

Redesign AlNajaf City Traffic Using Graph Theory

إعادة تصميم المخطط المروري لمدينة النجف الاشراف باستخدام نظرية البيان

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بحث مستل

Abstract:

Traffic flow and tours represent one of the most important issues in what is known as city planning since their results show how the main street, hi ways, and intersections look like and how they are connected to each other to give the maximum performance and traffic flow during the different time intervals including the rush hours.

In this paper we present a traffic model for AlNajaf City based on graph theory, Minimum Spanning Tree, and Shortest Path Algorithms. The model shows the best network paths and alternative tours for the traffic flow in the main streets and intersections in different rush hours. Different tools and software were used in the implementation of the proposed model, including MatLab(Matrix-Laboratory) , AutoCad, and others.

Keywords: *Traffic flow, Graph, Minimum Spanning Tree, Shortest Path, TSP.*

الخلاصة :

يمثل تدفق حركة المرور ومساراتها واحدة من أهم القضايا فيما يعرف باسم تخطيط المدن لان نتائجها تظهر كيف تبدو شوارع المدن الرئيسية وتقاطعاتها وكيف أنها ترتبط مع بعضها البعض لإعطاء أفضل قدر من الأداء وتدفق حركة المرور خلال فترات زمنية مختلفة بما في ذلك ساعات الذروة . تقدم هذه الورقة نموذجاً مقترحاً لحركة المرور لمدينة النجف الاشراف على أساس نظرية البيان، خوارزميات شجرة الامتداد ذات الحد الأدنى وخوارزميات أقصر مسار . يعطي النموذج المقترح أفضل مسارات شبكة الطرق والمسارات البديله لتدفق حركة المرور في الشوارع الرئيسية والتقاطعات في ساعات الذروة المختلفة . وقد استخدمت أدوات وبرامج مختلفة في تنفيذ النموذج المقترح ، بما في ذلك ماتلاب، أو اتوكاد وغيرها

I. Introduction

Graphs and graph theory are widely used in different applications, one of which is the city planning. This represents the most important applied concept of graph theory[1,2]. Minimum Spanning Trees, shortest path, Travelling Sales Person Problem, Maximum and Minimum flow are the critical concepts used in city planning. Graph theory applications can be listed and not limited to the following[2,3,4]:

1. Network Design
2. Route inspection problem
3. Travelling Sales Person Problem
4. Network flow
5. Maximum and Minimum flow
6. Networks of communication
7. Data organization
8. Computational devices
9. Flow of computation
10. Websites structures
11. Linguistics and Natural Languages
12. Study molecules in chemistry and Physics

13. Sociology, Biology, mathematics and others

I.1.Literature Review

The following are some related works in using graph theory for traffic flow control and city planning:

- 1- ZiyangLv, YanruCai, Jixin Chen, and Lurong Wu suggested an intelligent control for the traffic lights using graph theory, where traffic lights in intersections can be controlled depending on the flow in different directions [3].
- 2- Elvin J Moore, "Maximum flow in road networks with speed-dependent capacities application to Bangkok traffic", where traffic flow problem is studied, in which edge weights represent road capacities (maximum vehicles per hour) that are functions of the traffic speed (km/hr) and traffic density (vehicles per kilometer)"[4].
- 3- Manuel Appert and Laurent Chapelon, "Measuring urban road network vulnerability using graph theory : the case of Montpellier's road network", where the authors postulate that the network morphology, structure and level of congestion can be influencing factors. Two vulnerability indices which pinpoint accessibility loss in the city by removing links and vertices one by one, have been developed to assess the network's vulnerability" [5].
- 4- Mees, P. "A centenary review of transport planning in Canberra, Australia". The paper presents a study of specific type of transportation planning in the city of Canberra[6].
- 5- Jaworski, P., Edwards, T. , Moore, J. and Burnham, K."Cloud computing concept for Intelligent Transportation Systems". The paper presents a method of deploying cloud computing in designing and implementing an intelligent transportation system [7].
- 6- Liu Xi, Gong Li, Gong Yongxi, and Liu Yu "Revealing daily travel patterns and city structure with taxi trip data". The authors in this papers showed that data collected from taxi trip can be used to extract the knowledge of travel patterns that can be used in optimizing transportation system yield [8].
- 7- Tizghadam, A. "A graph theoretical approach to traffic engineering and network control problem". This paper looks at the problem of traffic engineering and network control from a new perspective. A graph-theoretical metric, betweenness, in combination with a network weight matrix is used to characterize the robustness of a network. Theoretical results lead to a definition of "criticality" for nodes and links. It is shown that this quantity is a global network quantity and depends on the weight matrix of the graph"[9].

