# **Effect of Position and Inclination Angle of Cutoff Wall on Seepage Control in the Foundation of Dam Structure**

Lecturer: Imad Habeeb Obead University of Babylon - College of Engineering [Eng\\_imad2005@yahoo.com](mailto:Eng_imad2005@yahoo.com)

### **Abstract:**

The present work presents finite element model which formulated to analyze the twodimensional steady state seepage of water through the foundation of dam structure in the presence of inclined cutoff as seepage control device. A computer program using FORTRAN 90 language developed to determine the pressure head at nodal points, the exit gradients and the seepage discharge behind inclined cutoff walls. The results were presented by performing a parametric study for various design parameter. In this work the inclination angle( $\beta$ ) of cutoff changed (from  $\beta = 30^{\degree}$  to  $\beta = 150^{\degree}$ ), with inclined cutoff located upstream, mid distance, and downstream parts along the floor length of the dam structure respectively. The applicability of the model were examined with relevant existing approximations from literature, results demonstrate that installing cutoff in upstream part of dam structure with  $\beta = 60^{\circ}$  consistent with analytical solution(AE=5.11 %), while installing cutoff in mid distance of dam length agree well (AE=2.03%, for  $\beta$ =105°), the case of cutoff installed in the downstream part of dam structure strongly convenient(AE=0.2% for  $\beta$ =30°). The average value for coefficient of determination  $R^2$ for above the range of inclination angles which corresponding the various positions of cutoff wall was (0.9918,0.9713 and 0.9949) respectively, which indicates good agreement with previous studies and many of the factors studied.

**الخالصة**

ان التشغيل والأداء الأمن والاقتصادي لبنية السد يعتمد على جملة من العوامل من بينها اختيار طريقة التصميم الهيدروليكي المناسبة بالإضافة الى مجابهة خطر حدوث ظاهرة الانبوية في اساس السد وغيرها. تم في الدراسة الحالية استخدام نموذج العناصر المحددة لنحليل الجريان المستقر وباتجاهين خلال قاعدة لبنية سد مزودة بجدار قاطع راسى مائل كوسيلة للسيطرة على التسرب . استخدم لهذا الغرض برنامج حاسوبي بلغة فورتران 90 لإيجاد شحنة الضغط الهيدروليكي عند العقد وحساب كل من تدرج المخرج وكُمية المياه المتسربة خلف الجدار القاطع المائل ولكل حالة. تم فحص امكانية تطبيق هذا النموذج مع بعض البحوث المتوفرة ذات العلاقة وعرضت النتائج عن طريق اجراء دراسة لمعلمات تصميمية مختلفة. لقد بينت النتائج انه في حالة كون الجدار القاطع الراسي يقع في منطقة مقدم جسم السد ولزاوية ميل مقدار ها (°60) فان نسبة الخطأ المطلّق (%AE=5.11)، اما في حالة وضع الجدار القاطع في منتصف قاعدة السد فقد انخفُض ْهذا المقدار الى (AE=2.03%) لزاوية ميل مقدار ها (105°) في حين اصبح هذا المقدار(AE=0.2%) عند وضع الجدار القاطع في منطقة مؤخر جسم السد ولزاوية ميل مقدارها (°30)،اما متوسط قيم معامل التحديد (R2) لمجموعة من زوايا المعيل المعتمدة في الدراسة والمقابلة لكل موضع للجدار القاطُع فكانت (0.9918 و0.9713 و0.9949)على النوالي،مما يدل على حصول توافق جيد مع الدراسات السابقة ولكثير من العوامل المدروسة

## **1.Introduction**

Many hydraulic structures, such as weirs, barrages, locks, and dams, which are built over permeable soils are provided with sheet piles as a protection measure against scour and undermining. However, the traditional design of these structures is based on using one or two rows of sheet piles that extend laterally along the width of the canal[1]. The seepage flow exerts pressure on the structure and generates erosive forces which tend to pull soil particles with the flow. This causes the formation of irregular passages like pipes which move beneath the structure. This process is known as the piping phenomenon. To design a safe hydraulic structure against seepage, the following two important points must be considered[2].

1- Safety against Uplift Pressure.

2- Safety against Piping.

