Local solvability of semi linear initial value control problem via Composite semi group approach.

Samer Kasim Hassan
Department of Math.,
Collage of education,

Ahmed Shawki Jaber
Department of Math.
Collage of science

Al-Mustansiryah University.

Abstract

The aim of this paper is to prove the local existence uniqueness of the mild solution of semi linear initial value control problem in suitable Hilbert space using semi group theory "composite semi group" and Banach fixed point theorem.

1 Introduction

In this Paper by using the theory of composite semigroup[1],[3],and "Banach contraction principle", the local existence, uniqueness of the mild solution to some control operator semi-linear initial value control problem which has been developed in an arbitrary Hilbert space H.

Our work is concerned the semi linear initial value control problem:

$$\begin{aligned} &\frac{d}{dt}(Z(t)+g(t,Z(t))) = A_1 Z(t) + Z(t)A_2 + Bu(t) \\ &Z(0)=\theta \end{aligned} \qquad (1)$$

where A_1 *and* A_2 are the infinitesimal generators of a C_0 - semi group $T_1(t), T_2(t)$ respectively and $D(A_1), D(A_2) \subset$ L (H). g is a non linear continuous map defined from [0,r) × L (H) into L (H) and B is a linear bounded control operator defined from U into L (H), Where U is a Banach space and u(.) be arbitrary control function is given in $L_2([0,r):U)$ is a Banach space of control function.

The mild solution will be developed as follows:

By using lemma(2.2),[3], we have $\mathbb{T}(t)Z$ is differentiable, that implies L (H) value function.

$G(s)=\mathbb{T}(t-s)(Z(s)+g(s,Z(s)))$ is differentiable for $0<s<t$

$$\frac{dG}{ds} = \mathbb{T}(t-s)\frac{d}{ds}(Z(t)+g(s,Z(s)))+\frac{d}{ds}\ \mathbb{T}(t-s)(Z(t)+g(s,Z(s)))$$

$$= \mathbb{T}(t-s)[A_1Z(t)+Z(t)A_2+Bu(s)]-\mathbb{A}\ \mathbb{T}(t-s)(Z(s)+g(s,Z(s)))$$

$$= \mathbb{T}(t-s)\ \mathbb{A}\ Z(s)+\mathbb{T}(t-s)Bu(s)-\ \mathbb{A}\ \mathbb{T}(t-s)Z(s)-\mathbb{A}\ \mathbb{T}(t-s)g(s,Z(s))$$

$$= \mathbb{T}(t-s)Bu(s)-\mathbb{A}\ \mathbb{T}(t-s)g(s,Z(s)) \tag{2}$$

By integrating (2), from 0 to t, yield:

$$G(t)-G(0)=\int_0^t \mathbb{T}(t-s)Bu(s)ds-\int_0^t \mathbb{A}\ \mathbb{T}(t-s)g(s,Z(s))ds$$

Since $G(s)=\ \mathbb{T}(t-s)(Z(s)+g(s,Z(s))),$ then:

$$Z(t)+g(t,Z(t))-\mathbb{T}(t)(\theta+g(0,\theta))=\int_0^t \mathbb{T}\ (t-s)Bu(s)ds\ -\int_0^t \mathbb{T}(t-s)\ \mathbb{A}\ g(s,Z(s))ds$$

Implise that:

$$Z(t)\ = \mathbb{T}(t)(\theta+g(0,\theta))\ -\ g(t,Z(t))+\int_0^t \mathbb{T}\ (t-s)Bu(s)ds-\int_0^t \mathbb{T}(t-s)\ \mathbb{A}\ g(s,Z(s))ds$$

(3)

2. Preliminaries

2.1 Definition [6] :

A family $\{T(t)\}_{t\geq 0}$ of bounded linear operators on a Banach space X is called a (one-parameter) semigroup on X if it satisfies the following conditions:

T(t + s) = T(t)T(s), ∀ t, s ≥ 0

T(0) = I

2.2 Definition [4]:

A semigroup $\{T(t)\}_{t\geq 0}$ on a Banach space X is called strongly continuous semigroup of a bounded linear operators or (C_0-semigroup) if the map $\mathbb{R}^+ \ni t \longrightarrow T(t) \in L(X)$, satisfies the following conditions:

1. $T(t + s) = T(t)T(s), \forall\, t, s \in \mathbb{R}^+$.
2. $T(0) = I$.
3. $\lim_{n\to\infty} \|T(t)x - x\|_X = 0$, for every $x \in X$.

