Artin's characters table of the group ($Q_{2m}\times D_3$) and AC($Q_{2m}\times D_3$) when m is a prime number

جدول شواخص ارتن للزمرة $Q_{2m}\times D_3$ والتجزئة الدائرية $Q_{2m}\times D_3$ عندما m عدد اولي

Ass.Prof.NaseerRasoolMahmood Naba Hasoon Jaber
University of Krufa\College of Education for Girls
Department of Mathematics

1.Abstract

The main purpose of this paper is to find Artin's characters table of the group ($Q_{2m}\times D_3$)when m is a prime number, which is denoted by Ar($Q_{2m}\times D_3$) where Q_{2m} is denoted to Quaternion group and D_3 is the Dihedral group of order 6 .Moreover we have found the cyclic decomposition of Artin's cokernel AC($Q_{2m}\times D_3$) when m is a prime number .

الملخص:

الهدف الرئيسي للبحث هو ايجاد جدول شواخص ارتن للزمرة $Q_{2m}\times D_3$ عندما m عدد اولي, ويرمز له Ar($Q_{2m}\times D_3$) عندما Q_{2m} تمثل زمرة Quaternion و D_3 تمثل زمرة Dihedral من الرتبة 6 بالاضافة الى ايجاد التجزئة الدائرية للزمرة $Q_{2m}\times D_3$ عندما m عدد اولي .

2.Introduction

let G be a finite group ,two elements of G are said to be Γ-conjugate if the cyclic subgroups they generate are conjugate in G and this defines an equivalence relation on G and its classes are called Γ-classes.

let H be a subgroup of G and let ø be a class function on H,the induced class function on G is given by:

$$\emptyset'(g) = \frac{1}{|H|}\sum_{r\in G}\emptyset^\circ(rgr^{-1})\ \forall g \in G$$

when $ø^\circ$ is defined by:

$$\emptyset^\circ(h) = \begin{cases} \emptyset(h) & \text{if } h \in H \\ 0 & \text{if } h \notin H \end{cases}$$

∅ be a character of H ,then ø´ is a character of G and it is called the induced charater on G.all the characters of G induced from a principale Artin's character.

Let $\bar{R}$ (G) denotes an abelian group generated by Z-valued characters of G under the pointwise addition . Inside this group there is a subgroup generated by Artin's characters ,which will be denoted by T(G) the factor group $\frac{\bar{R}(G)}{T(G)}$ is called the Artin Cokernel of G and denoted by Ac(G).

3.Preliminaries

3.1 <u>The Generalized Quaternion Group Q_{2m}</u> [5]

For each positive integer m,the generalized Quaternion Group Q_{2m} of order 4m with two generators x and y satisfies $Q_{2m}=\{x^h y^k, 0\le h \le 2m-1, k=0,1\}$ which has the following properties $\{x^{2m}=y^4=I, yx^my^{-1}=x^{-m}\}$.

3.2The Dihedral Group D_3 [6]

The Dihedrael Group D3 is generate by a rotation r of order 3 and reflection s of order 2 then 6 elements of D3 can be written as: $\{1,r,r^2,s,sr,sr^2\}$.

3.3The Rational valued characters table:

Definition(3.3.1) [3]

A rational valued character θ of G is a character whose values are in Z,which is θ(g)∈Z for all g∈ G.

Theorem (3.3.2)[6]

Every rational valued character of G be written as a linear combination of Artin's characters with coefficient rational numbers.

Corollary (3.3.3)[3]

The rational valued characters $\theta_i = \sum_{\sigma \epsilon Gal(\,Q(\chi_i)/Q)} \sigma(\chi_i)$Form a basis for $\bar{R}(G)$,whereχ_i are the irreducible characters of G and their numbers are equal to the number of conjugacy classes of a cyclic subgroup of G.

Proposition(3.3.4)[6]

The number of all rational valued characters of finite G is equal to the number of all distinct Γ-classis.

Definition (3.3.5)[3]

The information about rational valued characters of a finite group G is displayed in a table called a rational valued characters table of G.We denote it by ≡*(G)which is $l \times l$ matrix whose columns are Γ-classes and rows are the valuses of all rational valued characters where l is the number of Γ-classes.