II. Graph definition

A graph is an ordered pair $G = (V, E)$ comprising a set of vertices or nodes V together with a set of edges or lines E , which are 2-element subsets of V (i.e., an edge is related with two vertices).

The vertices belonging to an edge are called the ends, endpoints, or end vertices of the edge. A vertex may exist in a graph and not belong to an edge [1,2,5].For an edge $\{u, v\}$, graph theorists usually use the somewhat shorter notation uv . There are many types of graphs, some of which are:

1. Directed graph
2. Undirected graph
3. Weighted graph and unweighted graph
4. And many others

Graph can be represented in different ways such as:

1. Adjacency matrix

A two-dimensional matrix, in which the rows represent source vertices and columns represent destination vertices. Data on edges and vertices must be stored externally. Only the cost for one edge can be stored between each pair of vertices.

2. Incidence matrix

Atwo-dimensional Boolean matrix, in which the rows represent the vertices and columns represent the edges.The entries indicate whether the vertex at a row is incident to the edge at a column.

3. Adjacency list

Vertices are stored as records or objects, and every vertex stores a list of adjacent vertices. This data structure allows the storage of additional data on the vertices.

4. Incidence list

Vertices and edges are stored as records or objects. Each vertex stores its incident edges, and each edge stores its incident vertices. This data structure allows the storage of additional data on vertices and edges.

III. Problem Statement:

Solving the problem of traffic flow in AlNajaf city by applying graph theory such as Shortest Path, Travelling Sales Person Problem(TSP) (The travelling salesman problem or travelling salesperson problem asks the following question: Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?), Minimum Spanning Tree(MST) where a spanning tree of a graph is just a subgraph that contains all the vertices and is a tree, and the MST is a spanning tree with minimum weight, Minimum and Maximum flow and Graph Mining techniques to find out the best routes during the city rush hours. The city map is first converted into a graph where each intersection is represented by a vertex and the roads and streets were represented by the edges connecting the vertices.

IV. Proposed Model Methodology

1. Converting all the streets and intersections in AlNajaf City into a weighted graph (in both directed and undirected)
2. Allocating of the weights (Distances between intersections) according to the scaled map of the city.
3. Applying different graph algorithms including (MST, Shortest path, TSP and others) on the original city graph to find out which path to follow during the rush hours and the best vertices set connecting the city graph.
4. Different cut locations were studied and step 3 is repeated to get different route alternative The time schedule for the research project is show in figure(1).

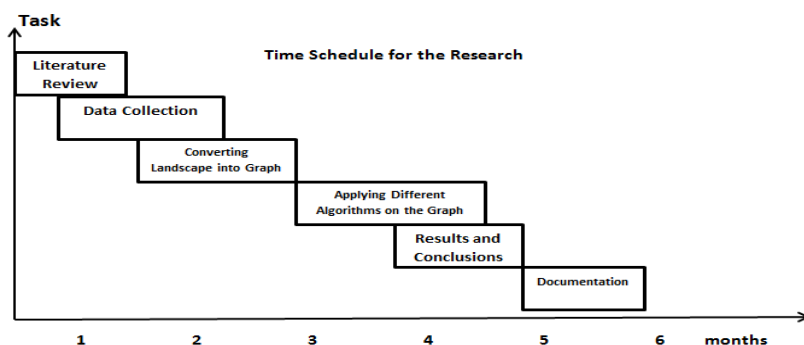


Figure (1). Time Schedule for the Research Project.