In particular, uplift pressure, which is the penetrating pressure, may accelerate structural corrosion and consequently increase seepage. Avoiding high water pressure is, therefore, one of the main concerns in the design of hydraulic structures. Reduction of uplift pressure is often necessary to secure structural safety, and is generally achieved by adopting minor filter drainage systems. In addition, it is not possible to install such a drainage systems for various types of structures. As an alternative measure, cutoffs, like sheet piles or concrete curtains, can be provided to reduce uplift and seepage forces resulting in appreciable saving in dimensions of the dam structure[3].

The existence of large exit gradients in regions downstream (D/S) dam where fine grained soils are inadequately filtered can sometimes lead to internal erosion and piping failure, therefore, seepage analysis of cutoff walls is useful in order to determine if high gradients develop at the base of the cutoff wall or on the D/S exit point. The hydraulic structure would be safe when the existing exit gradient ( $I_{Exit}$ ) is less than the critical gradient ( $I_{\text{Critical}}$ ) [4], i.e., if the exit gradient at the D/S side approaches the critical hydraulic gradient, then piping originates in the soil.

The exit gradient can be obtained as[5].

$$
I_{\scriptscriptstyle \text{Exit}} = \frac{H_i - H_{\scriptscriptstyle \text{ds}}}{\delta y_i} \qquad \qquad \ldots \ldots (1)
$$

In which;

 $H_i$ : the values of head at nodes vertically below exit nodes(L).

 $\delta y_i$ : the vertical distance along exit nodes(L).

*H*<sub>*ds*</sub>: the head loss D/S the dam structure(L).

The critical exit gradient( $I<sub>Critical</sub>$ ) is given by expression [6]:

 $I_{\text{Critical}} =$ *e G s*  $\ddot{}$  $\overline{\phantom{0}}$ 1 1

In which; Gs: the relative density of soil partials. e: the void ratio.

### **2.Review of Literatures**

The problem of seepage under dams was considered by many authors. **Abbas [7]** used conformal transformation and gave a solution for seepage flow beneath a flat bottom dam with an inclined sheet pile at its toe on a homogeneous and isotropic soil of infinite depth. **Mohamed and Agiralioglu [8]** used a two-dimensional finite difference model to analyze steady state seepage flow beneath a flat bottom dam with an inclined sheet pile at its toe on a homogeneous and anisotropic soil.

…..(2)

**Al-Senousi and Mohamed[9]** prepared a model to compute the piezometric head distribution under a hydraulic structure with inclined cut-off for different flow conditions and soil characteristics.

**Khassaf** *et al***.[2]** presented a numerical investigations on the effect of cut-off inclination angle on exit gradient and uplift pressure head under hydraulic structure and determines the optimum location and angle of inclination of cutoff by using (ANSYS11.0). They concluded that using D/S cutoff inclined towards the D/S side with angle of inclination less than 120º is beneficial in increasing the safety factor against the piping phenomenon.

**Esmat** [10] used (Geo-Studio 2007 SEEP/W) computer program to investigate the effect of the cutoff wall angle of inclination which varied from  $0^{\degree}$  to 180 $^{\degree}$  on the minimizing the flow quantity to conserve water in the reservoir, the uplift pressure under the dam to maintain stability of the dam, and the exit gradient to prevent quick condition to occur at the toe of the dam where piping may caused erosion of the soil. The results shows that the best angle to minimize the water flow is about

 $60^{\circ}$ , the best angle to minimize the uplift pressure was about  $120^{\circ}$  to  $135^{\circ}$ , and the best angle to minimize the exit gradient was about 45° to 75°.

**Ijam [3]** established An analytical solution for seepage flow below a dam floor with inclined cutoff located anywhere along the base of the dam. The derived equations have been used for computation of hydraulic gradient along the D/S bed and for the pressure at key points. The results obtained were presented for different values of cutoff location, inclination angle, and floor length/ depth of cutoff ratio.

In the present work, the influence of a cutoff position, inclination angles of cutoff are investigated using the finite element method. The applicability of the system are evaluated by performing a parametric study for various design parameters. A non-dimensional parameter approach has been used to simplify the analysis, and a set of graphs for non-dimensional parameters are presented as an alternative for design. Based on the analysis results, some design conclusions are made.

