Tikrit Journal of Engineering Sciences (2024); 31(2): 148-158
DOI: http://doi.org/10.25130/tjes.31.2.14

IRAQI Academic Scientific Journals

ISSN: 1813-162X (Print); 2312-7589 (Online)
Tikrit Journal of Engineering Sciences
available online at: http://www.tj-es.com

TJES Tikrit Journal of Engineering Sciences

Analysis of Dynamic Systems Through Artificial Neural Networks

Abdulsattar Abdullah Hamad [a*], Mamoon Fattah Khalf [a], Fadam M. Abdoon [b], M Lellis Thivagar [c]

a Department of Physics, College of Education, University of Samarra, Samarra, Iraq.
b Department of Chemistry, College of Science, Tikrit University, Tikrit, Iraq.
c School of Mathematics, Madurai Kamaraj University, Madurai, Tamil Nadu, India.

Keywords:
Adaline Networks; Artificial Neural Networks; Dynamic Systems; Multilayer Perceptron Networks; Parameter Identification; Regressive Models.

Highlights:

- *Parameter identification techniques for linear and non-linear dynamic systems currently.*
- *Artificial Neural Networks occupying a prominent - procedure identifying linear dynamic systems parameters in two stages: in the first, a regressive model is fitted from the excitation and response time records, and in the second, its parameters are identified (matrixes of stiffness and damping) and dynamic characteristics.*

ARTICLE INFO
Article history:

Received	11 Aug.	2023
Received in revised form	06 Sep.	2023
Accepted	18 Oct.	2023
Final Proofreading	08 Jan.	2024
Available online	18 May	2024

Citation: Hamad AA, Khalf MF, Abdoon FM, Thivagar ML. **Analysis of Dynamic Systems Through Artificial Neural Networks**. *Tikrit Journal of Engineering Sciences* 2024; **31**(2): 148-158.
http://doi.org/10.25130/tjes.31.2.14

*Corresponding author:
Abdulsattar Abdullah Hamad
Department of Physics, College of Education, University of Samarra, Samarra, Iraq.

Abstract: Parameter identification techniques for linear and nonlinear dynamic systems currently show a clear orientation toward black box models, with Artificial Neural Networks occupying a prominent place there. This paper presents a procedure for identifying linear dynamic systems parameters in two stages: in the first, a regressive model is fitted from the excitation and response time records, and in the second, its parameters are identified (matrixes of stiffness and damping) and dynamic characteristics (vibration frequencies and modes) based on the previous model. Artificial Neural Networks of the Adaline type and multilayer Perceptions are used for the first stage. The second stage is fully formulated through matrix algebra, which facilitates its systematic implementation and makes it independent of the complexity or dimension of the studied system. The proposed procedure is intended to operate from experimental records, so special attention is paid to the sensitivity of the results to the data interval and noise in the input signals. For the latter, various noise levels were incorporated into the correct responses obtained under ideal conditions, which respond to Gaussian distribution functions with a null mean and specified standard deviation. The proposed procedure justification, the results with the regressive models, and a study of the sensitivity of the results to the variation in the available data quality are presented.

تحديد معلمات الأنظمة الديناميكية من خلال الشبكات العصبية الاصطناعية

عبدالستار عبدالله حمد[1]، مأمون فتاح خلف [1]، فدعم متعب عبدون [2] , M Lellis Thivagar [3]
[1] قسم الفيزياء/ كلية التربية/ جامعة سامراء/ سامراء - العراق.
[2] قسم الكيمياء/ كلية العلوم/ جامعة تكريت/ تكريت - العراق.
[3] كلية الرياضيات/ جامعة مادوراي كامارج/ مادوراي/ تاميل نادو - الهند.

الخلاصة

تُظهر تقنيات تحديد المعلمات للأنظمة الديناميكية الخطية وغير الخطية حاليًا اتجاهًا واضحًا نحو نماذج الصندوق الأسود، مع احتلال الشبكات العصبية الاصطناعية مكانًا بارزًا هناك. يعرض هذا البحث إجراءً لتحديد معلمات الأنظمة الديناميكية الخطية على مرحلتين: في الأولى، يتم تركيب نموذج تراجعي من سجلات وقت الإثارة والاستجابة، وفي الثانية، يتم تحديد معلماته (مصفوفات الصلابة والتخميد) والديناميكية. الخصائص (ترددات وأوضاع الاهتزاز) بناءً على النموذج السابق. تم استخدام الشبكات العصبية الاصطناعية من نوع Adaline والتصورات متعددة الطبقات في المرحلة الأولى. أما المرحلة الثانية فقد تمت صياغتها بالكامل من خلال الجبر المصفوفي مما يسهل تنفيذها بشكل منهجي ويجعلها مستقلة عن تعقيد أو أبعاد النظام المدروس. يهدف الإجراء المقترح إلى العمل من السجلات التجريبية، لذلك يتم إيلاء اهتمام خاص لحساسية النتائج لفاصل البيانات والضوضاء في إشارات الإدخال. بالنسبة للأخيرة، تم دمج مستويات الضوضاء المختلفة في الاستجابات الصحيحة التي تم الحصول عليها في ظل ظروف مثالية، والتي تستجيب لوظائف التوزيع الغوسية بمتوسط فارغ وانحراف معياري محدد. يتم عرض مبررات الإجراء المقترح، والنتائج مع النماذج التراجعية، ودراسة حساسية النتائج للتغير في جودة البيانات المتاحة.

الكلمات الدالة: شبكات أدالين؛ الشبكات العصبية الاصطناعية؛ الأنظمة الديناميكية؛ شبكات بيرسبترون متعددة الطبقات؛ تحديد المعلمة؛ النماذج الرجعية.

1.INTRODUCTION

Parameter identification aims to develop and improve the mathematical representation of a physical system using experimental data. It is the way to establish a bridge between the domain of reality and the models that claim to represent it, contributing to a better understanding of the former and improving the latter. In this work, attention is focused on identifying the parameters of mechanical systems and determining the characteristic values of their models from their response to the action of loads that vary with time [1]. This approach constitutes an inverse approach to the usual one, the analytical one, which occurs when determining the response of a specific structure to applied loads. On the other hand, there is a second form of inverse approach, which aims to identify the loads necessary for a particular system to behave according to a previously established response, representing a classic control theory problem. It is worth clarifying here that when talking about inverse statements, it is not done with a strictly mathematical sense but rather with a conceptual understanding. Naturally, to identify the parameters of a dynamic system [2], it must first be dimensionally delimited, which is why this process implies the definition of an equivalent system, which is usually more straightforward than the system or structure it is expected to represent. Thus, identifying parameters is an appropriate means for obtaining equivalent systems, and the characteristic values obtained represent properties that could be called condensed, both mass, dissipative and elastic. The condensed qualifier originates in the problem's direct statement, where through the finite element method, the inertia, damping, and rigidity matrices of the model are obtained, and later, the number of degrees of freedom is reduced by the required study type. There is a growing number of lines of research that focus their attention on the modeling and identifying parameters of dynamic systems. In addition, recent developments in computer technology, such as data acquisition, signal processing, and massively parallel data processing, accentuate this trend. These parameter identification techniques recognize a first classification according to whether the physical models of the represented systems support them. The above finite element method is used for the first case, and for the second, models are called "black box" [3]. In the last case, defining the model is based on the system's dynamic behavior, i.e., on the temporal records of the excitations and their responses, regardless of the mathematical formulation of the phenomenon studied. Within these "black box" approaches, autoregressive models and artificial neural networks occupy a prominent place. Three aspects justify the increasing use of different variants of neural networks in the modeling of dynamic systems:

1) Its aptitude to approximate complex and highly nonlinear functions, developing models that will be used to identify parameters.
2) Its learning capacity, which makes it possible to adjust the model through processes that, with the algorithms Currently available, have significantly reduced the classic convergence problems.
3) The tolerance of neural networks to imprecise or incomplete data makes them especially appropriate for applications with data originating from measurements on the same system.

Concerning the first aspect, Kolmogorov demonstrated that any network that has at least one hidden layer and contains an adequate number of suitably trained units will acquire the character of a universal approximator [4], i.e., it can reproduce any continuous and nonlinear function that can be defined in some hyperspace. Secondly, it should be noted that artificial neural networks are massively parallel systems materialize their learning capacity by modifying the interconnection weights between their neurons. Furthermore, there is the possibility of dynamically altering the topology of a network, i.e., its number of layers, the number of neurons in each layer, or how they are interconnected, which offers other less conventional ways of learning and gives great flexibility to this process. All these techniques have their antecedent in the Backpropagation method [5, 6], which stimulated a veritable revolution in the neural networks and became a classic. The last aspect pointed out is the tolerance to imprecise or incomplete data. In effect, neural networks can recognize patterns from noisy, distorted, or incomplete signals because the information is distributed in the connections between neurons. Also, this type of storage has a certain degree of redundancy. Recent studies [7] have shown that redundancy is the conditioning factor of noise tolerance and generalization capacity and are mutually related properties. When the good performances exhibited by artificial neural networks as approximators of unknown functions are mentioned, their weaknesses must also be recognized. Mainly, these refer to specific aspects of its configuration and training, where multiple possibilities are presented, and there needs to be more definitive recommendations that allow the most convenient ones for each case to be selected. Here, the following must be mentioned:

- **a)** The architecture of the network, in terms of the number of layers, the units per layer, and the links between them.
- **b)** The activation functions, which can be linear, hyperbolic, sigmoidal, or a combination of them.
- **c)** The most appropriate initial weights.
- **d)** The training process techniques. Even though these aspects are the subject of intense study, for the moment, an exploration task can only be avoided once the most convenient combination is found for each case.

However, in cases where it is possible to anticipate the general form of the differential expressions that govern the problem, as occurs here, this knowledge can be applied to define models that are as specific as possible. In this way, when representing known phenomena, the options presented by the neural model are drastically reduced. The idea that the architecture of neural networks should be inspired by knowledge about the nature of the phenomena represented was postulated by numerous authors [8], who assured that this is the way to obtain simple, efficient, and accurate models. The aforementioned allows the authors to affirm that artificial neural networks offer a valid option to implement inverse models of dynamic systems. In the procedure presented here, they are adopted as a basis for the subsequent identification of their parameters. In this case, the type of networks used is called "forward" or "feedforward" because they establish a direct relationship between input and output that emulates the represented system behavior without feedback loops or recurring processes. In particular, Adaline units, Madeline networks, and Perceptions are used. This paper establishes a bridge between the realm of reality and that of the models that purport to represent it toward better understanding the former and improving the latter. It is helpful to understand parameter identification to improve the mathematical representation of a physical system using experimental data. This work emphasizes detecting mechanical system parameters and figuring out the model parameters based on how the mechanical systems react to time-varying loads. This paper proposes a procedure for identifying parameters of dynamic systems that recognize two stages, as shown in Fig. 1. The first stage is intended to fit a regressive model from excitation time records and the response of the natural system. This model aims to filter the noise of the response signals and reproduce the system's behaviors under initial conditions that would not be feasible to impose in the natural system. It is a model implemented with some of the types of neural networks already mentioned. In the second stage, the system parameters (stiffness and damping matrices) and their dynamic characteristics (vibration frequencies and modes) were identified from the response (displacements) obtained with the regressive model. To do this, the velocity and acceleration of the reaction were obtained, which were then incorporated into a matrix algebra procedure. The complete system block diagram is shown in Fig. 2.

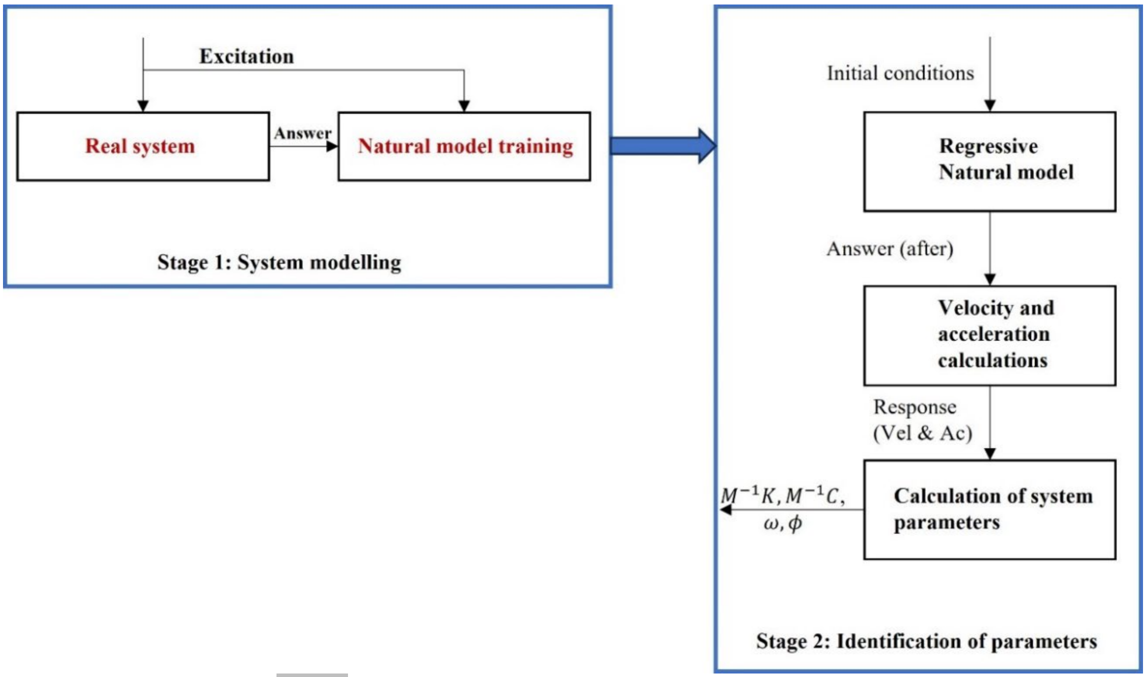


Fig. 1 Modelling and Identification of Parameters.