2.3 Definition [7] :

The weakest topology on $L(X, Y)$, such that $E_x : L(X, Y) \longrightarrow Y$ given by:

$E_x(T) = Tx$ are continuous for all $x \in X$

is called the strong operator topology.

2.4 Remark [6] :

A semigroup $\{\{T(t)\}_{t\geq 0}$ is called a continuous in the uniform operator topology, if:

(1) $\|T(t + \Delta)x - T(t)x\| \longrightarrow 0$, as $\Delta \longrightarrow 0$, $\forall\, x \in X$.

(2) $\|T(t)x - T(t - \Delta)x\| \longrightarrow 0$, as $\Delta \longrightarrow 0$, $\forall\, x \in X$

2.5 Definition [3]:

Let $L(H)$ be a Banach space, a one-parameter family

$\{\mathbb{T}(t)\}_{t\geq 0} \subset L(L((H))$, $t\in[0,\infty)$ of bounded linear operators defined by

$$\mathbb{T}(t)=T_1(t)ZT_2(t) \tag{3}$$

for generator $\mathbb{A}$,for any $Z\in L(H)$ and $t \in[0,\infty)$ is called composite semigroup ,where $T_1(t)$,$T_2(t)$ are two semigroups defined from H into H for A_1, A_2 respectively.

2.6 Definition[3] :

The infinitesimal generator $\mathbb{A}$ of $\mathbb{T}(t)$ on astrong operator topology defined as the limit:

$$\mathbb{A}\, Z = \tau \text{ - } \lim_{t\downarrow 0} \left\{\frac{\mathbb{T}(t)Zh - Z}{t}\right\},\ Z \in D(\mathbb{A})$$

where $D(\mathbb{A}) \subset L(H)$is the domain of $\mathbb{A} + \Delta\mathbb{A}$ and defined as follows:

$$D(\mathbb{A}) = \left\{ Z \in L(H) : \tau\text{- } \lim_{t\downarrow 0} \left\{\frac{\mathbb{T}(t)Zh - Z}{t}\right\} \text{ exist in } \{L(H),\tau\} \right\}.$$

where {L(H), τ} stands for L(H) equipped with the strong operator topology τ, i.e., topology induced by family of seminorms $\rho = \{\rho_h\}_{h\in H}$, where siminorms $\rho_h(Z) = \|Zh\|_H$, $Z \in L(H)$.

2.7 Remarks [1]:

a-The different between the usual strongly continuous semigroups of and the composite perturbation semigroup (3) it follows from the fact that in general for $Z \in L(H)$, the function $[0,\infty) \supseteq t \mapsto \mathbb{T}(t)Z \in L(H)$ is continuous in {L(H), τ}, and which cannot to be continuous in {L(H), ‖.‖} unless the semigroups $\{T_1(t)\}_{t\geq 0}$, $\{T_2(t)\}_{t\geq 0} \subset L(H)$ are uniformly continuous. However, this case only if their generators A_1, A_2 are bounded operators on H .

b-The generator $\mathbb{A}$ is densely define only in {L(H), τ} and dose not in {L(H), ‖.‖}.

This implies that $D(\overline{A})$ in L(H) is only a proper set and not the whole L(H).

