified new scalar

We will derive a new parameter by equaling the direction of (4) and direction (6) as follows :

$$-\nabla f^{-1} g_{k+1} - \lambda g_{k+1} = -(1+\theta^*)g_{k+1} + \beta^{HS} s_k \quad (10)$$

Multiplying both sides of (10) by $s_k^T \nabla f$

$$-s_k^T \nabla f \nabla f^{-1} g_{k+1} - \lambda s_k^T \nabla f g_{k+1} = -(1+\theta^*) s_k^T \nabla f \ g_{k+1} + \beta^{HS} s_k^T \nabla f \ s_k$$

From QN condition ($s_k^T \nabla f(x)$), we get

$$-s_k^T g_{k+1} - \lambda y_k^T g_{k+1} = -(1+\theta^*) y_k^T \ g_{k+1} + \beta^{HS} \ s_k^T \nabla f \ s_k \quad (11)$$

Using the Taylor expansion to second-order terms, f and $g_{k+1}^T s_k$ can be written as:

$$f_k = f_{k+1} - g_{k+1}^T s_k + \frac{1}{2!} s_k^T \nabla f_{k+1} s_k + 0(\|s_k\|^3),$$

$$2f_k = 2f_{k+1} - 2g_{k+1}^T s_k + s_k^T \nabla f_{k+1} s_k + 0(\|s_k\|^3),$$

Then we obtain

$$s_k^T \nabla f_{k+1} s_k = 2(f_k - f_{k+1}) + 2g_{k+1}^T s_k$$

$$\therefore s_k^T \nabla f_{k+1} s_k = s_k^T y_k + 2(f_k - f_{k+1}) + g_{k+1}^T s_k + \ g_k^T s_k \quad (12)$$

After we submit eq. (12) in eq. (11), we obtain

$$-s_k^T g_{k+1} - \lambda y_k^T g_{k+1} = -y_k^T g_{k+1} - \theta^* y_k^T g_{k+1} + \beta^{HS} (s_k^T y_k + 2(f_k - f_{k+1}) + g_{k+1}^T s_k + g_k^T s_k)$$

$$\theta^* y_k^T g_{k+1} = -y_k^T g_{k+1} + s_k^T g_{k+1} + \lambda y_k^T g_{k+1} + \beta^{HS} (s_k^T y_k + 2(f_k - f_{k+1}) + g_{k+1}^T s_k + g_k^T s_k)$$

$$\theta^* = \frac{-y_k^T g_{k+1} + s_k^T g_{k+1} + \lambda y_k^T g_{k+1} + \beta^{HS} (s_k^T y_k + 2(f_k - f_{k+1}) + g_{k+1}^T s_k + g_k^T s_k)}{g_{k+1}^T y_k}$$

$$d_{k+1}^{new} = -(1+(\frac{-y_k^T g_{k+1} + s_k^T g_{k+1} + \lambda y_k^T g_{k+1} + \beta^{HS} (s_k^T y_k + 2(f_k - f_{k+1}) + g_{k+1}^T s_k + g_k^T s_k)}{g_{k+1}^T y_k})) g_{k+1} + \frac{g_{k+1}^T y_k}{s_k^T y_k} s_k \qquad (13)”$$

2.1 The new algorithm

“Step1: given $x_0 \in R^n$, Set k= 0.

Step2: let $d_0 = -g_0$

Step3: Determine the positive step length, denoted as α_k, that satisfies equations (7) and (9), and then set: $x_{k+1} = x_k + \alpha_k d_k$.

Step4: if $\|g_k\| \leq 10^{-5}$, then stop

Step5: Otherwise, compute the new direction using (13)

Step6: if k= n , or Powell Resistant $\frac{|g_k^T g_{k+1}|}{\|g_{k+1}\|^2} \geq 0.2$, [14] then go to step 2 else set k=k+1and go to step 3.”

a. **Theorem [1]**:

In the case of a uniformly convex function f, one can find a constant μ such that:

$$f_k - f_{k+1} \geq -g_{k+1}^T s_k + \frac{\mu}{2}\|s_k\|^2 \qquad (14)$$

where μ is a small positive constant ,0< $\mu < 1$ [15]

b. Theorem:

If the line search defined in equation (2) satisfies the strong Wolfe condition, then the newly defined direction in equation (13) is shown to provide a sufficient descent condition.