## **3. Problem Statement**

The problem analyzed in this study represents two-dimensional confined flow problem. A detailed presentation of dam structure geometries and material properties considered in this study are demonstrated in figure(1) below. In primary simulation, a cut off is used in upstream (U/S) in order to study its effect on the reduction of uplift pressure.





#### **3.1 Theory and Numerical Formulation**

In seepage problems, Laplace's equation combines Darcy's law and the continuity equation into a single second order partial differential equation[11]. The two dimensional Laplace equation for steady state flow is:

$$
\frac{\partial}{\partial x}\left(K_x \frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial H}{\partial y}\right) = 0 \qquad \qquad \dots (3)
$$

In which; H:total head (L).  $K_X$  and  $K_Z$ : are the hydraulic conductivities in X and Y directions respectively ( $L/T$ ).

For unsteady or transient flow condition, the groundwater flow equation for two dimensional of incompressible flows in porous media can be expressed as [12]:

$$
\frac{\partial}{\partial x}(K_x \frac{\partial H}{\partial x}) + \frac{\partial}{\partial y}(K_y \frac{\partial H}{\partial y}) + Q = \frac{\partial \theta}{\partial t}
$$
 ......(4)

In which;

Q: the specified inflow/outflow rate or discharge  $(L^3/T)$ .

θ: the water volume content which represents the fraction of the total volume of soil that is occupied by the water contained in the soil $(L^3)$ .

#### t: the time (T).

If the coefficients of hydraulic conductivity are assumed to be independent of  $X$  and  $Y$ , that is if the region is assumed to be homogeneous. For steady flow, and setting Q equal to zero, then Eq.(4) transforms to the following form:

$$
K_x \frac{\partial^2 H}{\partial x^2} + K_y \frac{\partial^2 H}{\partial y^2} = 0 \qquad \qquad \dots (5)
$$

The basic concept of the finite element method is to divide the problem region(flow domain) into sub-domains(finite elements)connected at their common nodal points and that the unknown function of the field variable is defined approximately within each element. The approximate solution of each element expressed by continuous function. A spatial discretization of Eq.(5) within permeable foundation of dam structure, was applied with a **Galerkin** type finite element method [13].

The unknown field variable ( $H$ ) through the solution domain is approximated by  $H^e$ :

$$
H^{e}(x, y) = \sum_{i=1}^{N} N_{i}(x, y)H_{i}
$$
...(6)

In which;

*N<sup>i</sup>* : shape function associated with node *i*.

*H<sup>i</sup>* : unknown potential at node *i*.

*N* : total number of nodes.

The approximate solution for total head variation,  $H(x,y)$ , over the whole domain is specified as:

$$
H(x, y) = \sum_{e=1}^{n_e} H^e = \sum_{e=1}^{n_e} \sum_{i=1}^{N} N_i(x, y) H_i
$$
 ....(7)

In which;

 $n_e$  :total number of elements in the flow domain.

Substituting equation(7) into governing equation(5) yields:

$$
\left[K_x \frac{\partial^2}{\partial x^2} \sum_{i=1}^N N_i H_i\right] + \left[K_y \frac{\partial^2}{\partial y^2} \sum_{i=1}^N N_i H_i\right] = R^e \neq 0 \qquad \dots (8)
$$

in which  $R^e$  is the elemental residual.

There are more different approaches to formulate the approximate solution of this problem. In the present work, the weighted residual method with **Galerkin's** criterion is used:

$$
\int_{A^e} W_j R^e dA = 0 \qquad \qquad \dots (9)
$$

by minimizing or maintains residuals small at all points of the domain, in order to achieve this, Eq.(8) should be integrated on the problem domain after weighting by a certain function and should equal zero as follows:

$$
\sum_{1}^{ne} \int_{A^e} W_j R^e dA = 0 \qquad \qquad \dots (10)
$$

In which; *W<sup>j</sup>* : the weighting function.