2.8 Lemma [3]:

let $\mathbb{T}(t) = T_1(t)ZT_2(t)$, $t \geq 0$ be a composite perturbation semigroup defined on L(L(H)) , $T_1(t), T_2(t)$ are , perturbation semigroups defined on L(H) then

a- The family $\{\mathbb{T}(t)\}_{t\geq 0} \subseteq L(H)$, $t \geq 0$ is a semigroup, i.e.,

(1). $\mathbb{T}(0)Z = Z$, $\forall Z \in L(H)$

(2). $\mathbb{T}(t+s)Z = \mathbb{T}(t)(\mathbb{T}(s)z)$

$= \mathbb{T}(s)(\mathbb{T}(t))Z$

$Z \in L(H)$, $t, s \in [0, \infty)$.

b- $\|\mathbb{T}(t)\|_{L(H)} \leq M_1M_2 e^{t(w_1+w_2)}$, for $t \in [0, \infty)$.

c- $\mathbb{T}(t) \in L(L(H))$ is strong-operator and continuous at the origin, i.e.,

$$\tau\text{-}\lim_{t\downarrow 0} \|(\mathbb{T}(t)Z)h - (\mathbb{T}(0)Z)h\|_H = 0,\ h \in H,\ Z \in L(H).$$

2.9 Lemma [3]:

The operator $\mathbb{A}$ is infinitesimal generator for $\mathbb{T}(t)$ define on its domain $D(\mathbb{A})$ satisfies the following properties:

(a) $D(\mathbb{A})$ is strong-operator dense in $L(H)$.

(b) $\mathbb{A}$ is uniform-operator closed on $L(H)$.

(c) For $Z \in L(H)$

$$\int_0^t \mathbb{T}(r)Z\,dr \in D(\mathbb{A}), \text{ and}$$

$$\mathbb{A}\left(\int_0^t \mathbb{T}(r)Z\,dr\right) = \mathbb{S}(t)Z - Z.$$

(d) For $Z \in D(\mathbb{A})$

$\mathbb{T}(t)Z \in D(\mathbb{A})$, the function $[0,\infty) \ni t \longmapsto \mathbb{T}(t)Z \in L(H)$

is continuously differentiable in $\{L(H),\tau\}$ and

$$\frac{d}{dt}(\mathbb{T}(t)Z) = (\mathbb{A})\,(\mathbb{T}(t)Z)$$

$$= \mathbb{T}(t)(\mathbb{A}Z)$$

(e) For $Z \in D(\mathbb{A})$ and $h \in D(A_1)$

$$(\mathbb{A}\,Z)h = A_1Zh + ZA_2h.$$

2.10 ***Concluding Remark [2]*** :

Suppose $x(0) \in D(A)$ and the function $f(t)$ with range in H is strongly continuously differentiable in the open interval $(0,t_1)$ with derivative continuous in closed interval $[0,t_1]$, then

$\dot{x}(t) = Ax(t) + f(t),\ 0 \le t < t_1$ has a unique solution satisfying

$$A\int_0^t T(t-s)f(s)ds = \int_0^t T(t-\sigma)\dot{f}(\sigma)d\sigma - [f(t) - T(t)f(0)]. \qquad (4)$$

By using concluding remark (2.10) the mild solution (3),becomes :

$Z(t)= \mathbb{T}(t)(\theta+g(0,\theta)) - g(t,Z(t))$

$$+\int_0^t \mathbb{T}(t-s)Bu(s)ds - [\int_0^t \mathbb{T}(t-s)\dot{g}(\sigma,Z(\sigma))d\sigma - g(t,Z(t)) + \mathbb{T}(t)g(0,\theta)]$$

Hence

$$Z_u(t) = \int_0^t \mathbb{T}(t-s)Bu(s)ds - \int_0^t \mathbb{T}(t-s)\dot{g}(\sigma,Z(\sigma))d\sigma \qquad (5)$$

For every given $u \in L_2([0,r):U)$.

2.11 Schauder fixed point theorem [5]:

Let M be a nonempty closed bounded convex subset of a Banach space X and the map T:M→M is compact then T has a fixed point

.