Proof:

After multiplying both sides of eq (13) by $\frac{g_{k+1}}{\|g_{k+1}\|^2}$,we get:

$$\frac{d_{k+1}^T g_{k+1}}{\|g_{k+1}\|^2} = -1 - \left(\frac{-y_k^T g_{k+1} + s_k^T g_{k+1} + \lambda y_k^T g_{k+1} + \beta^{HS} (s_k^T y_k + 2(f_k - f_{k+1}) + g_{k+1}^T s_k + g_k^T s_k)}{g_{k+1}^T y_k}\right)\frac{g_{k+1}^T g_{k+1}}{\|g_{k+1}\|^2} + \frac{g_{k+1}^T y_k}{s_k^T y_k}\left(\frac{s_k^T g_{k+1}}{\|g_{k+1}\|^2}\right)$$

$$\frac{d_{k+1}^T g_{k+1}}{\|g_{k+1}\|^2} = -1 + 1 - \frac{s_k^T g_{k+1}}{g_{k+1}^T y_k} - \lambda - 1 - \frac{2(f_k - f_{k+1})}{s_k^T y_k} - \frac{g_{k+1}^T s_k}{s_k^T y_k} - \frac{g_k^T s_k}{s_k^T y_k} + \frac{g_{k+1}^T y_k}{s_k^T y_k}\left(\frac{s_k^T g_{k+1}}{\|g_{k+1}\|^2}\right) \qquad (15)$$

After, we multiple eq. (14) by -2 and submit in eq. (15) , we obtain

$$\frac{d_{k+1}^T g_{k+1}}{\|g_{k+1}\|^2} \leq -\frac{s_k^T g_{k+1}}{g_{k+1}^T y_k} - \lambda - 1 + \frac{2 g_{k+1}^T s_k}{s_k^T y_k} - \frac{\mu\|s_k\|^2}{s_k^T y_k} - \frac{g_{k+1}^T s_k}{s_k^T y_k} - \frac{g_k^T s_k}{s_k^T y_k} + \frac{g_{k+1}^T y_k}{s_k^T y_k}\left(\frac{s_k^T g_{k+1}}{\|g_{k+1}\|^2}\right) \qquad (16)$$

Since $g_{k+1}^T y_k \leq \|g_{k+1}\|\|y_k\|$ and from strong wolfe condition, we get

$$\frac{d_{k+1}^T g_{k+1}}{\|g_{k+1}\|^2} \leq -\frac{s_k^T g_{k+1}}{\|g_{k+1}\|\|y_k\|} - \lambda - 1 + \frac{2 g_{k+1}^T s_k}{s_k^T y_k} - \frac{\mu\|s_k\|^2}{s_k^T y_k} - \frac{g_{k+1}^T s_k}{s_k^T y_k} - \frac{g_k^T s_k}{s_k^T y_k} + \frac{\|g_{k+1}\|\|y_k\|}{s_k^T y_k}\left(\frac{s_k^T g_{k+1}}{\|g_{k+1}\|^2}\right) \qquad (17)$$

from strong Wolfe condition, we get

Since $s_k^T g_{k+1} \leq -\sigma s_k^T g_k$

$$\frac{d_{k+1}^T g_{k+1}}{\|g_{k+1}\|^2} \leq -\frac{(-\sigma s_k^T g_k)}{\|g_{k+1}\|\|y_k\|} - \lambda - 1 + \frac{2(-\sigma s_k^T g_k)}{s_k^T y_k} - \frac{\mu\|s_k\|^2}{s_k^T y_k} - \frac{(-\sigma s_k^T g_k)}{s_k^T y_k} - \frac{g_k^T s_k}{s_k^T y_k} + \frac{\|g_{k+1}\|\|y_k\|}{s_k^T y_k}\left(\frac{-\sigma s_k^T g_k}{\|g_{k+1}\|^2}\right) \quad (18)$$

$$s_k^T y_k = s_k^T g_{k+1} - s_k^T g_k$$

$$\leq -\sigma s_k^T g_k - s_k^T g_k$$

$$s_k^T y_k \leq -(\sigma+1) s_k^T g_k$$

$$-s_k^T y_k \geq (\sigma+1) s_k^T g_k \rightarrow s_k^T g_k \leq \frac{-s_k^T y_k}{(\sigma+1)}$$