Based on **Galerkin's** technique, the weighted function is taken equal to the shape function(i.e., W<sub>i</sub>  $=N_i$ ). Applying this principle in finite element method over the whole problem domain, Eq.(10) can be written as:

$$
\sum_{1}^{N_E} \left[ \int_{A^e} N_j^e \left[ K_x \frac{\partial^2}{\partial x^2} \sum_{i=1}^N N_i H_i + K_y \frac{\partial^2}{\partial y^2} \sum_{i=1}^N N_i H_i \right] dA \right] = 0 \qquad \dots (11)
$$

In which:  $dA=dx\times dy$ ; ( $j=1,2,3,...,N$ ); N =number of nodes per element, and  $N_E$  = total number of elements all over the flow domain.

By applying integration by parts with **Green's** theorem to the second order derivatives terms, the continuity requirements for the shape function, (N), from  $(C^1$ -continuity) to  $(C^2$ -continuity) were reduced. Whereas ( $C^1$  &  $C^2$ ) are the continuity for the shape function for the first and zero stage, respectively(Senda, 2003). The second derivative terms in Eq.(11) can be substituted by applying the product rule for differentiation as follows:

$$
N_j^e \frac{\partial^2 H}{\partial x^2} = \frac{\partial}{\partial x} \left( N_j^e \frac{\partial H}{\partial x} \right) - \frac{\partial N_j^e}{\partial x} \frac{\partial H}{\partial x}
$$
 ......(12)

Accordingly, the first term of Eq.(11) will be:

$$
\int_{A^e} N_j^e \left( K_x \frac{\partial^2}{\partial x^2} \sum_{i=1}^N N_i H_i \right) dA = \int_{A^e} K_x \left[ \left( \frac{\partial}{\partial x} N_j^e \frac{\partial}{\partial x} \sum_{i=1}^N N_i H_i \right) - \frac{\partial N_j^e}{\partial x} \frac{\partial}{\partial x} \sum_{i=1}^N N_i H_i \right] dA
$$
...(13)

Applying the *Green's* theorem to the first integral in right hand side of Eq.(13) by performing the integration around element boundary yields:

$$
\int_{A^e} K_x \left( \frac{\partial}{\partial x} N_j^e \frac{\partial}{\partial x} \sum_{i=1}^N N_i H_i \right) dA = \int_{\Omega^e} N_j^e K_x \frac{\partial}{\partial x} \sum_{i=1}^N N_i H_i dy \qquad \qquad \dots (14)
$$

Substituting Eq.(14) into Eq.(13) gives:

$$
\int_{A^e} K_x \left( \frac{\partial}{\partial x} N_j^e \frac{\partial}{\partial x} \sum_{i=1}^N N_i H_i \right) dA = \int_{\Omega^e} N_j^e K_x \frac{\partial}{\partial x} \sum_{i=1}^N N_i H_i dy - \int_{A^e} \frac{\partial N_j^e}{\partial x} K_x \frac{\partial}{\partial x} \sum_{i=1}^N N_i H_i dA
$$
...(15)

The second term of Eq.  $(11)$  will be:

$$
\int_{A^e} N_j^e \left( K_y \frac{\partial^2}{\partial y^2} \sum_{i=1}^N N_i H_i \right) dA = \int_{A^e} K_y \left[ \left( \frac{\partial}{\partial y} N_j^e \frac{\partial}{\partial y} \sum_{i=1}^N N_i H_i \right) - \frac{\partial N_j^e}{\partial y} \frac{\partial}{\partial y} \sum_{i=1}^N N_i H_i \right] dA
$$
...(16)

A similar procedures presented by Eqs.(14and 15) were adopted as follows:

$$
\int_{A^e} K_y \left( \frac{\partial}{\partial y} N_j^e \frac{\partial}{\partial y} \sum_{i=1}^N N_i H_i \right) dA = \int_{\Omega^e} N_j^e K_y \frac{\partial}{\partial y} \sum_{i=1}^N N_i H_i dx \qquad \qquad \dots \tag{17}
$$

Substituting Eq.(17) into Eq.(16) gives:

$$
\int_{A^e} N_j^e \left( K_y \frac{\partial^2}{\partial y^2} \sum_{i=1}^N N_i H_i \right) dA = \int_{\Omega^e} N_j^e K_y \frac{\partial}{\partial y} \sum_{i=1}^N N_i H_i dx - \int_{A^e} \frac{\partial N_j^e}{\partial y} K_y \frac{\partial}{\partial y} \sum_{i=1}^N N_i H_i dA
$$
...(18)