3.Main results:

It should be notice that the local existence and uniqueness of a mild solution defined in (5) to the semi intial value control problem defined in (1) have been developed, by assuming the following assumptions:

1- $\mathbb{A}Zh=A_1Zh + ZA_2h$, for any $Z \in L(H)$ and $h \in H$,is infinitesimal generator of a C_o composite semigroup $\{\mathbb{T}(t)\}_{t\geq 0}$ with domain $D(\mathbb{A}) \subseteq L(H)$.

2- Let O be an open subset of $[0, r)\times L(H)$ for $0 < r \leq \infty$, .

3- For every $(t, Z) \in O$, there exists a neighborhood $G \subset O$ of (t, Z); the nonlinear maps g: $[0, r)\times L(H) \longrightarrow L(H)$ satisfy the locally Lipschitize

condition:

$$\| \dot{g}(t, Z) - \dot{g}(s, Z_1)\|_{L(H)} \leq L\|Z - Z_1\|_{L(H)}, \ L_1>0, \text{ for all } (t, Z) \text{ and } (s, Z_1) \in G.$$

4- For t′ > 0, $\|g(t, z(t))\|_{L(H)} \le k_0$, for $0 \le t \le t'$ and for every $z(t) \in L(H)$.

5-u (.) be an arbitrary given control function in $L_2([0,r):U)$, a Hilbert space of control functions and B is bounded linear B is bounded linearcontrol operator from U into { L(*H)* , τ } with $\|B\|_{L_2([0,r):U)} \le k_1$ and $\|u(t)\|_U \le k_2$, for $0<t<r$.

6-. Let $t_1>0$ such that $t_1= \min\{r, t'\}$ and satisfy the following conditions:

$$t_1 \le \frac{1}{W_1+W_2}\log\frac{(w_1+w_2)(\delta-\delta')}{[k_1k_2+L\delta+k_0](M_1M_2)}$$,where $\delta' < \delta$ is a positive constant.

3.1 Theorem :

Assume that hypothesis (1- 6) are hold, then for every $Z_0 \in L(H)$, there exists a fixed number t_1, $0 < t_1 < r$, such that the initial value control problem has a unique local continuous mild solution $Z_u \in C((0, t_1]:L(H))$, for every control function $u(.) \in L^2((0, r):U)$.

Proof:

Without losing of generality, we may suppose that $r < \infty$, because we are concerned here with the local existence only.For a fixed point (0,Θ) in the open subset O of [0, r)×L(H),we choose δ > 0 such that the neighborhood G of the point (0, Θ) is defined as follows:

$$G = \{(t, Z) \in O: 0 \le t \le t', \|Z - \Theta\|_{L(H)} \le \delta\} \subset O.$$

Since O is an open subset of [0, r)×L(H) and,set $Y = C([0, t_1]: L(H))$, then Y is a Banach space with the supremum norm:

$$\|y\|_Y = \sup_{0\le t\le t_1} \|y(t)\|_{L(H)} .$$

Let S_u be the nonempty subset of Y, defined as follows:

$$S_u = \{Z_u \in Y: Z_u(0) = \Theta, \|(Z_u(t) - \Theta)\|_{L(H)} \le \delta, 0 \le t \le t_1\} . \tag{6}$$

To prove the closedness of S_u as a subset of Y ,

Let $Z_u^n \in S_u$, such that $Z_u^n \xrightarrow{p.c.} Z_u$ as $n \longrightarrow \infty$,we must prove that $Z_u \in S_u$ where (p.c) stands for point wise convergence.