$$\therefore s_k^T g_k \leq \frac{-s_k^T y_k}{(\sigma+1)} \quad (19)$$

Now, we submit eq. (19) in eq.(18) and we get the following

$$\frac{d_{k+1}^T g_{k+1}}{\|g_{k+1}\|^2} \leq -\frac{(\sigma s_k^T y_k)}{(\sigma+1)\|g_{k+1}\|\|y_k\|} - \lambda - 1 + \frac{2(\sigma s_k^T y_k)}{(\sigma+1) s_k^T y_k} - \frac{\mu\|s_k\|^2}{s_k^T y_k} - \frac{(\sigma s_k^T y_k)}{(\sigma+1) s_k^T y_k} + \frac{s_k^T y_k}{(\sigma+1) s_k^T y_k} + \frac{\|g_{k+1}\|\|y_k\|}{s_k^T y_k}\left(\frac{\sigma s_k^T y_k}{(\sigma+1)\|g_{k+1}\|^2}\right)$$

$$\frac{d_{k+1}^T g_{k+1}}{\|g_{k+1}\|^2} \leq -\frac{(\sigma s_k^T y_k)}{(\sigma+1)\|g_{k+1}\|\|y_k\|} - \lambda - 1 + \frac{2\sigma}{(\sigma+1)} - \frac{\mu\|s_k\|^2}{s_k^T y_k} - \frac{\sigma}{(\sigma+1)} + \frac{1}{(\sigma+1)} + \frac{\|y_k\|}{\|g_{k+1}\|}\left(\frac{\sigma}{(\sigma+1)}\right)$$

$$\frac{d_{k+1}^T g_{k+1}}{\|g_{k+1}\|^2} + 1 \leq \frac{2\sigma}{(\sigma+1)} + \frac{1}{(\sigma+1)} + \frac{\|y_k\|}{\|g_{k+1}\|}\left(\frac{\sigma}{(\sigma+1)}\right) = \frac{1}{\sigma+1}\left(2\sigma + 1 + \frac{\|y_k\|}{\|g_{k+1}\|}\sigma\right) = C$$

$$\therefore d_{k+1}^T g_{k+1} \leq -(1-C)\|g_{k+1}\|^2 \quad , \quad 0 < C < 1 \quad (20)$$

3.1 Assumption ”

- The set S, defined as $S = \{x: f(x) \leq f(x_0)\}$, is bounded, implying the existence of a positive scalar $b > 0$ such that $\|x\| \leq b, \forall x \in s$.
- In a neighborhood N of S, function f is continuously differentiable, and its gradient satisfies the Lipschitz condition, as given by

$$\|g(x) - g(y)\| \leq L\|x - y\|, \quad \forall x, y \in N \quad (21)$$

Under these assumptions on f, we can conclude that there exists a positive constant $\gamma > 0$ such that

$$\gamma \leq \|\nabla f(x)\| \leq \bar{\gamma} \quad (22)$$

Furthermore, the inequality

$$\big(g(x) - g(y)\big)(x - y) \geq \hat{\mu}\|x - y\|^2, \forall x, y \in S, \mu > 0 \quad (23)$$

holds, where $\hat{\mu}$ is a positive constant. [15]

3.2 Lemma:

Assuming that assumption (3.1) is met and considering any conjugate gradient method where d_{k+1} is a descent direction and also α_k satisfies conditions (7) and (9), if: "

$$\sum_{k\geq 1}\frac{1}{\|d_{k+1}\|^2}=\infty \qquad (24)$$

Then

$$\lim_{k\to\infty}(\inf\|g_k\|)=0 \text{"... [16]} \qquad (25)$$

3.3 Theorem:

If we assume that assumption (3.1) is fulfilled and the direction d_{k+1} defined by equation (13) qualifies as a descent direction, with α_k being computed using equations (7) and (9), then:

$$\lim_{k\to\infty}(\inf\|g_k\|)=0 \text{ . “}$$

Proof

By taking the absolute value of β^{HS}, we get

$$|\beta^{HS}| \leq \frac{|g_{k+1}y_k|}{s_k^T y_k} \quad , \qquad s_k^T y_k \geq \hat{\mu}\|s\|^2$$

Since $|g_{k+1}y_k| \leq \|g_{k+1}\|\|y_k\|$ and from eq. (23) , we obtain

$$|\beta^{HS}| \leq \frac{\|g_{k+1}\|\|y_k\|}{\hat{\mu}\|s\|^2}=\varphi_1$$

$$\therefore |\beta^{HS}| \leq \varphi_1$$

$$|\theta^*| = \left|\frac{-y_k^T g_{k+1} + s_k^T g_{k+1} + \lambda y_k^T g_{k+1} + \beta^{HS}\,(s_k^T y_k + 2(f_k - f_{k+1}) + g_{k+1}^T s_k + g_k^T s_k)}{g_{k+1}^T y_k}\right|$$

$$|\theta^*| \leq 1 + \left|\frac{s_k^T g_{k+1}}{g_{k+1}^T y_k}\right| + \lambda + \varphi_1\left[\left|\frac{s_k^T y_k}{g_{k+1}^T y_k}\right| + \left|\frac{-2(-f_k + f_{k+1})}{g_{k+1}^T y_k}\right| + \left|\frac{g_{k+1}^T s_k}{g_{k+1}^T y_k}\right| + \left|\frac{g_k^T s_k}{g_{k+1}^T y_k}\right|\right]$$