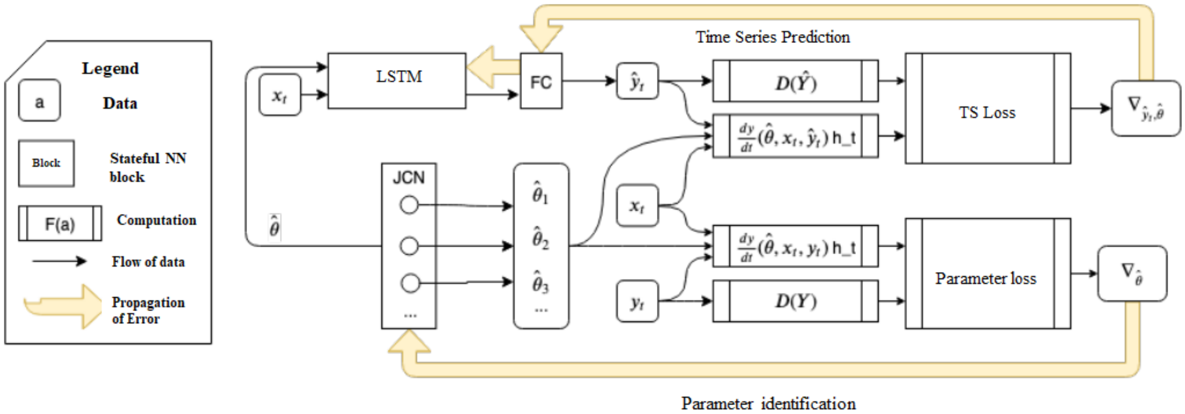


Fig. 2 Block Diagram of the Complete System.

2.REGRESSIVE MODELS

As already mentioned, in the first stage, the behaviors of an unknown function were emulated from the values of its response to the action of a specific excitation condition. This method is usually called ARX (Auto Regressive and Exogenous) the fact that 1) each response value "y" is determined from other values obtained by the same model, 2) these correspond to previous instants, and 3) represents the response of a system to an external action "u." The general expression for the response is as follows:

$$y_k = \sum_{j=1}^{n} \propto_j y_{k-j} + \sum_{i=1}^{m} \beta_j \mu_{k-j} + e_k \quad \textbf{(1)}$$

Where "n" is the number of response values corresponding to previous instants, and "m" is the number of excitation values. Note that ek represents the model's error, and "n" means its order. It must also be borne in mind that matters of m ≤ n are commonly used and that in the case of a system with "q" degrees of freedom, with q > 1, the response "y" and the action "u" are represented by vectors, while matrices represent a and β. In literature, it is common to find references about the impossibility of predicting the number of values of "n" and "m" most convenient to solve each specific problem [9]. However, the two cases are essentially different in representing functions, such as representing completely unknown phenomena and modeling systems that respond to clearly established physical principles. In the first case, these are accurate black box models, only expressible through their input-output relations. In contrast, in the second, it is possible to anticipate, at least, the general form of the differential expressions that govern the problem. As already expected, it is advisable to apply this knowledge to guide the definition of the model and make it as specific as possible. Considering the linear elastic system case with several degrees of freedom, its dynamic equilibrium is represented by a system of differential equations that have the following general form:

$$My + Cy + Ky = u \quad \textbf{(2)}$$

Where M, C, and K represent the inertia, damping, and stiffness matrices. To solve this problem numerically, 4th-order finite difference formulas were used to express the velocity and acceleration as a function of the displacements in four previous intervals:

$$y'_{t+\triangle t} = (11y_{t+\triangle t} - 18y + 9y_{t-\triangle t} - 2y_{t-2\triangle t})/(6 \triangle t)|$$
$$y_{t+\triangle t}=(2\, y_{t+\triangle t} - 5y_t + 4\, y_{t-\triangle t} - y_{t-2\triangle t})(\triangle t^2) \quad \textbf{(3)}$$

Then, the dynamic equilibrium Eq. (2) is considered at instant t+Δt. Acceleration and velocity were replaced by Eq. (3), the terms were finally grouped, and an equation is reached attributed to Houbolt (1950) [10], which has the following general form:

$$y_{t+\triangle t} = Ay_t + By_{t-\triangle t} + Dy_{t-2\triangle t} + E\mu_{t+\triangle t} \quad \textbf{(4)}$$

Where the matrices A, B, D, and E are expressed as a linear combination of the M, C, and K matrices, as shown in the following equations.

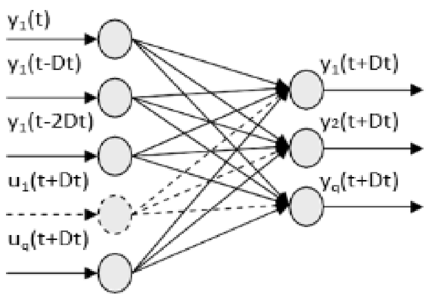
$$A = H^{-1}[(5/\triangle t^2)M + \left(\frac{3}{\triangle t}\right) C]$$
$$B = H^{-1}[(4/\triangle t^2)M + \left(\frac{3}{2\triangle t}\right) C]$$
$$D = H^{-1}[(1/\triangle t^2)M + \left(\frac{1}{3\triangle t}\right) C]$$
$$E = H^{-1}$$
$$H = [(2/\triangle t^2)M + \left(\frac{1}{6\triangle t}\right) C] \quad \textbf{(5)}$$

When adopting this approach, two objectives were pursued: the first was to delimit the number of terms of the general expression (1) based on a rational criterion, eliminating the uncertainty above regarding the most convenient values for "n" and "m," where m, are represent the inertia damping, and stiffness matrices, In this case, they are three and one, respectively. The second objective was to have information on the behavior that could be expected from a model of these characteristics. For the latter, some of the numerous works that evaluated Houbolt's proposal were used, several of which are summarized in a study by Yotov et al. [11], from which valuable information could be extracted, such as the following:

- **a)** This approach has proved to be one of the most efficient for integrating equations corresponding to initial value problems of dynamic elastoplastic systems.
- **b)** The numerical process is unconditionally stable in the resolution of linear systems.
- **c)** It introduces slight damping and distortion in the frequencies, as occurs with most numerical methods that integrate equations of motion.