The rational character table of Q_{2m} where m is an odd number(3.3.6) [5]

	Γ-classes of c_{2m}								[y]
	X^{2r}				X^{2r+1}				
Θ_1	1 1				1 1				1
Θ_2	1				1				0
⋮									⋮
$\Theta_{l/2}$	≡*(C_m)				≡*(C_m)				0
$\Theta_{(l/2)+1}$	1 1				1 1				-1
⋮	1				1				0
Θ_{l-1}									⋮
Θ_l	≡*(C_m)				H				0
Θ_{l+1}	2	2	⋯	2	-2	-2	⋯	-2	0

Table(1)

Where $0 \le r \le l$,l is the number of Γ-classes C_{2m} ,θj such that $1 \le j \le l+1$ are the rational valued characters of group Q_{2m} and if we denote Cij the elements of ≡*(C_m) and hij the elements of H as defined by:

$$H_{ij}=\begin{cases} C_{ij} & if \quad i = l \\ -C_{ij} & if \quad i \neq l \end{cases}$$

The rational character table of Q_{2m} when m=p, p is a prime number(3.3.7)[5]
$\equiv^*(Q_{2p})$

Γ-classes	[1]	$[x^2]$	$[x^p]$	[x]	[y]
Θ_1	1	1	1	1	1
θ_2	p-1	-1	p-1	-1	0
θ_3	1	1	1	1	1
θ_4	p-1	-1	1-p	1	0
θ_5	2	2	-2	-2	0

Table(2)

The rational character table of D_3(3.3.8)[4]
$\equiv^*(D_3)$

classesΓ-	[I]	[r]	[s]
$\lvert CL_\alpha \rvert$	1	2	3
$\lvert C_{D_3}(cl_\alpha) \rvert$	6	3	2
θ_1	2	-1	0
θ_2	1	1	1
θ_3	1	1	1

Table(4)

4.Artin's Character Tables:

Theorem(4.1):[3]

Let H be a cyclic subgroup of G and $h_1,h_2,\ldots,h_m$ are chosen representatives for the m-conjugate classes of H contained in CL(g) in G,then:

$$\varphi'(g)=\begin{cases}\frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & if\ h_i \in H\cap CL(g)\\ 0 & if\ H\cap CL(g)=\phi\end{cases}$$

Propostion(4.2)[3]

The number of all distinct Artin's characters on a group G is equal to the number of Γ-classes on G.Furthermore , Artin's characters are constant on each Γ-classes.

Theorem(4.3) [8]

The Artin's characters table of the Quaternion group Q_{2m} when m is odd number is given as follows:

	Γ-classes of C_{2m}								
Γ-classes	X^{2r}				X^{2r+1}				[y]
$\lvert CL_\alpha \rvert$	1	2	⋯	2	1	2	⋯		2m
$\lvert C_{Q_{2m}}(CL_\alpha) \rvert$	4m	2m	⋯	2m	4m	2m	⋯		2
Φ_1		2Ar(C_{2m})							0
Φ_2									0
⋮									⋮
Φ_l									0
Φ_{l+1}	m	0	⋯	0	m	0	⋯	0	1

Table(5)

Where $0 \leq r \leq m-1$, l is the number of Γ-classes of C_{2m} and Φj are the Artin characters of the Quaternion group Q_{2m},for all $1 \leq j \leq l+1$.

The Artin characters table of Q_{2m} when m=p, p is prime number (4.4)

The general form of Artin's characters of Q_{2m} when m=p,p is prime number

Γ-classes	[1]	[x^2]	[x^p]	[x]	[y]
$\lvert CL_\alpha \rvert$	1	2	1	2	2p
$\lvert C_{Q_{2p}}(CL_\alpha) \rvert$	4p	2p	4p	2p	2
Φ_1	4p	0	0	0	0
Φ_2	4	4	0	0	0
Φ_3	2p	0	2p	0	0
Φ_4	2	2	2	2	0
Φ_5	P	0	P	0	1

Table(6)

The Artin characters of D_3 (4.5)[6]

Γ-classes	[I]	[r]	[s]
$\lvert CL_\alpha \rvert$	1	2	3
$\lvert C_{D_3}(CL_\alpha) \rvert$	6	3	2
Φ_1	6	0	0
Φ_2	2	2	0
Φ_3	3	0	1

Table(7)

5.The main resulte

Propostion(5.1)

If p is a prime number and, then The Artin's character table of the group ($Q_{2p} \times D_3$) is given as:

The general form of the Artin characters of the group($Q_{2p} \times D_3$)when p is prime number