Since $Z_u^n \in S_u$, then we have $Z_u^n(0) = \Theta$ and:

$$\|(Z_u^n(0) - \Theta \|_{L(H)} \le \delta,\ 0 \le t \le t_1 .$$

Since $Z_u^n \xrightarrow{u.c} Z_u$, hence $Z_u \in Y$, where (u.c) stands for the uniform convergence, and also since $Z_u^n \xrightarrow{u.c} Z_u$, then $\| Z_u^n - Z_u\|_Y \longrightarrow 0$ and therefore:

$$\sup_{0\le t\le t_1} \| Z_u^n (t) - Z_u(t)\|_{L(H)} \longrightarrow 0 \text{ as } n \longrightarrow \infty ,$$

which implies that:

$$\| Z_u^n(t) - Z_u(t)\|_{L(H)} \longrightarrow 0 \text{ as } n \longrightarrow \infty ,$$

for every $0 \le t \le t_1$, i.e., and

$$\tau\text{-}\lim_{n\to\infty} Z_u^n(t) = Z_u(t),\ \forall\ 0 \le t \le t_1, \tag{7}$$

hence:

$$\tau\text{-}\lim_{n\to\infty} Z_u^n(0) = Z_u(0) \qquad \text{(by (5))} .$$

since $Z_u^n \in S_u$ and $Z_u^n(0) = \Theta$, that yields:

$$\tau\text{-}\lim_{n\to\infty} Z_u^n(0) = \Theta$$

Hence $Z_u(0) = \Theta$.

Now:

$$\|Z_u(t) - \Theta \|_{L(H)} = \|\tau\text{-}\lim_{n\to\infty} Z_u^n(t) - \Theta \|_{L(H)}$$

$$= \tau\text{-}\lim_{n\to\infty} \| Z_u^n(t) - \Theta \|_{L(H)}$$

$$\le \tau\text{-}\lim_{n\to\infty} \delta .$$

Thus $Z_u(t) \in S_u$.

Since $Z_u(t)$ arbitrary element in S_u, hence S_u is a closed subset of Y .

Now, define a map F_u: $S_u \longrightarrow Y$, given by:

$$(F_u Z_u)(t) = \int_0^t \mathbb{T}(t-s)B\, u(s)\, ds + \int_0^t \mathbb{T}(t-s)\, \dot{g}(s, Z(s)ds \quad , \qquad (8)$$

for arbitrary $u(.) \in L^2([0, \infty): U)$.To show that $F_u(S_u) \subseteq S_u$,

Let Z_u be an arbitrary element in S_u and let $F_uZ_u \in F_u(S_u)$, to prove that $F_uZ_u \in S_u$ for an arbitrary element Z_u in S_u.

From (6), notice that $F_uZ_u \in Y$ {by the definition of the map F_u} and $(F_uZ_u)(0) = \Theta$ {by (8)}. Notice also that

$$\|(F_uZ_u)(t) - \Theta \|_{L(H)} = \| \int_0^t \mathbb{T}(t-s)Bu(s)\, ds - \int_0^t \mathbb{T}(t-s)\dot{g}(s, Z(s))ds - \Theta\|_{L(H)}$$

$$= \| \int_0^t \mathbb{T}(t-s)Bu(s)ds - \int_0^t \mathbb{T}(t-s)\dot{g}(s, Z(s))ds \; +$$

$$\int_0^t \mathbb{T}(t-s)\dot{g}(s, \Theta)ds - \int_0^t \mathbb{T}(t-s)\dot{g}(s, \Theta)ds \|_{L(H)} \qquad (9)$$

$$= \| \int_0^t \mathbb{T}(t-s)Bu(s)\, ds + \int_0^t \mathbb{T}(t-s)[\dot{g}(s, Z(s)$$

$$- \dot{g}(s, \Theta)]ds - \int_0^t \mathbb{T}(t-s)\dot{g}(s, \Theta)ds\|_{L(H)}$$

$$\leq \int_0^t \|\mathbb{T}(t-s)\|_{L(H)} \| \; \|B\| \, \|u(s)\| \, ds$$

$$+ \int_0^t \|\mathbb{T}(t-s)\|_{L(H)} \, \|\dot{g}(s, Z(s) - \dot{g}(s, \Theta)\|_{L(H)} \, ds$$

$$+ \int_0^t \|\mathbb{T}(t-s)\|_{L(H)} \, \|\dot{g}(s, \Theta)\|_{L(H)} \, ds.$$