These considerations allowed the researchers to anticipate that the model proposed in Eq. (4) was apt to represent the solution of the system of differential equations shown in Eq. (2), provided that the elements of the matrices A, B, D, and E could be determined. Note that if it were a direct statement, the matrices M, C, and K would all be known, and the solution to the problem would be expressed by Eq. (5). On the contrary, this is an inverse approach in which M, C, and K are unknown. For this reason, artificial neural networks were used to develop a black box model that represents Eq. (4). As a linear problem, the most convenient option is a set of Adaline units (Adaptive Linear Element), or more precisely, as many Adaline units as degrees of freedom the represented system has. In other words, for a plan with "q" dynamic degrees of freedom, there will be "4q" inputs that will affect "q" Adaline units, as shown in Fig.3(a). The alternatives of a Madeline network (Multiple Adaline) or a Multilayer Perceptron network (shown in Fig.3 (b)) may offer some advantages, which will be discussed later, but prevent a direct relationship with Eq. (4). For the training of these networks, a process of successive adjustments of the synaptic weights, generically called "descending gradient," is used, for which it is necessary to have records of the system's responses to certain excitation conditions. In the case of multilayer networks, the output errors are projected backward with the already mentioned "backpropagation" method. These processes seek to find the minimum error function ε to determine the synaptic weights *wi* of the networks, which in the case of the Adaline units are the elements of the matrices A, B, D, and E. To do this, ε is defined from Eq. (4):

$$\mathcal{E} = y'_{t+\triangle t} - (Ay'_t + By'_{t-\triangle t} + Dy'_{t-2\triangle t} + EU'_{t+\triangle t}) \quad \textbf{(6)}$$



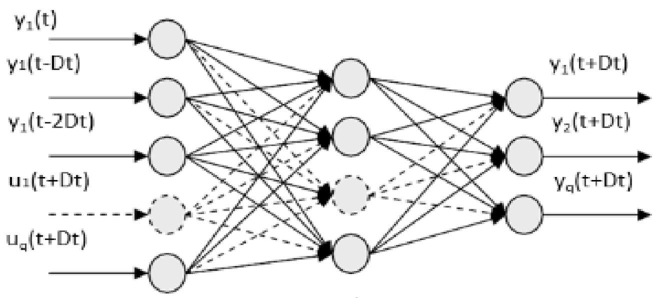


Fig. 3 (a) Schematics of an Adaline Unit Array and (b) a Perceptron Network.

The sets of training pairs {u', y'} are formed by the vectors of known values of the excitation and the response vectors of the system. Then, the scalar J that represents the global error of the network is defined as follows:

$$J = \mathcal{E}^T \mathcal{E}\ , \qquad \frac{\partial J}{\partial w} = 0 \qquad \textbf{(7)}$$

Whose minimum is searched in the hyperspace w of the synaptic weights of the neural network through the descending gradient method.

3.IDENTIFICATIONS OF PARAMETERS

3.1.Lack of a Single Solution

In the case of using Adaline units, the problem has been solved. Once the matrices A, B, D, and E are known, there should be no major drawbacks in deducing the parameters represented by the matrices M, C, and K since both groups of matrices are linked by Eq. (5). However, there is an unexpected difficulty here. A, B, D, and E do not represent a unique solution for the problem formulated in Eqs. (6) and (7). In other words, there are many different matrices A, B, D, and E that can represent the unknown function with very good approximation via Eq. (4) but whose values will not lead to correct values for the matrices M, C, and K because the functional J that is to be minimized presents multiple local minima, which have their origin in the inappropriate excitation used to determine the response of the system. Jha and An [12,13] introduced the concept of persistent excitation to typify the requirements that excitation signals must meet for a system to be identifiable. This condition establishes that the excitation signals must have a sufficiently broad spectrum of frequencies to perturb the system appropriately, and it is recognized as an indispensable requirement for the absolute minimum to be possible by minimizing J. However, it is not easy to ensure such a condition, much less when dealing with the excitation of natural structures, so the practical feasibility of identifying the parameters in this way had to be discarded.

3.2.Determination of Characteristic Values

As a result of the inconvenience above, it was necessary to look for other alternatives to identify the system parameters. For this purpose, the procedure described below was developed. Using the regressive neural model obtained in the previous stage, it is subjected to carefully chosen initial conditions to obtain a "y" response in which its main natural vibration modes are present. Once the response vector "y" is known, Eq. (3) is used to calculate the vectors of velocity y˙ and acceleration ˙y˙, all of them of dimension "q." These displacements and velocities are grouped into a 2q-dimensional state vector called z:

$$z = \begin{Bmatrix} y' \\ y \end{Bmatrix} \qquad \textbf{(8)}$$

Then, Eq. (2) is reformulated as a function of z:

$$z' = Fz + Gu \qquad \textbf{(9)}$$

Where

$$F = \begin{bmatrix} -M^{-1}C & M^{-1}K \\ 1 & 0 \end{bmatrix}, \mathrm{G} = \begin{bmatrix} -M^{-1} \\ 0 \end{bmatrix} \qquad \textbf{(10)}$$

The "p" column vectors z and u, corresponding to as many instants "p" of the response of the system, can be grouped in the matrices Z and U in such a way that:

$$Z = [\, Z_1\, Z_2\, Z_3\ \ldots\ldots Z_P\,]\ , \text{‘}U = [\, U_1\, U_2\, U_3\ \ldots\ldots U_P\,] \qquad \textbf{(11)}$$