Γ-classes	[1,I][x^2,I][x^p,I][x,I][y,I]	[1,r][x^2,r][x^p,r][x,r][y,r]	[1,s][x^2,s][x^p,s][x,s][y,s]
$\lvert CL_\alpha \rvert$	1 2 1 2 2p	1 2 1 2 2p	1 2 1 2 2p
$\lvert C_{Q_{2p} \times D_3}(CL_\alpha) \rvert$	24p 24p 12p 12	24p 12p 24p 12p 12	24p 12p 24p 12p 12
$\Phi_{(1,1)}$ $\Phi_{(2,1)}$ ⋮ $\Phi_{(l+1,1)}$	6Ar(Q_{2p})	0	0
$\Phi_{(1,2)}$ $\Phi_{(2,2)}$ ⋮ $\Phi_{(l+1,2}$	2Ar(Q_{2p})	2Ar(Q_{2p})	0
$\Phi_{(1,3)}$ $\Phi_{(2,3)}$ ⋮ $\Phi_{(l+1,3)}$	3Ar(Q_{2p})	0	Ar(Q_{2p})

Table(8)

which is (5×5) square matrix .

Proof: Let g∈ $(Q_{2p}{}^{x}D_3)$; g=(q,d),q∈Q_{2p},d∈D_3

Case(I):

If H is a cyclic subgroup of (Q_{2p}×{I}),then 1- H=<(x,I)> 2- H=<(y,I)>

And φ the principle character of H, Φ_j Artin's characters of Q_{2p},1≤j≤l+1, then by using theorem (4.1)

$$\Phi_j(\mathrm{g}) = \begin{cases} \frac{|C_G(\mathrm{g})|}{|C_H(\mathrm{g})|}\sum_{i=1}^{p}\varphi(hi) & if\ hi \in H\cap CL(\mathrm{g}) \\ 0 & if\ H\cap CL(\mathrm{g}) = \phi \end{cases}$$

1- H=$<(x,I)>$

(i) If g=(1,I)

$\Phi_{(j,1)}(1,I)=\frac{|C_{Q_{2p}{}^{x}D_3}(g)|}{|C_H(\mathrm{g})|}.\ \varphi\ (\mathrm{g})=\frac{24p}{|C_H(\mathrm{g})|}.1=\frac{6.4p}{|C_H(\mathrm{g})|}.1=\frac{6|C_{Q_{2p}}(1)|}{|C_{<x>}(1)|}.\varphi(1)=6.\ \Phi_j(1)$ since H∩ $CL(1,I) = \{(1,I)\}$

(ii) If g=(x^p,I),g∈ H then

$\Phi_{(j,1)}(\mathrm{g})=\frac{|C_{Q_{2p}{}^{x}D_3}(\mathrm{g})|}{|C_H(\mathrm{g})|}\varphi(g)=\frac{24p}{|C_H(\mathrm{g})|}.1=\frac{6|C_{Q_{2p}}(x^p)|}{|C_{<x>}(x^p)|}\ \varphi(x^p)=6.\ \Phi_j(x^p)$ since H∩ $CL(g) = \{\mathrm{g}\}, \varphi(\mathrm{g}) = 1$

(iii) If g=(x^2,I) or g=(x,I) and g∈ H then

$\Phi_{(j,1)}(\mathrm{g})=\frac{\left|C_{Q_{2p}{}^{x}D_3}(g)\right|}{|C_H(g)|}(\varphi(\mathrm{g})+\varphi(\mathrm{g}^{-1}))=\frac{12p}{|C_H(\mathrm{g})|}(1+1)=\frac{3.4p}{|C_H(\mathrm{g})|}.2=\frac{3|C_{Q2p(q)}|}{|C_{H(q)}|}.2=6.\ \Phi_j(q)$

since H∩ $CL(g) = \{\mathrm{g}, \mathrm{g}^{-1}\}$ and $\varphi(\mathrm{g})=\varnothing(\mathrm{g}^{-1})=1$ and since g=(q,I),q∈Q_{2p},q≠x^m

(iv) if g∉H then

$\Phi_{(j,1)}$(g)=0 since H∩CL(g)=ϕ

2- If H=<(y,I)>={(1,I),(y,I)(y^2,I)(y^3,I)} then

(i) If g=(1,I) then

$\Phi_{(l+1,1)}(\mathrm{g})=\frac{|C_{Q_{2p}{}^{x}D_3}(\mathrm{g})|}{|C_H(\mathrm{g})|}\varphi(\mathrm{g})=\frac{24p}{4}.1=6.p=6.\Phi_{l+1}(1)$ since H∩CL(1,I)={(1,I)}

(ii) If g=(x^p,I)=(y^2,I) and g∈ H then

$\Phi_{(l+1,1)}(\mathrm{g})=\frac{|C_{Q_{2p}{}^{x}D_3}(\mathrm{g})|}{|C_H(\mathrm{g})|}\varphi(\mathrm{g})=\frac{24p}{4}.1=6.p=6.\Phi_{l+1}(x^p)$ since H∩CL(g)={g},φ(g)=1