Since $y_k = g_{k+1} - g_k$ and from wolfe condition, and Lipchitz condition $y_k \leq L\|s\|$,
and $s_k^T y_k = L\|s_k\|^2$ we have

$$|\theta^*| \leq 1 + \left|\frac{-\sigma g_k^T s_k}{g_{k+1}^T(g_{k+1} - g_k)}\right| + \lambda + \varphi_1\left[\left|\frac{L\|s_k\|^2}{g_{k+1}^T(g_{k+1} - g_k)}\right| + \left|\frac{-2(g_k^T s_k - \frac{\mu}{2}\|s_k\|^2)}{g_{k+1}^T(g_{k+1} - g_k)}\right| + \left|\frac{-\sigma g_k^T s_k}{g_{k+1}^T(g_{k+1} - g_k)}\right| + \left|\frac{g_k^T s_k}{g_{k+1}^T(g_{k+1} - g_k)}\right|\right]$$

$$|\theta^*| \leq 1 + \frac{|-\sigma g_k^T s_k|}{\|g_{k+1}\|^2 - |g_{k+1}^T g_k|} + \lambda + \varphi_1\left[\frac{|L\|s_k\|^2|}{\|g_{k+1}\|^2 - |g_{k+1}^T g_k|} + \frac{|-2g_k^T s_k|}{\|g_{k+1}\|^2 - |g_{k+1}^T g_k|} + \frac{\mu\|s_k\|^2}{\|g_{k+1}\|^2 - |g_{k+1}^T g_k|} + \frac{|-\sigma g_k^T s_k|}{\|g_{k+1}\|^2 - |g_{k+1}^T g_k|} + \frac{|g_k^T s_k|}{\|g_{k+1}\|^2 - |g_{k+1}^T g_k|}\right]$$

From Powell restart and since $d_k = -g_k$, and we get $s_k = \alpha_k d_k$

$$|\theta^*| \le 1 + \frac{\alpha_k \sigma \|g_k\|^2}{\|g_{k+1}\|^2 - 0.2\|g_{k+1}\|^2} + \lambda + \varphi_1 \left[\frac{L\|s_k\|^2}{\|g_{k+1}\|^2 - 0.2\|g_{k+1}\|^2} + \frac{2\sigma\|g_k\|^2}{\|g_{k+1}\|^2 - 0.2\|g_{k+1}\|^2} + \frac{\mu\|s_k\|^2}{\|g_{k+1}\|^2 - 0.2\|g_{k+1}\|^2} + \frac{\sigma\|g_k\|^2}{\|g_{k+1}\|^2 - 0.2\|g_{k+1}\|^2} + \frac{-\sigma\|g_k\|^2}{\|g_{k+1}\|^2 - 0.2\|g_{k+1}\|^2} \right]$$

$$|\theta^*| \le 1 + \frac{\alpha_k \sigma \|g_k\|^2}{0.8\|g_{k+1}\|^2} + \lambda + \varphi_1 \left[\frac{L\|s_k\|^2}{0.8\|g_{k+1}\|^2} + \frac{2\sigma\|g_k\|^2}{0.8\|g_{k+1}\|^2} + \frac{\mu\|s_k\|^2}{0.8\|g_{k+1}\|^2} + \frac{\sigma\|g_k\|^2}{0.8\|g_{k+1}\|^2} + \frac{-\sigma\|g_k\|^2}{0.8\|g_{k+1}\|^2} \right]$$

$$|\theta^*| \le 1 + \frac{\alpha_k \sigma \|g_k\|^2}{0.8\|g_{k+1}\|^2} + \lambda + \varphi_1 \left[\frac{(L+\mu)\|s_k\|^2}{0.8\|g_{k+1}\|^2} + \frac{2\sigma\|g_k\|^2}{0.8\|g_{k+1}\|^2} \right] = \varphi_2$$

By taking the absolute value of eq (13), we get

$$\|d_{k+1}\| \le (1+\varphi_2)\|g_{k+1}\|^2 + \varphi_1\|s_k\|^2 \le \gamma$$

$$\sum_{k\ge 1} \frac{1}{\|d_{k+1}\|} \ge \frac{1}{\sum \gamma} \sum_{k \ge 1} 1 = \infty$$

$$\therefore \lim_{k\to\infty} \|g_{k+1}\| = 0$$

4. Numerical results and comparisons

We present numerical experiments aimed at comparing our new algorithm with the HS method on a common set of unconstrained optimization test functions[17], known for providing efficient solutions to such problems.