IV.1. Minimum Spanning Tree Algorithms

1. Prim's algorithm [1]

The algorithm starts with a tree consisting of a single vertex, and continuously increases its size one edge at a time, until it spans all vertices.

- Input:

A non-empty connected weighted graph with vertices V and edges E

- Initialize:

$V_{new} = \{x\}$, where x is an arbitrary node from $V, E_{new} = \{\}$

- Repeat until $V_{\text{new}} = V$:
 - Choose an edge $\{u, v\}$ with minimal weight such that u is in V_{new} and v is not (if there are multiple edges with the same weight, any of them may be picked)
 - Add v to V_{new} , and $\{u, v\}$ to E_{new}
- Output:
 V_{new} and E_{new} describe a minimal spanning tree

The time complexity for Prim's algorithm with adjacency matrix representation of the graph is given by $O(|V|^2)$, whereas a binary heap and adjacency list representation have time complexity $O((|V| + |E|) \log |V|) = O(|E| \log |V|)$.

2. Kruskal Algorithm [1]

KRUSKAL(G):

```
1 A =  $\emptyset$ 
2 foreach  $v \in G.V$ :
3 MAKE-SET(v)
4 foreach (u, v) ordered by weight(u, v), increasing:
5 if FIND-SET(u)  $\neq$  FIND-SET(v):
6 A = A  $\cup$  {(u, v)}
7 UNION(u, v)
8 return A
```

The time complexity is given by $O(E \log V)$

IV.2. Dijkstra's Algorithm (1)

The shortest path from one vertex to another

```
1 S := empty sequence
2 u := target
3 while previous[u] is defined:
4 insert u at the beginning of S
5 u := previous[u]
6 end while ;
```

IV.3. Dijkstra's Algorithm (2)

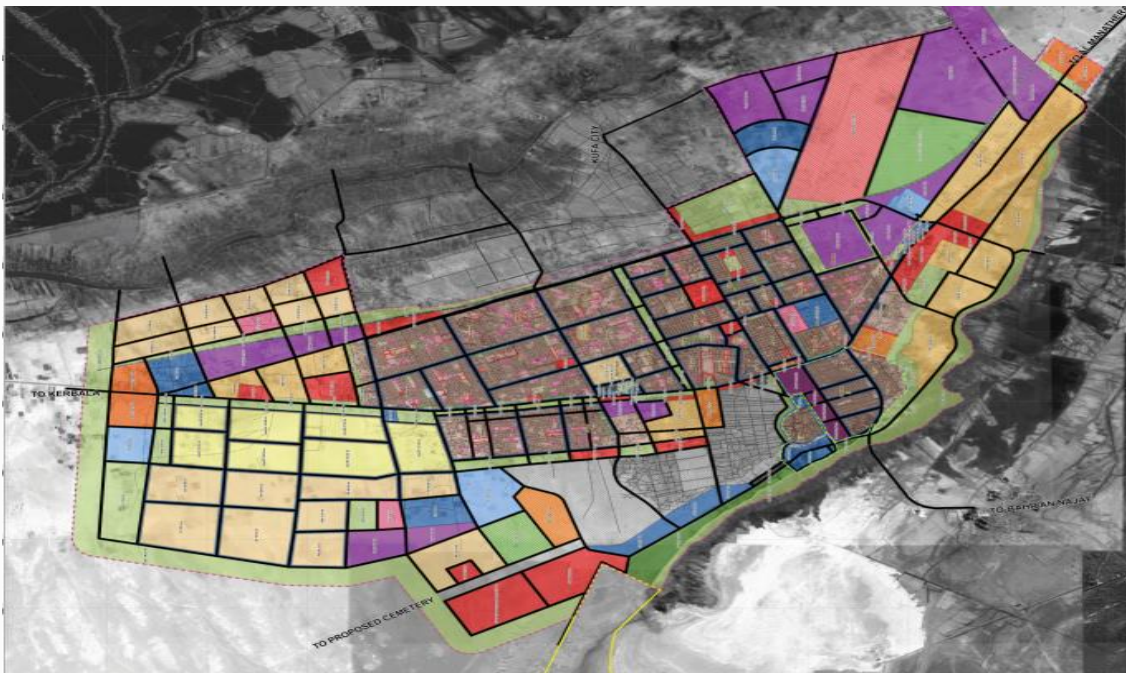
```
1 function Dijkstra(Graph, source):
2 for each vertex v in Graph:
3 dist[v] := infinity ;
4
5 previous[v] := undefined ;
6 end for
7
8 dist[source] := 0 ;
9 Q := the set of all nodes in Graph ;
10
11 while Q is not empty:
12 u := vertex in Q with smallest distance in dist[] ;
13 remove u from Q ;
14 if dist[u] = infinity:
15 break ;
16 end if
17
18 for each neighbor v of u:
```

```
19
20 alt := dist[u] + dist_between(u, v) ;
21 if alt < dist[v]:
22 dist[v] := alt ;
23 previous[v] := u ;
24 decrease-key v in Q;
25 end if
26 end for
27 end while
28 return dist;
29 end function
```