From Eqs. (9) and (11), the following expression is obtained that considers the behavior of the system at all times "p":

$$Z' = FZ + GU \qquad \textbf{(12)}$$

Rearranging and post-multiplying both members by the pseudoinverse of the Z matrix, an expression is obtained that determines F:

$$F = (Z' - GU)Z^+ \qquad \textbf{(13)}$$

Where

$$Z' = (ZZ')Z^T \qquad \textbf{(14)}$$

Considering that the excitation U is null, the system only responds to certain initial conditions in the absence of external charges, yielding:

$$F \ = Z'Z^+ \qquad \textbf{(15)}$$

Moreover, considering the definition of F from Eq. (10), it follows that:

$$F_{11} = -M^{-1}C$$
$$F_{12} = -M^{-1}K \qquad \textbf{(16)}$$

Here, it can be verified that to implement the calculation of matrix F, it is not convenient to use Eq. (15) but to carry out a similar analysis from the transpose of Eq. (12), which leads to an equivalent expression:

$$F^T = (Z^T)^+ Z'^T \qquad \textbf{(17)}$$

Note that although Eqs. (15) and (17) are algebraically equivalent. The latter is numerically advantageous. Indeed, calculating the pseudoinverse of the first equation involves the inversion of a matrix of order "p." In contrast, determining the pseudoinverse of Eq. (17) involves inversing a matrix of order 2q, and always q << p since the number of degrees of freedom "q" of the model is always much less than the number of time intervals "p" in which the response of the system is considered. Thus, determining the F12 submatrix is reached, which opens the doors to calculating the system's frequencies and normal vibration modes. In effect, going back to Eq. (2), omitting the damping forces and assuming a harmonic response, the classic eigenvalue problem is posed, which is expressed as:

$$(M^{-1}K - \lambda \mathrm{I})\ \bar{Y} = 0, \text{ where } \lambda = \omega^2 \qquad \textbf{(18)}$$

which is equivalent to say that

$$(-F_{12} - \omega^2 I)\ \bar{y} = 0, \qquad \textbf{(19)}$$

where ω represents the normal frequencies and modes of vibration. For cases where the inertia matrix M is known, which is usually diagonal, the stiffness K and damping C matrices can be determined from the equations below.

$$C = -M\,F_{11}$$
$$K = -M\,F_{12} \qquad \textbf{(20)}$$

Furthermore, when it is possible to assume that the damping is proportional to the inertia and stiffness of the system (Rayleigh damping), a matrix φ contains the vibration modes as its columns, and the generalized damping matrix Γ of the system is determined by the system. Thus, the damping factors ξ_i corresponding to each vibration mode can be known:

$$\Gamma = \phi^T\, C\, \phi = -\phi\, MA_{11}\phi \qquad \textbf{(21)}$$

Where

$$\Gamma = \begin{bmatrix} 2\xi_1\omega_1 & \dots & 0 \\ \cdots & \ddots & \dots \\ 0 & \dots & 2\xi_n\omega_n \end{bmatrix} \qquad \textbf{(22)}$$

3.3.Procedure

Once the solution to the problem has been formulated, the proposed procedure is summarized:

- **a)** Develop a neural model capable of reproducing the studied system's behavior in a specific time interval. This model will consist of as many Adaline units as degrees of freedom "q" has the system, with a total of 4q inputs and "q" outputs. The inputs corresponding to each degree of freedom are the displacements in three successive time intervals and the external action in the last interval, as indicated by Eq. (4). The output of each unit will be displacement in the following time interval.
- **b)** Use the previous model to determine the response "y" of the system to certain initial conditions in the absence of external charges, which ensures that there will be components of all the frequencies of interest in this response.
- **c)** Using Eq. (3), obtain the velocity and acceleration vectors in the same intervals in which the displacements were determined.
- **d)** Select displacements, speeds, and accelerations corresponding to a certain number of points "p," representing the system's response.
- **e)** With these vectors, build the matrices Z and $\dot{Z}^{\cdot}$. Then, calculate the matrix F according to Eq. (17).
- **f)** Formulate the eigenvalue problem, Eq. (19), and calculate frequencies and normal vibration modes.
- **g)** Calculate K and C with Eq. (20) if the inertia matrix M can be known.
- **h)** Determine the damping factors with Eqs. (21) and (22) in cases where the Rayleigh damping model is valid.

4.RESULTS

4.1.Neural Models

For adjusting the neural models, a basic version of the Backpropagation method was implemented, in which the use of more sophisticated algorithms to have the highest sensitivity to the different conditions of the input signals was avoided. The only improvement was to allow the adjustment of the learning factor η during the training process [14] while keeping the momentum factor θ constant. The regressive models reproduced the systems' studied behavior through Adaline units and Perceptron networks with a hidden layer (the number of hidden-in-out layers is one). Although, in all cases, these networks demonstrated excellent performance, the results obtained confirmed the advisability of resorting to the simplest neural models since these allow for establishing parallelism with the numerical models that represent the problem studied, which was already the subject of a previous study by the same authors [15]. Models of linear mechanical systems with one and three degrees of freedom were used, with a single excitation signal and natural frequency periods greater than 0.2 seconds. The training sets corresponded to response segments of between 10 and 20 seconds, with intervals between Δt=0.1 and Δt=0.01 seconds, which gave rise to several known arousal and response points of the brain system that varied between 100 and 2000. A load condition was used to train the model, and a different load condition was used for validation. Thus, it was possible to verify, as expected, that neural models are not limited to memorizing behavior patterns of the real system but capture its properties so that once trained, they correctly reproduce their response to any excitation condition and initial conditions. For this, ensuring that the system's excitation causes all vibration modes in its response is necessary.

4.2.Neural Models

The same mechanical systems with which the neural models were developed were used to validate the presented procedure, from one to three degrees of freedom. The difference was that, in this case, the responses corresponded to initial displacement and speed conditions without external loads to make Eq. (17) applicable. As mentioned, these initial conditions had to be carefully defined to ensure all vibration modes in the system responses. Considering this critical condition, verifying that the proposed method allowed the researchers to obtain excellent results in all cases was possible. When studying the quality of the results, the inertia matrix was considered known, and the stiffness and damping matrices were determined with Eq. (20). Then, the mean square errors of the nonzero elements of these matrices and the most significant individual errors in each were considered as comparison values. The results reproduced below correspond to a system with three degrees of freedom, with natural periods of vibration T = 1.708, 0.654, and 0.293 sec, respectively. In one of the studies conducted, the sensitivity of the results to different △t intervals used in the training of the neural model and in the subsequent calculation of the velocity and acceleration vectors, based on the derivatives of their response, was evaluated. Fig. 4 shows evolving errors in the stiffness matrix with time intervals that increase values of Δt/T between 0.035 and 0.35 (blue line), considering T = 0.293 sec. Here, it was possible to verify that with a value Δt/T of the order of 0.1 (value recommended by the literature for processes of numerical integration of differential equations), the errors reach 2% and that for Δt/T > 0.1 grow steadily Mean square error is below 0.8 %, which is an acceptable range based on literature [McCormick and Salvadori (1976)].