(iii) If g≠(x^p,I) and g∈H ,i.e.{g=(y,I) or g=(y^3,I)} then

$\Phi_{(l+1,1)}(\mathrm{g})=\frac{|C_{Q_{2p}xD_3}(\mathrm{g})|}{|C_H(\mathrm{g})|}(\varphi(\mathrm{g})+\varphi(\mathrm{g}^{-1}))=\frac{12}{4}(1+1)=3.2=6.\Phi_{l+1}(y)$since H∩CL(g)={g,$g^{-1}$} and φ(g)= φ(g^{-1})=1

Otherwise

$\Phi_{(l+1,I)}$(g)=0 since H∩CL(g)= ϕ

Case(II):

If H is a cyclic subgroup of (Q_{2p}x{r}) then:

1- H=<(x,r)> 2- H=<(y,r)>

1-H=<(x,r)>

and φ the principle character of H, then by using theorem (4.1)

$$\Phi_j(\mathrm{g}) = \begin{cases} \frac{|C_G(\mathrm{g})|}{|C_H(\mathrm{g})|}\sum_{i=1}^{p}\varphi(h_i) & if\ h_i \in H\cap CL(\mathrm{g}) \\ 0 & if\ H\cap CL(\mathrm{g}) = \phi \end{cases}$$

(i) If g=(1,I),(1,r) then

$\Phi_{(j,2)}(\mathrm{g})=\frac{|C_{Q_{2p}xD_3}(\mathrm{g})|}{|C_H(\mathrm{g})|}\varphi(\mathrm{g})=\frac{24.p}{|C_H(1,I)|}.1=\frac{6.4p}{|C_H(1,I)|}.1=\frac{6|C_{Q_{2p}}(1)|}{3|C_{<x>}(1)|}\varphi(1)=2.\Phi j(1)$

since H∩CL(g)={(1,I),(1,r),(1,r^2)}

(ii) g=(1,I),(x^p,I),(x^p,r),(1,r); g$\in H$

if g=(1,I),(1,r) then

$\Phi_{(j,2)}(g)=\frac{|C_{Q2p}xD_3(g)|}{|C_H(g)|}\varphi(g)=\frac{24p}{|C_H(g)|}.1$ since H∩CL(g)={g} and φ(g)=1

$=\frac{6.3p}{|C_H(g)|}.1=\frac{6|C_{Q2p}(1)|}{3|C_{<x>}(1)|}\varphi(1)=2\Phi_j(1)$

(iii) if g= (x^p,I),(x^p,r) then

$\Phi_{(j,2)}(g)=\frac{|C_{Q2p}xD_3(g)|}{|C_H(g)|}\varphi(g)=\frac{24p}{|C_H(g)|}.1=\frac{6.3p}{|C_H(g)|}.1=\frac{6|C_{Q2p}(x^p)|}{3|C_{<x>}(x^p)|}\varphi(1)=2\Phi_j(x^p)$

(iv) if g≠ (x^p,I),(x^p,r) and g$\in H$ then

$\Phi_{(j,2)}(g)=\frac{|C_{Q2p}xD_3(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{12p}{|C_H(g)|}(1+1)$ since H∩CL(g)={g,g^{-1}} and φ(g)= φ(g^{-1})=1

$=\frac{3.4p}{|C_H(g)|}(1+1)=\frac{3|C_{Q2p}(q)|}{3|C_{<x>}(q)|}.2=2\Phi_j(q)$

Since g=(q,r),q∈Q_{2p},q≠x^p

(v) *if* g $\notin H$ then

$\Phi_{(j,2)}(g)=0=\Phi_j(q)$ since H∩CL(g)= ϕ

2- if H=<(y,r)>={(1,I),(y,I),(y^2,I),(y^3,I),(1,r),(y,r),(y^2,r),(y^3,r)}

(i) if g=(1,I),(1,r) then

$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24p}{12}.1=2p=2\Phi_{l+1}(g)$

(ii) if g=(y^2,I)=(x^p,I),(y^2,r) and g$\in H$ then

$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24p}{12}.1=2p=2\Phi_{l+1}(g)$ since H∩CL(g)={g} and φ(g)=1

(iii) if g≠ (x^p,I) and g$\in H$ i.e. g={(y,I),(y,r)} or g={(y^3,I),(y^3,r)} then