Substituting Eqs. $(15)$  and  $(18)$  in Eq. $(11)$  results in:

$$
\sum_{1}^{N_{E}} \left[ \int_{A^{e}} \left( \frac{\partial N_{j}^{e}}{\partial x} K_{x} \sum_{i=1}^{N} N_{i} H_{i} + \frac{\partial N_{j}^{e}}{\partial y} K_{y} \sum_{i=1}^{N} N_{i} H_{i} \right) dxdy - \int_{\Omega^{e}} \left( N_{j}^{e} K_{x} \frac{\partial}{\partial x} \sum_{i=1}^{N} N_{i} H_{i} \ell_{x} + N_{j}^{e} K_{y} \frac{\partial}{\partial y} \sum_{i=1}^{N} N_{i} H_{i} \ell_{y} \right) d\Omega \right] = 0 \quad ....(19)
$$

In which;

 $\Omega^e$ : represent the surface boundaries of the element (*i*).

$$
dx = \ell_x d\Omega
$$

$$
dy = \ell_y d\Omega
$$

And in matrix form Eq. (19) can be written as:

$$
\sum_{1}^{N_E} [K^e][H] = 0 \qquad \qquad \dots (20)
$$

In which;

 $[K<sup>e</sup>]$ : represent the element stiffness matrix and knowing as:

$$
[Ke] = \int_{Ae} [Be]T [De][Be] dxdy
$$
...(21)  
In which:

In which;

$$
\sum_{i}\left[\int_{A^{e}}\left(\frac{x_{i}+y_{i}}{\partial x}K_{x}\sum_{i=1}^{N_{i}}N_{i}H_{i} + \frac{x_{i}+y_{i}}{\partial y}K_{y}\sum_{i=1}^{N_{i}}N_{i}H_{i}\right)dx dy - \int_{\Omega^{e}}\left(N_{j}^{e}K_{x}\frac{\partial}{\partial x}\sum_{i=1}^{N_{i}}N_{i}H_{i} \ell_{x} + N_{j}^{e}K_{y}\frac{\partial}{\partial y}\sum_{i=1}^{N_{i}}N_{i}H_{i} \ell_{y}\right)d\Omega\right] = 0 \quad .....(19)
$$
\nIn which;  
\n $\Omega^{e}$ : represent the surface boundaries of the element (*i*).  
\n $dx = \ell_{x}d\Omega$   
\n $dy = \ell_{y}d\Omega$   
\nAnd in matrix form Eq. (19) can be written as:  
\n $\sum_{i=1}^{N_{e}}[K^{e}][H] = 0 \qquad .....(20)$   
\n $\frac{1}{N_{e}}$   
\n $[K^{e}] = \int_{i}^{N_{e}}[B^{e}]^{T}[D^{e}][B^{e}]dxdy \qquad .....(21)$   
\n $[K^{e}] = \int_{i}^{N_{e}}[B^{e}]^{T}[D^{e}][B^{e}]dxdy \qquad .....(21)$   
\nIn which;  
\n $[B^{e}] = \begin{bmatrix} \frac{\partial N^{e}}{\partial x} & \frac{\partial N^{e}N_{e}}{\partial x} \\ \frac{\partial N^{e}}{\partial y} & \frac{\partial N^{e}N_{e}}{\partial y} \end{bmatrix}$   
\n $N^{e}$ : the interpolation or shape function of element *e*.  
\nIn this study, two-dimensional quadratic isoparametric elements were used  
\nas shown in figure (2). The interpolation function for such elements can be  
\n $N_{i} = a_{i} + b_{i}x + c_{i}y + d_{i}xy$   
\n $N^{e}$ : the derivations of the above functions are shown in Eq. [22].  
\n $[D^{e}] = \begin{bmatrix} K_{x} & 0 \\ 0 & K_{y} \end{bmatrix}$   
\n $[D^{e}] = \begin{bmatrix} K_{y} & 0 \\ 0 & K_{z} \end{bmatrix}$   
\n $[D^{e}] = \begin$ 

*N e* : the interpolation or shape function of element *e*.