The running time is given by $O(|E|+|V|^2) = O(|V|^2)$.

V. Results

Figure (2) shows the basic landscape of AlNajaf city as approved by the official authority, this landscape is first converted into a graph where the intersections are represented by vertices and the roads and streets are represented by the Graph Edges. All the weights (Distances) were allocated on each edge as shown in figure(3) with the distances shown in table(I).



Figure(2). AlNajaf City landscape.

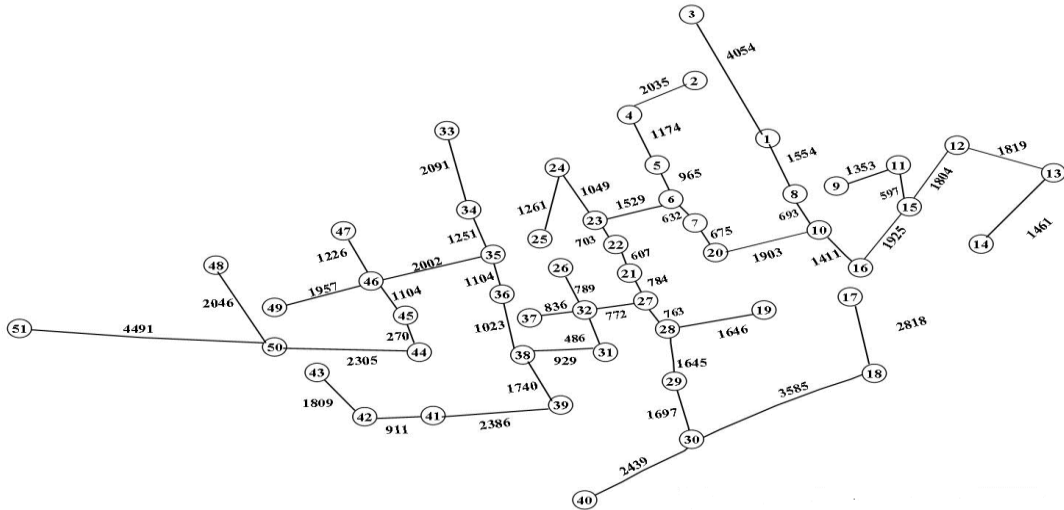


Figure(3). Graph Representation of AlNajaf Landscape

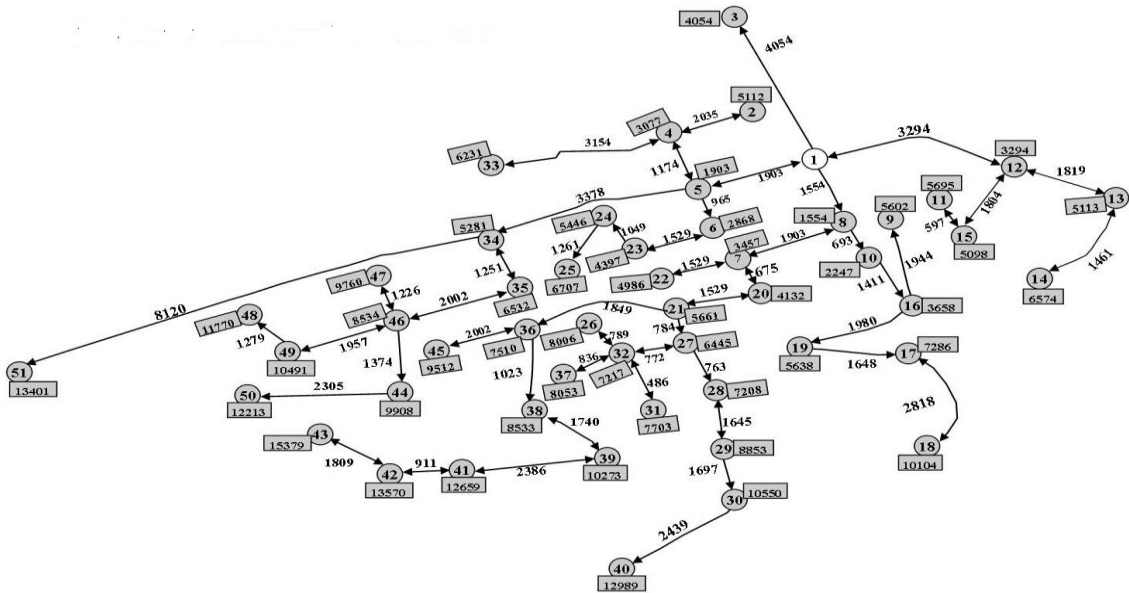
Table (I) Samples of the Distances Between Different Nodes (Intersections)of AlNajaf City Graph.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0	0	4054	0	1903	0	0	1554	0	0	0	3294	0	0	0	0	0	0	0	0
2	1388	0	0	2035	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	2666	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	1174	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1903	0	0	1174	0	965	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	632	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	1597	0	0	1903	0	0	0	0	0	0	0	0	0	0	0	675
8	0	0	0	0	0	0	1903	0	693	0	0	0	0	0	0	0	0	0	0	0
9	1714	0	0	0	0	0	0	0	0	1353	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1411	0	0	0	1903
11	0	0	0	0	0	0	0	0	1353	0	0	0	0	0	597	0	0	0	0	0
12	3294	0	0	0	0	0	0	0	0	0	0	0	1819	0	1804	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	1819	0	1461	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	1461	0	1878	0	0	4417	0	0
15	0	0	0	0	0	0	0	0	0	597	1804	0	1878	0	1925	0	0	0	0	0
16	0	0	0	0	0	0	0	0	1944	0	0	0	0	1925	0	0	0	1980	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	303	0	2818	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	4417	0	0	2818	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1648	0	0	1679	0
20	0	0	0	0	0	0	675	0	1903	0	0	0	0	0	0	0	0	1679	0	0

One of the most powerful technique in city planning technique is the use of graph theory for designing network systems, one of which is connecting the city intersections with one traffic light controller, finding different alternative paths for traffic blocks, redesigning the city traffic flow and others. To achieve such applications, the graph representation of the city is processed using Minimum Spanning Tree and Shortest Path Techniques to get the best paths and tours. Figures(4) and (5) show the output for the best paths. The subtotal and the final distances are shown on the diagrams, and the distances with the routes are shown in Table(II)..