Both our new and HS CG algorithms are implemented with cubic line search. The comparison is primarily based on three factors: the number of iterations (NOI), the number of function evaluations (NOF), and computational time. Our algorithms achieve convergence promptly, typically within a short time frame $\|g_k\|_2 \le 10^{-5}$. For larger problem dimensions n, we employ the Powell restart technique by using the Dolan-More performance profile.

Figures (1,2) in this study illustrate our technique's performance through the Dolan-More graph[18], a key metric assessed by NOF, particularly for problem dimensions around 100,1000.

In Figures (3,4), a similar pattern is observed as in the figures below , demonstrating the performance of our method in comparison to the baseline methods, with a focus on the number of iterations (NOI) for problem dimensions around 100, 1000.

able(1)

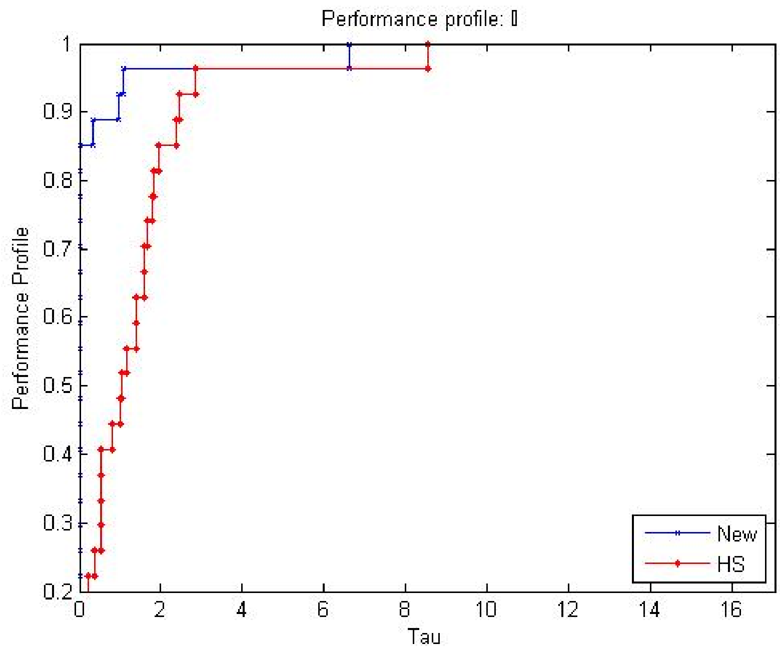
Comparison between new algorithm and standard HS CG algorithm with (n=100)

No.	**Dim.**	**New Algorithm**			**HS Algorithm**		
		NOF	**NOI**	**CPU**	**NOF**	**NOI**	**CPU**
1	100	26	14	10	29	15	19
2	100	12	7	5	14	8	8
3	100	78	35	30	72	31	29
4	100	15	6	6	40	24	20
5	100	20	11	8	13	6	6
6	100	9	4	4	5	2	2
7	100	86	35	34	29	18	10
8	100	19	10	8	19	18	0
9	100	5	2	2	5	2	2
10	100	81	17	16	17	10	6
11	100	100	49	46	53	27	21
12	100	9	4	4	14	28	30
13	100	195	75	60	170	78	79