In this study, two-dimensional quadratic isoparametric elements were used which have four nodes, as shown in figure (2). The interpolation function for such elements can be expressed as:

$$
N_i = a_i + b_i x + c_i y + d_i xy
$$
...(23)  
When a, b, c, and d, represents unknown constants corresponding to the element's (c) four

Where  $a_i$ ,  $b_i$ ,  $c_i$ , and  $d_i$  represents unknown constants corresponding to the element's  $(e)$  four nodes (*i*).

The derivations of the above functions are shown in Eq.[22].

$$
[De] = \begin{bmatrix} K_x & 0 \\ 0 & K_y \end{bmatrix} \tag{24}
$$

the formulation of seepage phenomenon by using finite element method produces a sets of linear equations. From assemblage :

$$
[K]_{N \times N} \{H\}_{N \times 1} = 0
$$
 ....(25)  
In which  $[K] = \sum_{1}^{N_E} K^e$ , and  $\{H\}_{N \times I}$  is unknown vector with N rows.



**Figure(2): A quadratic isoparametric element**

Computation of the hydraulic gradient directly performed, and the gradient vector is defined as:

$$
\begin{Bmatrix}\n\frac{\partial H}{\partial x} \\
\frac{\partial H}{\partial y}\n\end{Bmatrix} = \begin{bmatrix}\n\frac{\partial}{\partial x}[N] \\
\frac{\partial}{\partial y}[N]\n\end{bmatrix} \{H^e\} = [B] \{H^e\} \quad \dots (26)
$$

The boundary conditions should be specified before starting the solution. For the case considered in this study, a steady state of a confined flow, The bottom boundary of the dam structure was treated as no flow boundary. All the nodes along the U/S&/D/S that lie below the water level were treated as a reservoir boundaries. Generally the following conditions were concerned in the present study:

**A-** reservoir boundaries  $\Psi_1$ , and  $\Psi_2$ : since the water depth above such boundaries are known  $(H_t)$ , and (*Hds*) respectively; so, the pressure head distribution on these boundaries would be constant i.e., Equipotential lines, at these boundaries:

$$
H = \frac{P_{\gamma}}{\gamma_w} + Z \tag{27}
$$

**B-** impermeable boundaries  $\Phi_1$ ,  $\Phi_2$ , and  $\Phi_3$ : in which the velocity component normal to the boundary is equal to zero, i.e. these boundaries represents stream lines of constant stream function, at these boundaries:

$$
\frac{\partial H}{\partial n} = 0 \tag{28}
$$

In which; n is the perpendicular direction to this boundary. The boundary conditions are defined as shown in figure(3).



**Figure(3): Boundary Conditions for Application problem**

#### **4.Solution and Validation of Model**

A finite element code was developed by utilizing the power of standard of FORTRAN 90 language, the compiler used is the FORTRAN Power Station-Microsoft Developer Studio-94. A major modification conducted to the computer program [5] to solve seepage problem beneath dam structure in the presence of inclined cutoff, this modification was presented by generation of unstructured quadratic meshes to simulate the flow domain beneath dam and around inclined cutoff boundaries, these meshes are suitable for subsequent finite element method. Here coarse mesh of idealization comprising of quadrilateral elements is employed. 180 quadrilateral elements interconnected through 306 nodes. The flow domain should extend laterally on both sides of the dam structure and vertically to the impervious layer to a distance in which the maximum value of exit gradient that will be modeled smaller than I<sub>Critical</sub>. Often these values will not be known until the model is complete. But data must be collected in order to complete the model. Therefore, the lateral extent must be estimated for the data gathering effort. In some cases, it may be appropriate to set up a preliminary model based on limited data. The lateral extent can be estimated from other previous studies of the problem.

**Khassaf** *et al***.[2]** considered a general case study with lateral extent for each side twice the dam length(BS), and the vertical extent is(nBS). In the present work this assumptions were adopted as following:

**A**- in the analysis it is assumed that the lateral extent of  $(L_{\text{d/s}}=2L_{\text{d}})$  beyond the toe of dam, and (**Lu/s=2Ld**) to the heel of dam, represents the indefinite lateral extent of the soil strata adequately.

**B-** the vertical extent of flow domain beneath dam structure which represent the effective depth of (**3Ld**) to be thought-out sufficiently.