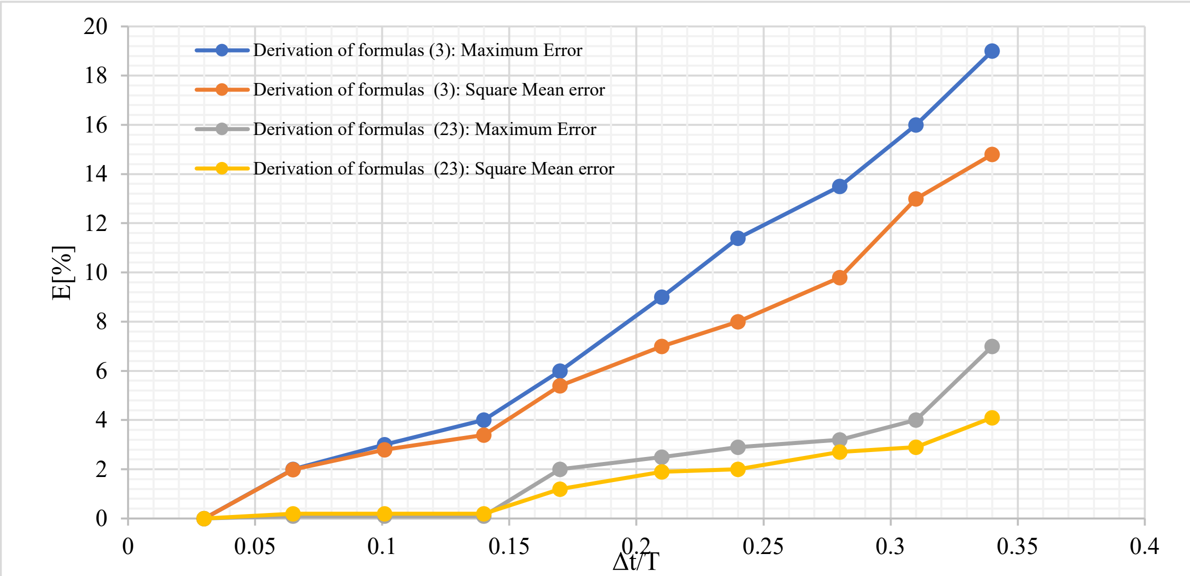


Fig. 4 Evolution of the Error of the Stiffness Matrix when Increasing △t/T.

Several options were explored to improve these results, which verified the need to increase the order of the numerical derivation formulas to obtain the velocity and acceleration vectors. Thus, Eq. (23) proposed by [16-19] was adopted, which produced a drastic improvement in the results (red line). It can be observed in Fig.3 that with values of Δt/T as high as 0.2, the maximum error in the stiffness matrix is of the order of 2%, and its mean square error is below 0.8 %. Thus, the sensitivity of the method to the quality of the velocity and acceleration vectors is recognized, and the new numerical derivation formulas are:

$$y'_{t+\triangle t} = (25y_{t+\triangle t} - 48y + 36y_{t-\triangle t} - 16y_{t-2\triangle t} + 3y_{t-3\triangle t})/(12 \triangle t)|$$

$$y_{t+\triangle t} = (35\, y_{t+\triangle t} - 104y_t + 11\,4\, y_{t-\triangle t} - 56\, y_{t-2\triangle t} + 11y_{t-3\triangle t})/(12 \triangle t^2) \quad \textbf{(23)}$$

Here, it is necessary to highlight that the number of points "p" of the response of a system used in this stage of parameter identification bears no relation to the issues used in the previous stage, model training. In this regard, studies were performed using 500, 200, 100, and 50 points, consistently equally spaced, obtaining identical results. As the system's complexity increases and the data quality used when training the neural model decreases, more "p" points will likely be beneficial. However, this has not yet been sufficiently studied to make recommendations in the user's favor. Next, with the same stiffness matrices used to study the evolution of the errors, the natural vibration frequencies of the system were calculated, posing the eigenvalue problem represented by Eq. (19). Here, it was possible to verify that, even though the errors of the stiffness matrix show a significant difference depending on whether the numerical derivation formulas, Eq. (3) or Eq. (23) are used, and their incidence in determining the frequencies is significantly reduced. As shown in Fig. 5, the values of the natural periods of vibration remained practically stationary for values of △t/T between 0.05 and 0.35. The actual values are those that correspond to △t/T = 0.

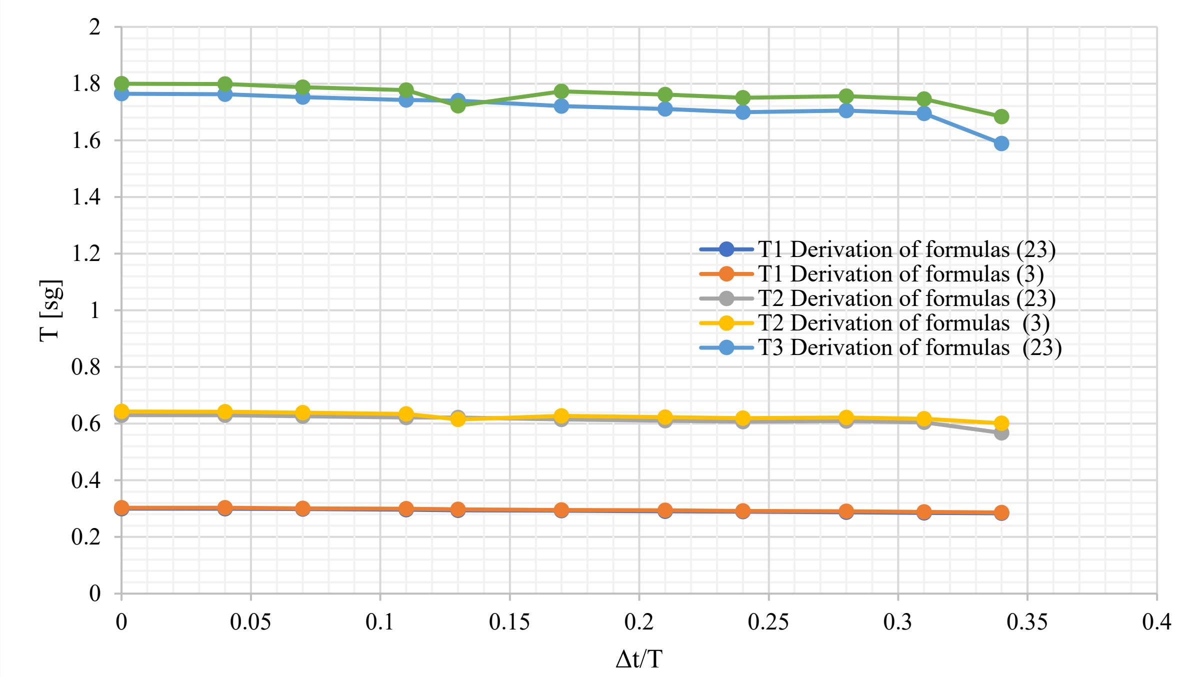


Fig. 5 Evolution of the Natural Periods of Vibration when Increasing Δt/T

These results indicate that the elements of the stiffness matrix suffer distortions as the time interval between the data increases. Still, they do so in such a way that the dynamic characteristics of the model are preserved. The damping matrices of the system were also determined under the same conditions in which the stiffness matrices were determined. The results are presented in Fig. 6, and as can be seen, the quality of the results is very good for low values of △t/T, but the error proliferates as the time interval increases. It could also be verified that the quality of these results is susceptible to the initial conditions imposed on the system, much more than what the stiffness matrix proved to be. Higher-order numerical differentiation formulas were used in this study. Because this procedure will be used to identify parameters of natural systems from measurements of their response, it was very important to know their sensitivity to eventual errors in the input signals. Thus, responses obtained numerically were used to which random noise was added, with Gaussian distribution, null mean, and specified standard deviation. Thus, using different noise amplitudes in the network training data, the procedure's sensitivity to determine the stiffness and damping matrices was studied. Fig. 7 represents the evolution of the errors in the stiffness matrix as a function of the amplitude of the error in the training data of the network, expressed as a percentage of the oscillation maximum amplitude.