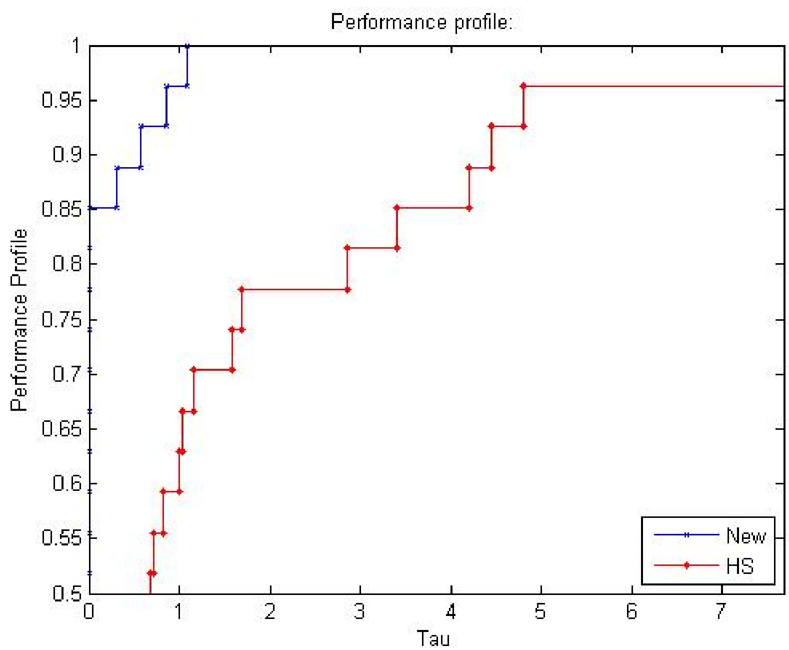
No.	Dim.	New Algorithm			HS Algorithm		
14	100	80	37	32	70	30	20
15	100	19	4	3	84	53	28
16	100	84	50	30	506	390	177
17	100	20	18	0	120	55	47
18	100	120	60	43	13	7	7
19	100	149	50	51	35	13	12
20	100	50	28	20	4	2	2
		1177	516	412	1312	817	525

Table(2)
Comparison between new algorithm and standard HS CG algorithm with (n=1000)

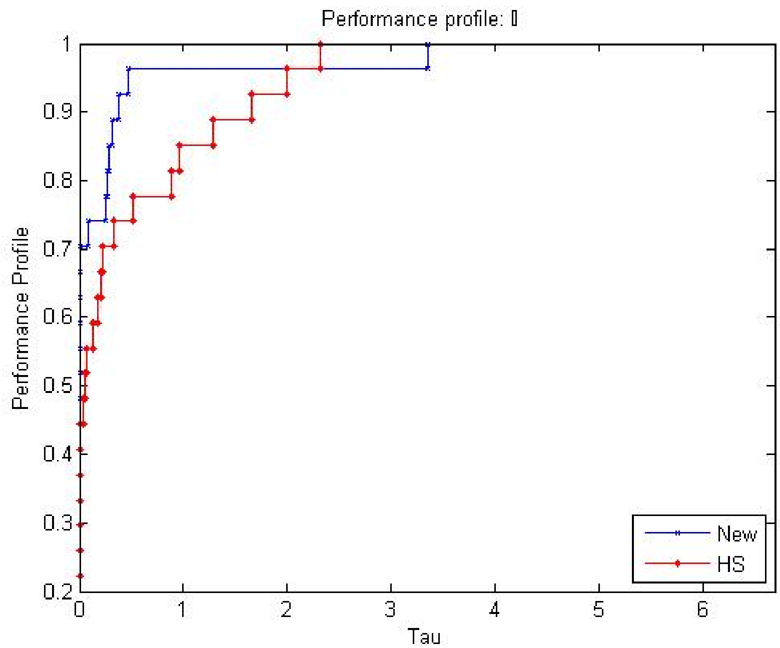
No.	Dim.	New Algorithm			HS Algorithm		
		NOF	**NOI**	**CPU**	**NOF**	**NOI**	**CPU**
1	1000	30	17	12	37	18	16
2	1000	79	30	24	88	39	34
3	1000	60	30	28	78	34	29
4	1000	18	8	6	28	10	10
5	1000	87	50	20	90	53	34
6	1000	30	10	8	34	13	12
7	1000	50	26	23	60	30	28
8	1000	46	30	15	61	37	25
9	1000	12	9	5	30	16	12
10	1000	20	17	10	30	15	13
11	1000	21	10	6	56	25	23
12	1000	59	18	18	34	17	15
13	1000	14	6	6	19	8	9
14	1000	50	28	20	51	29	18
15	1000	32	17	14	25	12	11
16	1000	5	2	2	5	2	2
17	1000	9	4	4	9	4	4
18	1000	7	4	3	8	4	3
19	1000	88	50	27	39	25	10
20	1000	90	30	32	87	36	30
		807	396	283	869	427	338