Preliminary tests performed for the this study and the results with analytic solution from the literature. Analytical solution for seepage beneath an impervious dam with inclined sheetpile has been presented by **Ijam** [3], the solution of this particular case (see figure (4)) was derived exactly by using the Schwarz-Christoffel transformation.



**Figure(4): Problem Layout(After Ijam [3])**

The pressure distribution equation along the solid boundary of the dam was given by:

$$
\frac{P_r}{\gamma_w} = H_1 + y - \frac{H}{2} - \frac{H}{\pi} Sin^{-1} \left[ \frac{(2t - t_B - t_F)}{(t_F - t_B)} \right] \tag{29}
$$
\nIn which;

 $P_r$ : the pressure (F/ $L^2$ ).

 $\gamma_w$ : unit weight of water(F/L<sup>3</sup>).

$$
t = 0.5(t_F - t_B)Cos\left[\frac{\pi w}{KH}\right] + \frac{(t_F + t_B)}{2}
$$
 ....(30)

In which;

$$
w = \frac{KH}{\pi} \sin^{-1} \left[ \frac{(2t - t_B - t_F)}{(t_F - t_B)} \right] - \frac{KH}{2}
$$
 ....(31)

Where,  $K$  is the coefficient of permeability  $(L/T)$ .

To find the pressure, it is necessary to find *t* corresponding to a given *z* (complex potential) along the solid boundary of the dam.

Table (1) indicates the absolute percentage error (*AE* %) which used to evaluate the performance of present model in compare with exact one. It was defined as:

$$
AE\% = \left| \frac{h_E - H_m}{h_E} \right| \times 100 \tag{32}
$$

where  $(H_m)$  and  $(h_E)$  are present model and exact solution values, respectively.

Furthermore, several statistical methods, relative root-mean-squared error, *RMSE*, and the coefficient of determination,  $R^2$ , are used to compare predicted and testing values for computing the model validation. The smaller  $AE$ % , *RMSE* and the larger  $R^2$  are indicative of better performance generality. The *RMSE* and  $R^2$  are defined by using the following equations[14]:

$$
RMSE = \sqrt{\frac{n_t \sum_{i=1}^{n_t} (h_{E_i} - H_{m_i})^2}{(n_t - 1) \sum_{i=1}^{n_t} (h_{E_i})^2}}
$$
...(33)

$$
R^{2} = 1 - \left(\frac{\sum_{i=1}^{n_{t}}(h_{E_{i}} - H_{m_{i}})^{2}}{\sum_{i=1}^{n_{t}}H_{m_{i}}^{2}}\right)
$$

 $....(34)$ 

Where  $(n_t)$  is the number of testing samples.



Table(1): Statistical values for computing present work validity compare with analytical solution

#### **5.Result and Discussion**

The numerical results show that the using inclined cutoff is important in increasing the safety factor against the piping phenomenon.

#### **5.1 Effect of inclined cutoff position (** $X_m/L_d$ **) on the uplift head ratio(** $H_m/H_t$ **)**

As shown in figure(5), as the cutoff is in U/S part of dam, high values of uplift head ratio are developed when the cutoff inclined in the direction of U/S side ( $\beta$  less than 90°). The uplift head ratio(H<sub>m</sub>/H<sub>t</sub>) decreases as( $\beta$ ) increases toward the dam toe,( $\beta \ge 90^{\degree}$ ) until the angle of inclination almost equal (120°), then the uplift head ratio starts increasing with ( $\beta$ ).



**Figure(5):** Uplift head ratio through dam foundation vs. a range of  $(\beta)$  values for cutoff in U/S **part of dam**

From figure(6), when the cutoff is in mid-base part of dam structure, the uplift head ratio are higher than those of vertical cutoff ( $\beta = 90^{\circ}$ ). For distance of ( $X_m/L_d \approx 0.5$ ) D/S the dam toe, rapid decreasing in the uplift head ratio occurs along the dam base for various values of inclination angle( $\beta$ ), for example, when the cutoff inclined by angle of ( $\beta=120^{\degree}$ ), the uplift head ratio declined from 0.7 to 0.13. As cutoff positioned in D/S part of dam structure as shown figure(7), the uplift reduced strongly if this cutoff was inclined toward the D/S side. For cutoff with  $\beta$ <90°, the uplift head ratio decreased by (11.4 %) when  $\beta$  inclined from 30° to 90°.