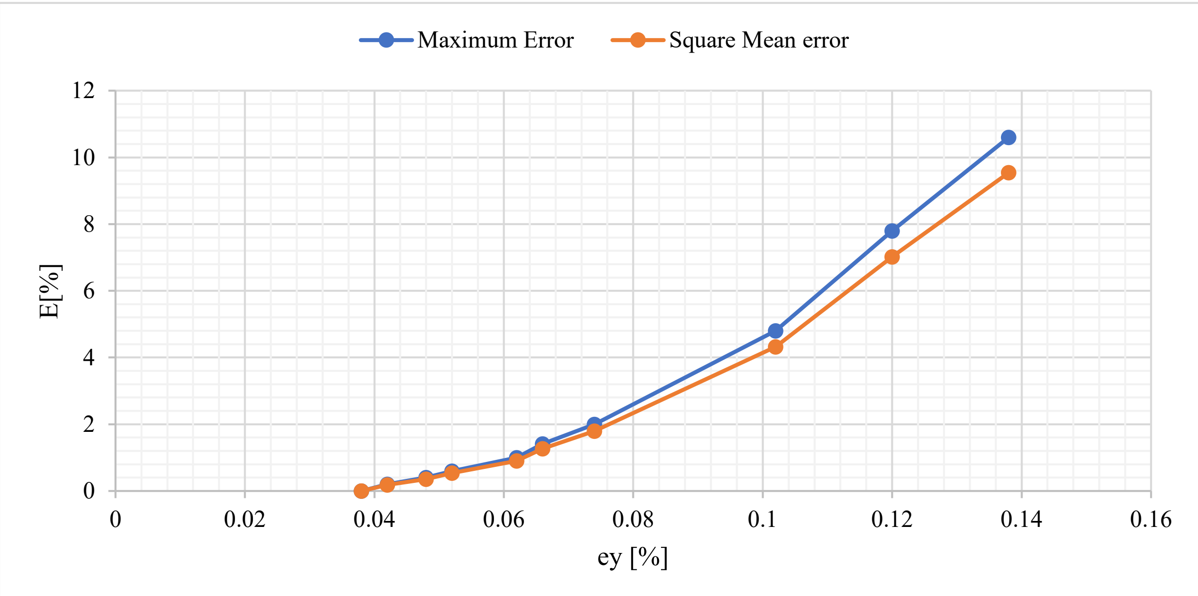


Fig. 6 Evolution of the Error (Derivation of Eq. (23)) of the Damping Matrix when Increasing Δt/T

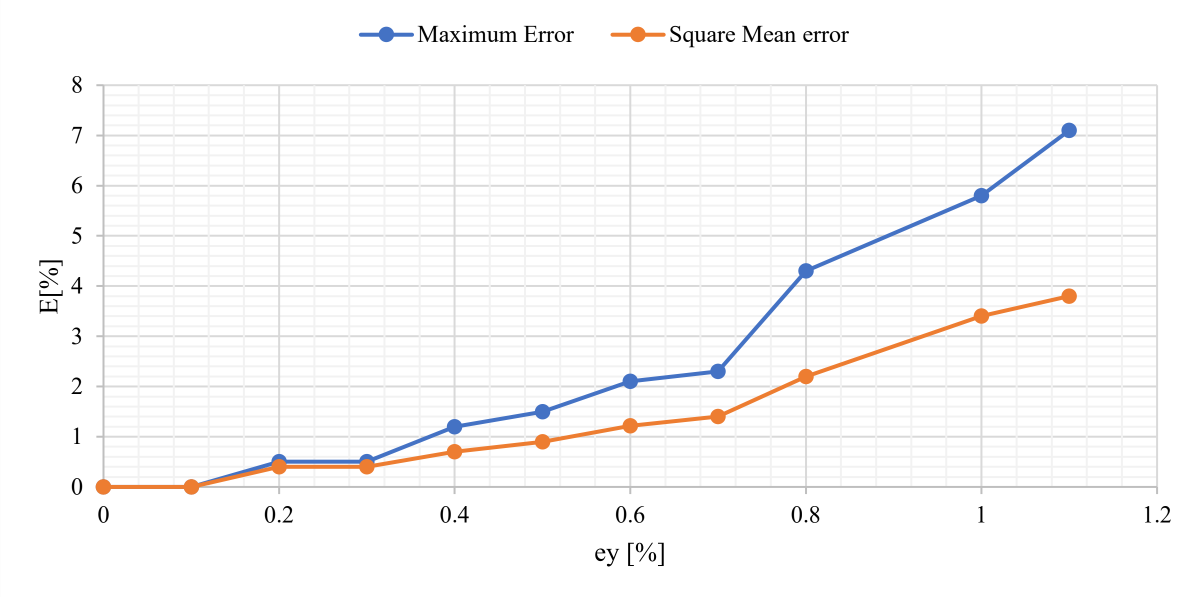


Fig. 7 Evolution of the Stiffness Matrix's Error (Derivation of Eq. (23)) as the Input Error Increases.

One aspect that deserves to be highlighted is that the procedure presented for the second stage is formulated through matrix algebra, which makes it independent of the complexity or dimension of the system studied, facilitating its systematic implementation, which distinguishes it from other procedures that have been presented for the same purpose, as is the case of the works by [20-25], Where it does not seem that the proposals are easily extensible to the treatment of more complex systems than those of the examples shown there.