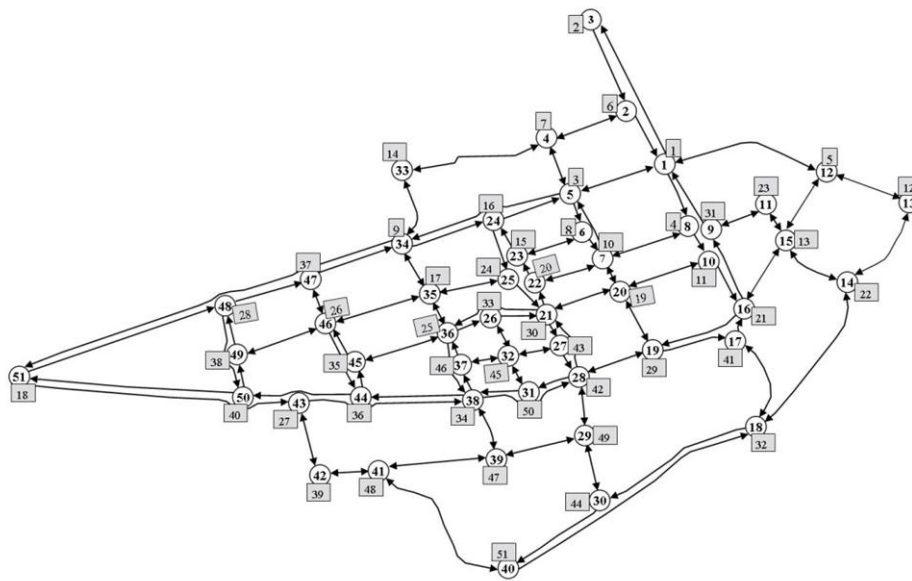
Figure(4). Minimum Spanning Tree for the Traffic Plan (with total distance 76109 m).



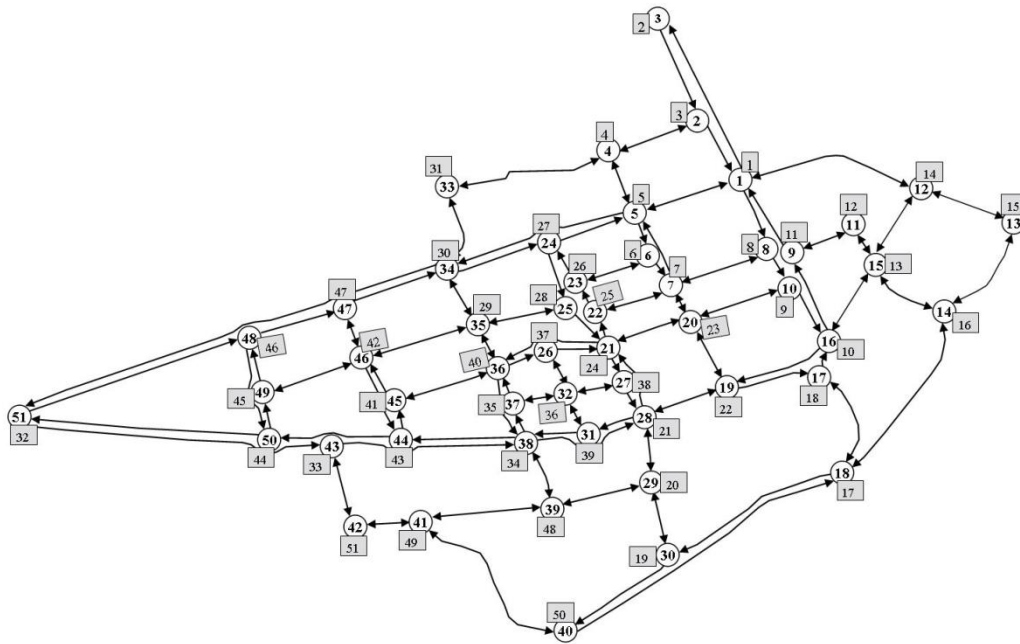
Figure(5). Shortest Path From Vertex (1) to all Other Vertices.

Table (II). Distances for shortest path from intersection (1) to all other intersection .