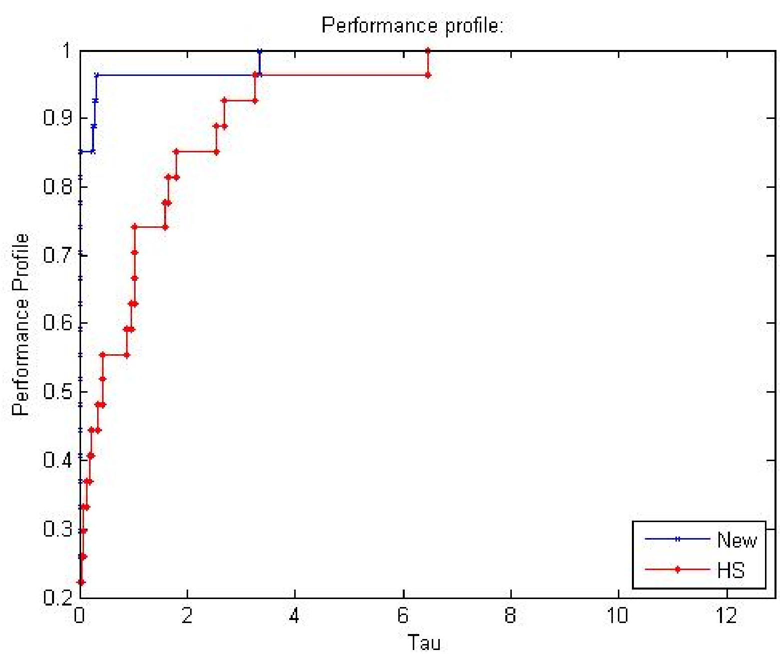
Figure(1) : performance profiles of NOF with (n=100)



Figure(2) : performance profiles of NOF with (n=1000)



Figure(3) : performance profiles of NOI with (n=100)



Figure(4) : performance profiles of NOI with (n=1000)

5. Conclusion

Regarding the theoretical aspects of our new algorithm, we have ensured both sufficient descent and global convergence under certain assumptions. The numerical results presented in the figures above depict the effectiveness of our algorithm as we make a direct comparison with the standard HS CG method.

6. Acknowledgments

The authors would like to thank the University of Mosul / College of Computer Science and Mathematics for their facilities, which have helped to enhance the quality of this work.

7. References

[1] B. M. Ayyub and R. H. McCuen, "Numerical Optimization," in *Numerical Analysis for Engineers*, Chapman and Hall/CRC, 2015, pp. 395–422. https://doi.org/10.1201/b19115

[2] N. Andrei, *Nonlinear conjugate gradient methods for unconstrained optimization*. Springer, 2020. https://doi.org/10.1007/978-3-030-42950-8

[3] Y. Dai and Q. Ni, "Testing different conjugate gradient methods for large-scale unconstrained optimization," *J. Comput. Math.*, pp. 311–320, 2003. https://www.jstor.org/stable/43693071

[4] M. R. Hestenes and E. Stiefel, *Methods of conjugate gradients for solving linear systems*, vol. 49, no. 1. NBS Washington, DC, 1952. https://www.math.unipd.it/~alvise/AN_2018/LETTURE/hestenes-stiefel.pdf

[5] R. Fletcher and C. M. Reeves, "Function minimization by conjugate gradients," *Comput. J.*, vol. 7, no. 2, pp. 149–154, 1964. https://doi.org/10.1093/comjnl/7.2.149

[6] H. I. Ahmed, E. T. Hamed, and H. T. Saeed Chilmeran, "A modified bat algorithm with conjugate gradient method for global optimization," *Int. J. Math. Math. Sci.*, vol. 2020, 2020. https://doi.org/10.1155/2020/4795793

[7] B. A. Hassan and H. M. Sadiq, "A new formula on the conjugate gradient method for removing impulse noise images," *Вестник Южно-Уральского государственного университета. Серия «Математическое моделирование и программирование»*, vol. 15, no. 4, pp. 123–130, 2022. https://doi.org/10.14529/mmp220412

[8] E. T. Hamed, R. Z. Al-Kawaz, and A. Y. Al-Bayati, "New investigation for the liu-story scaled conjugate gradient method for nonlinear optimization," *J. Math.*, vol. 2020, pp. 1–12, 2020. https://doi.org/10.1155/2020/361520

[9] H. T. S. Chilmeran, H. I. Ahmed, E. T. Hamed, and A. Y. Al-Bayati, "A new modification of the quasi-newton method for unconstrained optimization," *Indones. J. Electr. Eng. Comput. Sci.*, vol. 21, no. 3, pp. 1683–1691, 2021. (PDF) A new modification of the quasi-newton method for unconstrained optimization (researchgate.net)

[10] H. T. Chilmerane, "New Scale Dai-Yaun Conjugate Gradient Method for Unconstrained Optimization," *AL-Rafidain J. Comput. Sci. Math.*, vol. 10, no. 4, pp. 29–41, 2013. https://www.iasj.net/iasj/download/a3eb44db3297a340