**Figure(6): Uplift head ratio through dam foundation vs. a range of () values for cutoff in mid base part of dam** 



Figure(7): Uplift head ratio through dam foundation vs. a range of ( $\beta$ ) values for cutoff in D/S **part of dam** 

#### **5.2 Effect of inclined cutoff position (** $X_m/L_d$ **) on the exit gradient(** $I_{exit}$ **)**

The exit gradient was studied at the end of the dam structure for the cases defined previously, and the results are represented graphically as shown in figures below. When the cutoff is at U/S part of dam structure as shown in figure(8), large values for exit gradient are developed if the cutoff inclined toward the U/S direction( $\beta$  is less than 90°).



**Figure(8): Exit gradient vs. a range of () values for cutoff in U/S part of dam** 

As the cutoff located in the in mid-base part of dam structure as shown in figure (9), the same trends will come out almost as for U/S cutoff noticed above. Figure (10) shows that exit gradient in the D/S part of dam structure decreases as  $(\beta)$  increases toward the D/S side  $(\beta \ge 90)$ until the angle of cutoff inclination roughly approaches to  $(120^{\degree})$ , beyond this value greater exit gradient starts developed with increasing  $(\beta)$ .



**Figure(9): Exit gradient vs. a range of () values for cutoff in mid-base part of dam**



**Figure**(10): **Exit gradient vs. a range of**  $(\beta)$  **values for cutoff D/S part of dam** 

#### **5.3 Effect of inclined cutoff position on the seepage quantity behind dam**

The variation of seepage quantity behind the dam structure as the cutoff located at U/S part of dam, was shown in figure(11), the seepage decreases while  $(\beta)$  increases, and the least quantity of seepage occurred when ( $\beta$ ) value around ( $60^{\degree}$ ), then the seepage increases rapidly for ( $\beta \ge 90^{\degree}$ ).



Figure(11): Seepage quantity vs. a range of ( $\beta$ ) values for cutoff in U/S part of dam

Figure (12) shows that seepage behind the dam was decreases as  $(\beta)$  inclined toward U/S side, and the value of seepage quantity when  $(\beta)$  inclined to the direction of D/S part of dam was greater than those of  $\beta = 90^{\degree}$ . As the cutoff located in D/S part of dam (see figure 13), the seepage quantity decreases slightly and reaches to minimum value when  $(\beta)$  approximately (120<sup>°</sup>).



**Figure(12): Seepage quantity vs. a range of () for cutoff in mid-base part of dam** 



**Figure(13): Seepage quantity vs. a range of () for cutoff in D/S part of dam**

Throughout the model of this study, the pressure head ratio( *t m H H* ) was found to be influenced by a

several variables. A general non-dimensional equation can be written as:

$$
\frac{H_m}{H_i} = \Phi(\beta, \frac{X_m}{L_d})
$$
\n(35)

A multiple nonlinear regression analysis using Data-Fit Version 9.0 Engineering Software to correlate the values of( *t m H H* ) to geometric parameters yields following equation fits the data with

coefficient of multiple determination (R<sup>2</sup>=0.8333) and the value of standard error of estimate (SEE=<br>0.13).<br> $\frac{H_m}{H} = 0.6912 - 1.2920 E - 3 \times \pi \beta \cos(4.2795 \frac{X_m}{L})$  ....(36) 0.13).

$$
\frac{H_{m}}{H_{t}} = 0.6912 - 1.2920 E - 3 \times \pi \beta Cos(4.2795 \frac{X_{m}}{L_{d}})
$$
 ......(36)

#### **6. Conclusions**

- 1.The exit gradient downstream of an inclined cutoff wall is in general causes a considerable reduction in exit gradient along the end base of structure. This increases the safety against soil disturbance and changes the danger of piping further downstream from the toe point of the dam.
- 2.The uplift pressure head is decreased when the inclination of the cutoff wall is towards the downstream part of the dam.
- 3. The location of inclined cutoff in upstream side of dam at all values of  $(\beta)$  or in downstream side of dam with inclination angle toward upstream part was improper situation which involves unstable soil condition in the dam toe.

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