From	To	Distance	Path	From	To	Distance	Path
1	2	5112	1,5,4,2	1	27	6445	1,8,7,20,21,27
1	3	4054	1,3	1	28	7208	1,8,7,20,21,27,28
1	4	3077	1,5,4	1	29	8853	1,8,7,20,21,27,28,29
1	5	1903	1,5	1	30	10550	1,8,7,20,21,27,28,29,30
1	6	2868	1,5,6	1	31	7703	1,8,7,20,21,27,32,31
1	7	3457	1,8,7	1	32	7217	1,8,7,20,21,27,32
1	8	1554	1,8	1	33	6231	1,5,4,33
1	9	5602	1,8,10,16,9	1	34	5281	1,5,34
1	10	2247	1,8,10	1	35	6532	1,5,34,35
1	11	5695	1,12,15,11	1	36	7510	1,8,7,20,21,36
1	12	3294	1,12	1	37	8053	1,8,7,20,21,27,32,37
1	13	5113	1,12,13	1	38	8533	1,8,7,20,21,36,38
1	14	6574	1,12,13,14	1	39	10273	1,8,7,20,21,36,38,39
1	15	5098	1,12,15	1	40	12989	1,8,7,20,21,27,28,29,30,40
1	16	3658	1,8,10,16	1	41	12659	1,8,7,20,21,36,38,39,41
1	17	7286	1,8,10,16,19,17	1	42	13570	1,8,7,20,21,36,38,39,41,42
1	18	10104	1,8,10,16,19,17,18	1	43	15379	1,8,7,20,21,36,38,39,41,42,43
1	19	5638	1,8,10,16,19	1	44	9908	1,5,34,35,46,44
1	20	4132	1,8,7,20	1	45	9512	1,8,7,20,21,36,45
1	21	5661	1,8,7,20,21	1	46	8534	1,5,34,35,46
1	22	4986	1,8,7,22	1	47	9760	1,5,34,35,46,47
1	23	4397	1,5,6,23	1	48	11770	1,5,34,35,46,49,48
1	24	5446	1,5,6,23,24	1	49	10491	1,5,34,35,46,49
1	25	6707	1,5,6,23,24,25	1	50	12213	1,5,34,35,46,44,50
1	26	8006	1,8,7,20,21,27,32,26	1	51	13401	1,5,34,51



Figure(6). Breadth First Search Technique for AlNajaf City Graph Landscape.



Figure(7). Depth First Search Technique for AlNajaf City Graph Landscape.

One of the most important graph theory application in city planning is what is known as Depth First Search(DFS) and Breadth First Search(BFS), which can be used for searching or traversing a graph. Figures (6) and (7) show the results of applying these techniques on AlNajaf graph. The result of traversing is show in the following:

Breadth First Search (BFS)

Starting from a specific node(intersection) and visit the children of that node and then repeat that recursively. The output is given by the following sequence of intersections:

- 1
- 3 5 8 12 (The children of node 1)
- 2 4 6 34 7 10 13 15 (the children of nodes 3, 5, 8, and 12)
- 33 23 24 35 51 20 22 16 14 11
- 25 36 46 43 48 19 21 9 18
- 26 38 45 44 47 49 42 50 17 28 27 30
- 32 37 39 41 29 31 40

Depth First Search (DFS)

Starting from a specific node (1) and go through a specific path to visit the nodes until the last node in the path, then moving backward to a previous node and repeat recursively. The result is given by the following sequence of nodes (intersections):

- 1
- 3 2 5 6 7 8 10 16 9 11 15 12 13 14 18 17 30 29 28 19 20 21 22 23 24 25
- 35 34 33
- 51 43 38 37 32 26
- 27
- 31
- 36 45 46 44 50 49 48 47
- 39 41
- 40

VI. Conclusions

Graph theory can be expressed as one of the most crucial technique that is used in city planning in general and traffic flow in specific. The work in this paper aims to study the traffic flow in AlNajaf city throughout the use of graph theory such as Minimum Spanning Tree and Shortest Path Techniques. The diagrams in figure(2) shows AlNajaf city landscape, which has been converted into a weighted directed graph in figure(3), where the intersections are represented by vertices and the roads are represented by the graph edges. Figures(4) through (7) show the results of applying different graph theory algorithms.

VII. Discussion

All the weights shown on the graphs represent real distances. Applying Prim's algorithms to find the MST gives the tree shown in figure (4) with total distances 76109 m which can be used as a guide to redesign and construct the city traffic and to give a good estimation for the total cost required to connect the whole city. Figure(5) shows the shortest path from intersection (vertex 1) to all other vertices with the total shortest distance attached to each vertex from which alternative paths can be estimated in case of traffic blocks or any other cases. DFS and BFS were applied of AlNajaf graph which gave the results show in Figures(6) and (7).

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