[11] H. T. Chilmeran, S. A. Al-Taie, and E. G. Saeed, "New Parameter of CG-Method for Unconstrained Optimization," *AL-Rafidain J. Comput. Sci. Math.*, vol. 13, no. 1, pp. 22–31, 2019. DOI: 10.33899/csmj.2020.163500

[12] L. Zhang, W. Zhou, and D.-H. Li, "A descent modified Polak–Ribière–Polyak conjugate gradient method and its global convergence," *IMA J. Numer. Anal.*, vol. 26, no. 4, pp. 629–640, 2006. https://doi.org/10.1093/imanum/drl016

[13] R. Jaafar, M. Mamat, and I. Mohd, "A new scaled hybrid modified BFGS algorithms for unconstrained optimization," *Appl. Math. Sci.*, vol. 7, no. 6, pp. 263–270, 2013. hikari.com/ams/ams-2013/ams-5-8-2013/mamatAMS5-8-2013-3.pdf

[14] M. J. D. Powell, "Restart procedures for the conjugate gradient method," *Math. Program.*, vol. 12, pp. 241–254, 1977. https://doi.org/10.1007/BF01593790

[15] X. Chen and J. Sun, "Global convergence of a two-parameter family of conjugate gradient methods without line search," *J. Comput. Appl. Math.*, vol. 146, no. 1, pp. 37–45, 2002. https://doi.org/10.1016/S0377-0427(02)00416-8

[16] H. Yabe and M. Takano, "Global convergence properties of nonlinear conjugate gradient methods with modified secant condition," *Comput. Optim. Appl.*, vol. 28, pp. 203–225, 2004.

https://doi.org/10.1023/B:COAP.0000026885.81997.88

[17] N. Andrei, "An unconstrained optimization test functions collection," *Adv. Model. Optim*, vol. 10, no. 1, pp. 147–161, 2008. https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=0aa7264b4c5b14ddf091bfdc328c4fcb4049d4f4

[18] E. D. Dolan and J. J. Moré, "Benchmarking optimization software with performance profiles," *Math. Program.*, vol. 91, pp. 201–213, 2002. https://doi.org/10.1007/s101070100263

Appendix
The Test Function For Unconstrained Optimization

No.	The Test Function
1	ARWHEAD
2	Beale
3	Broyden Tridiagonal
4	Diagonal 1
5	Diagonal 2
6	Diagonal 3
7	Diagonal 4
8	Diagonal 5
9	DIXMAANE
10	DIXMAANI
11	DIXMAANJ
12	Dixmaan K
13	DQDRTIC
14	Extended BD1
15	Extended Cliff
16	Extended Powell
17	Extended Himmelblau
18	Extended Hiebert
19	Extended PSC1
20	Extended Three Expo Terms

تعزيز طريقة التدرج المترافق من خلال معلمة ترافق جديدة للتحسين غير المقيد

عبد احمد حسن[1]*، ا.د.همسه ثروت سعيد[2]

[1، 2] قسم الرياضيات، كلية علوم الحاسوب و الرياضيات، جامعة الموصل، الموصل، العراق

الخلاصة:
تأتي خوارزميات التدرج المترافق في مجموعة واسعة من التطورات . تركز تقنية التدرج المترافق في المقام الأول على المعلمة الطيفية. باتباع الطريقة القياسية التي اقترحها هستن وستيفل، في هذه الدراسة، قدمنا طريقة جديدة لطرق التدرج الطيفي المترافق وحصلنا على اتجاه جديد لطريقة التدرج المترافق لحل مشاكل التحسين غير المقيدة، و التي تعتمد على دالة غير خطية باستخدام خط بحث غير مضبوط . قدم اتجاهًا جديدًا. و في صياغة محددة، لا يضمن هذا الاتجاه الجديد التقارب العالمي فحسب ، بل يضمن أيضًا مسارًا هبوطيا . توضح فيه تجاربنا العددية بشكل لا لبس فيه ، وانه عند مقارنتها بتقنيات (CG) القياسية ، اعتمادا على عدد الدوال (NOF) ، و عدد التكرارات (NOI) ، و الوقت (CUP) ، وتقيمها بستخدام ملف تعريف اداء Dolan - More. فان طريقتنا الجديدة تظهر باستمرار اداءا فائقا عبر مجموعة متنوعة من اختبارات الدوال غير المقيدة . كما تضمن التقارب تحت بعض الفرضيات باستخدام خط البحث و شرط ولف القوي .