5.CONCLUSIONS AND FUTURE WORK

This work presents a procedure that allows the parameters (stiffness and damping matrices) and dynamic characteristics (frequencies and modes of vibration) of linear mechanical systems to be obtained. The method recognizes two stages; in the first one, a model was adjusted using artificial neural networks; in the second, this neural model was used to identify the parameters of the real system. In this last stage, the procedure was fully formulated through matrix algebra, which makes it appropriate to address complex problems with many degrees of freedom. The results with simple systems allowed the researchers to confirm the presented procedure's advantages and robustness before relatively large time intervals and noisy input signals. This method will likely help predict structural damage and linearize and identify parameters of complex mechanisms in robotics. In future work, the sensitivity of the calculated parameters to different initial conditions and the quality of the input signals will continue to be studied. Progressively, larger and more complex problems will be dealt with. Variants in the neural models will also be analyzed to have a greater capacity to filter noise and other disturbances.

ACKNOWLEDGEMENTS

The authors are grateful and feel a sense of relief and accomplishment. An acknowledgment for College of Education /Department of Physics /University of Samarra, College of Sciences, Department of Chemistry, Tikrit University and School of Mathematics, Madurai Kamaraj University India, for assignment, whether directly or indirectly.

REFERENCES

[1] Yavich R, Malev S, Volinsky I, Rotkin V. **Configurable Intelligent Design Based on Hierarchical Imitation Models**. *Applied Sciences* 2023; **13**(13):7602.

[2] Liu K, Mao Y, Chen X, He J, Dong M. **Research on Dynamic Modeling and Parameter Identification of the Grid-Connected PV Power Generation System**. *Energies* 2023; **16**(10):4152.

[3] Buhrmester V, Münch D, Arens M. **Analysis of Explainers of Black Box Deep Neural Networks for Computer Vision: A Survey**. *Machine Learning and Knowledge Extraction* 2021; **3**(4):966-989.

[4] Sopelsa Neto NF, Stefenon SF, Meyer LH, Bruns R, Nied A, Seman LO, Gonzalez GV, Leithardt VRQ, Yow K-C. A **Study of Multilayer Perceptron Networks Applied to Classification of Ceramic Insulators Using Ultrasound**. *Applied Sciences* 2021; **11**(4):1592, (1-19).

[5] Jasim FT, Jwaid MF, Karthick M. **Artificial Intelligence Innovation**

and Human Resource Recruitment. *Tamjeed Journal of Healthcare Engineering and Science Technology* 2023; **1**(2): 20–29.

[6] Rumelhart D, Hinton G. Williams R. **Learning Internal Representations by Error Propagation**. *Parallel Distributed Processing: Explorations in the Microstructures of Cognition, MIT Press* 1986; I:318-362

[7] Singh A, Kushwaha S, Alarfaj M, Singh M. **Comprehensive Overview of Backpropagation Algorithm for Digital Image Denoising.** *Electronics* 2022; **11**(10):1590.

[8] Shihab S, Shrooq Bahjat Smeein S, Delphi M. **Operational Spline Scaling Functions Method for Solving Optimal Control Problems**. *Samarra Journal of Pure and Applied Science* 2023; **5**(2): 160–172.

[9] Sildir H, Aydin E, Kavzoglu T. **Design of Feedforward Neural Networks in the Classification of Hyperspectral Imagery Using Superstructural Optimization**. *Remote Sensing* 2020; **12**(6):956, (1-19).

[10] Jumaa HA, Abed MM. **Co-retractable Modules and Semi-Simplicity Property**. *Samarra Journal of Pure and Applied Science* 2023; **5**(2): 151–159.

[11] Yotov K, Hadzhikolev E, Hadzhikoleva S, Cheresharov S. **Finding the Optimal Topology of an Approximating Neural Network**. *Mathematics* 2023; **11**(1):217, (1-18).

[12] Jha S, Khalf MF, Karthick M. **Convolutional Neural Networks for Breast Cancer Detection Using Regions of Interest from Infrared Images**. *Tamjeed Journal of Healthcare Engineering and Science Technology* 2023; **1**(2): 44–53.

[13] An S, Zang J, Yan M, Zhu B, Liu J. **Research on Adaptive Prescribed Performance Control Method Based on Online Aerodynamics Identification**. *Drones* 2023; **7**(1):50, (1-18).

[14] Houbolt J. **A Recurrent Matrix Solution for the Dynamic Response of Elastic Aircraft.** *Journal of Aeronautical Science* 1950; **17**:540-550.

[15] Hamad AA, Ahmed FM, Khalf MF, Thivagar ML. **Dynamic System Linear Models and Bayes Classifier for Time Series Classification in Promoting Sustainabilitys**. *Heritage and Sustainable Development* 2023; **5**(2), 183-198.

[16] Semler C, Gentleman C, Paidoussis M. **Numerical Solutions of Second Order Implicit Non-Linear Ordinary Differential Equations**. *Journal of Sound and Vibration* 1996; **195**(4):553- 574.

[17] Hamad AA, Ahmed FM, Kumar CL, Donipati S, Sreekrishna T, Bandhu D, Tayyeh AM. **Development of Cellulose Nanocomposites for Electromagnetic Shielding Applications by Using Dynamic Network**. *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering* 2023; **0**(0): 09544089231202913.

[18] Khan Q, Akmeliawati R. **Review on System Identification and Mathematical Modeling of Flapping Wing Micro-Aerial Vehicles**. *Applied Sciences* 2021; **11**(4):1546.

[19] Plagianakos Magoulas, Vrahatis. **Deterministic Nonmonotone Strategies for Effective Training of Multilayer Perceptrons**. *IEEE Transactions on Neural Networks* 2002; **13**(6):1268- 1284.

[20] Anaz SS, Alabbasi AM, Tayyeh AM. **Stratifying Transformer Defects Through Modelling and Simulation of Thermal Decomposition of Insulating Mineral Oil**. *Automatika* 2023; **64**(4): 733-747.

[21] Giró J, Olariaga S, Paez N, Garcia A. Vuirli A. **Definition and evaluation of Neural Models for Control Systems.** *Argentine Conference on Robotics (JAR). National Technological University. Cordoba, Argentina*, 2006.

[22] Raoof AG, Jassim TH. **A Double Intuitionistic Compact Space in Intuitionistic Topological Spaces**. *Samarra Journal of Pure and Applied Science* 2022; **4**(4): 78–87.

[23] Xu B, Wu Z, Chen G, Yokohama K. **Direct Identification of Structural Parameters from Dynamic Responses with Neural Networks**. *Engineering Applications of Artificial Intelligence* 2004; **17**(8):931-943, 2004.

[24] Bash AM, Othman TT, Oleiwi JK. **Improving Mechanical Properties of Laminated Biocomposites for Artificial Lower Limb Socket**. *Tikrit Journal of Engineering Sciences* 2023; **30**(3): 9–16.

[25] Saleh SM, Muhammad SH, Abo AA. **Effect of Pooled and Flat Stepped Spillway on Energy Dissipation Using Computational Fluid Dynamics**. *Tikrit Journal of Engineering Sciences*, 2022; **29**(2): 75–79.