From lemma (2.8)(b) and conditions (3),(4),(5), we have that:

$$\|(F_u Z_u)(t) - \Theta\|_{L(H)} \leq \int_0^t M_1 M_2 e^{(t-s)((W_1+W_2)} K_1 K_2 ds + \int_0^t M_1 M_2 e^{(t-s)((W_1+W_2)} \|\dot{g}(s, Z(s) - \dot{g}(s, \Theta)\|_{L(H)} \, ds$$

$$+ \int_0^t M_1 M_2 e^{(t-s)((W_1+W_2)} K_0 \, ds.$$

$$\|(F_u Z_u)(t) - \Theta\|_{L(H)} \leq \frac{M_1 M_2}{(W_1 + W_2)} e^{t(W_1+W_2)} K_1 K_2 + \frac{M_1 M_2}{(W_1 + W_2)} e^{t(W_1+W_2)} L\| Z(s) - \Theta)\|_{L(H)} + \frac{M_1 M_2}{(W_1 + W_2)} e^{t(W_1+W_2)} K_0$$

$$\leq \frac{M_1 M_2}{(W_1 + W_2)} e^{t(W_1+W_2)} K_1 K_2 + \frac{M_1 M_2}{(W_1 + W_2)} e^{t(W_1+W_2)} L\delta + \frac{M_1 M_2}{(W_1 + W_2)} e^{t(W_1+W_2)} K_0$$

$$\leq \frac{M_1 M_2}{(W_1 + W_2)} [K_1 K_2 + L\delta + K_0] \, e^{t_1(W_1+W_2)}$$

By using condition (6),we get

$$\|(F_u Z_u)(t) - \Theta\|_{L(H)} \leq \delta - \delta' < \delta, \text{ for } 0 \leq t \leq t_1 .$$

Thus, we have that $F_u: S_u \longrightarrow S_u$.

Now, we need to show that F_u is a strict contraction on S_u, this will ensure the existence of a unique mild solution to the semilinear initial value control problem.

Let $\overline{\overline{Z}}_u, \overline{Z}_u \in S_u$, then:

$$\|(F_u \overline{\overline{Z}}_u)(t) - (F_u \overline{Z}_u)(t)\|_{L(H)} = \| \int_0^t \mathbb{T}(t-s)Bu(s)\, ds$$

$$- \int_0^t \mathbb{T}(t-s)\dot{g}(s, \overline{\overline{Z}}_u(s))ds$$

$$- \int_0^t \mathbb{T}(t-s)Bu(s)ds$$

$$+ \int_0^t \mathbb{T}(t-s)\dot{g}(s, \overline{Z}_u(s))ds) \|_{L(H)}$$

$$= \| - \int_0^t \mathbb{T}(t-s)\dot{g}(s, \overline{\overline{Z}}_u(s))ds$$

$$+ \int_0^t \mathbb{T}(t-s)\dot{g}(s, \overline{Z}_u(s))ds) \|_{L(H)}$$

$$\leq \int_0^t \|\mathbb{T}(t-s)\|_{L(H)} \; \| \dot{g}(s, \overline{\overline{Z}}_u(s))$$

$$- \dot{g}(s, \overline{Z}_u) \|_{L(H)}\, ds$$

$$\leq \int_0^t M_1 M_2$$

$$\| \dot{g}(s, \overline{\overline{Z}}_u(s)) - \dot{g}(s, \overline{Z}_u(s)) \|_{L(H)} \, ds$$

$$\leq \int_0^t M_1 M_2 \; e^{(t-s)((W_1+W_2)} L \| \overline{\overline{Z}}_u(s) - \overline{Z}_u(s) \|_{L(H)}$$

$$\|(F_u \overline{\overline{Z}}_u)(t) - (F_u \overline{Z}_u)(t)\|_{L(H)} \leq \frac{1}{\delta} \frac{M_1 M_2}{(W_1 + W_2)} e^{t_1 (W_1 + W_2)} L \delta .$$

$$\| \overline{\overline{Z}}_u(s) - \overline{Z}_u(s) \|_{L(H)}$$

$$\leq \frac{1}{\delta} \frac{M_1 M_2}{(W_1 + W_2)} e^{t_1 (W_1 + W_2)} [K_1 K_2 + L\delta + K_0] \| \overline{\overline{Z}}_u(s) - \overline{Z}_u(s) \|_{L(H)} .$$

From condition(6),we have that:

$$\|(F_u \overline{\overline{Z}}_u)(t) - (F_u \overline{Z}_u)(t)|_{H_{-1}} \leq (1 - \frac{\delta'}{\delta}) \| \overline{\overline{Z}}_u - \overline{Z}_u \|_Y . \qquad (10)$$

Taking the supremum over [0, t_1] of both sides to (9), we get:

$$\sup_{0 \leq t \leq t_1} \|(F_u \overline{\overline{Z}}_u)(t) - (F_u \overline{Z}_u)(t)\|_{H_{-1}} \leq (1 - \frac{\delta'}{\delta}) \| \overline{\overline{Z}}_u - \overline{Z}_u \|_Y .$$

We obtain:

$$\|(F_u \overline{\overline{Z}}_u)(t) - (F_u \overline{Z}_u)(t)\|_Y \leq (1 - \frac{\delta'}{\delta}) \| \overline{\overline{Z}}_u - \overline{Z}_u \|_Y$$

Hence from condition (6) we have that $\delta' < \delta$, then F_u is a strict contraction map from S_u into S_u and therefore by Banach contraction principle, there exists a unique fixed point Z_u of Fu in S_u, i.e., there is a unique $Z_u \in S_u$, such that $F_u Z_u = Z_u$.

The fixed point satisfies the integral equation:

$$Z_u(t) = \int_0^t \mathbb{T}(t-s)B\,u(s)\,ds + \int_0^t \mathbb{T}(t-s)\,\dot{g}(s, Z(s))ds$$

for $0 \le t \le t_1$ and $\forall$ u(.) $\in$ u(.) $\in$ $L^2([0, \infty): U)$.

<u>Refrences</u>

[1]:Alber, J .,"On Implemented Semigroups.Semigroup ". Forum, Vol.63,pp.371-386,2001.

[2]:Balakrishnan, A.V., "Applied functional analysis ", by Springer- Verlag New York, Inc., 1976.

[3]:Emirsajlow, Z., "A composite semigroup for the infinite-dimensional differential sylvester equation", Proceedings of the 7th Medierranean Conference on Control and Automation, Haifa, Jun 28-30,1999.

[4]: Klaus, J. E and Rainer N., "One-parameter semigroup for linear evolution equation ", by springer-verlag, New York, Inc., 2000.

[5]:Marsden, J.E.,"Applied mathematical sciences",1995.

[6]: Pazy, A., "Semigroup of linear operator and applications to partial differential equation", Springer- Verlag, New York, Inc., 1983.

[7]:Sarfaty, S., "Functional analysis notes fall 2004", By Yevgeny V. Lensky Curnt instiution, 2004

"الحل العام(محلي)لمسألة سيطرة شبه خطيةّ ذات قيمة ابتدائية باستخدامممنهجية شبه زمرة مركبة"

سمير قاسم حسن أحمد شوقي جابر
كلية التربية /قسم الرياضيات كلية العلوم/قسم الرياضيت
الجامعة المستنصرية

المستخلص:

الهدف من هذا البحث هو اثبات وجود وحدانية الحل العام (محلي) لمسألة سيطرة شبه خطية ذات قيمة ابتدائية في فضاء هلبرت مناسب بأستخدام منهج شبه زمرة (شبه الزمرة المركبة) ونظرية النقطة الثابتة لبناخ.