$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{12}{12}(1+1)=2\Phi_{l+1}(g)$

since H∩CL(g)={g,g^{-1}} and φ(g)= φ(g^{-1})=1

otherwise $\Phi_{(l+1,2)}(g)=0$ since H∩CL(g)= ϕ

case(III):

if H is a cyclic subgroup of (Q_{2px}{s}) then

1- H=<(x,s)>, 2- H=<(y,s)>

and φ the principle character of H, then by using theorem (4.1)

$$\Phi_j(g)=\begin{cases}\frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{p}\varphi(h_i) & if\ h_i\in H\cap CL(g)\\ 0 & if\ H\cap CL(g)=\phi\end{cases}$$

1- H=<(x,s)>

(i) If g=(1,I) then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24p}{|C_H(1,I)|}.1=\frac{6.4p}{|C_H(1,I)|}.1=\frac{6|C_{Q2p}(1)|}{2|C_{<x>}(1)|}.1=3\Phi_j(1)$ since H∩CL(g)={(1,I)}

If g={(1,s)} then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{8p}{|C_H(1,s)|}.1=\frac{2.4p}{|C_H(1,s)|}.1=\frac{2|C_{Q2p}(1)|}{2|C_{<x>}(1)|}.1=\Phi_j(1)$ since H∩CL(g)={(1,s)}

(ii) If g=(1,I),(x^p,I),(x^p,s),(1,s); g$\in H$ then

If g=(1,I) then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24p}{|C_H(g)|}.1$ since H∩CL(g)={g} and φ(g)=1

$=\frac{6.4p}{|C_H(g)|}.1=\frac{6|C_{Q_{2p}}(1)|}{2|C_{<x>}(1)|}\varphi(1)=3\Phi_j(1)$

If g={(1,s)} then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{8p}{|C_H(g)|}.1=\frac{2.4p}{|C_H(g)|}.1=\frac{2|C_{Q_{2p}}(1)|}{2|C_{<x>}(1)|}.1=\Phi_j(1)$ since H∩CL(g)={g} and φ(g)=1

(iii)If g= (x^p,I) then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24p}{|C_H(g)|}.1=\frac{6.4p}{|C_H(g)|}.1=\frac{6|C_{Q_{2p}}(x^p)|}{2|C_{<x>}(x^p)|}\varphi(1)=3\Phi_j(x^p)$

If g= (x^p,s) then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{8p}{|C_H(g)|}.1=\frac{2.4p}{|C_H(g)|}.1=\frac{2|C_{Q_{2p}}(x^p)|}{2|C_{<x>}(x^p)|}\varphi(1)=\Phi_j(x^p)$

(iv)If g≠ $(x^p,I),(x^p,s)$ and g∈ H

If g≠ (x^p,I) then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))$

$=\frac{12p}{|C_H(g)|}(1+1)$ since H∩CL(g)={g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$

$=\frac{3.4p}{|C_H(g)|}(1+1)=\frac{3|C_{Q_{2p}}(q)|}{2|C_{<x>}(q)|}.2=3\Phi_j(q)$

Since g=(q,I),q∈Q_{2p}, q≠x^p

If g≠ (x^p,s) then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))$

$=\frac{8p}{|C_H(g)|}(1+1)$ since H∩CL(g)={g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$

$=\frac{2.4p}{|C_H(g)|}(1+1)=\frac{2|C_{Q_{2p}}(q)|}{4|C_{<x>}(q)|}.2=\Phi_j(q)$

Since g=(q,s),q∈Q_{2p}, q≠x^p

(v)*if* g ∉ H then

$\Phi_{(j,3)}(g)=0=\Phi_j(q)$ since H∩CL(g)= ϕ

2-if H=<(y,s)>={(1,I),(y,I),(y^2,I),(y^3,I),(1,s),(y,s),(y^2,s),(y^3,s)} then

(i)If g=(1,I) then

$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24p}{8}.1=3.p=3\Phi_{l+1}(g)$

If g=(1,s) then

$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{8p}{8}.1=p=\Phi_{l+1}(g)$

(ii)If g=(y^2,I)=(x^p,I) and g∈ H then

$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24p}{8}.1=$

$3.m=3\Phi_{l+1}(g)$ since H∩CL(g)={g} and φ (g)=1

If g=(y^2,s) and g∈ H then

$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{8p}{8}.1=p=\Phi_{l+1}(g)$ since H∩CL(g)={g} and φ (g)=1

(iii)If g≠ (x^p,I) and g∈ H i.e. g={(y,I),(y,s)} or g={(y^3,I),(y^3,s)} then

$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{12}{8}(1+1)=3\Phi_{l+1}(g)$

since H∩CL(g)={g,g^{-1}} and $\varphi(g)=\varphi(g^{-1})=1$

(iv)If g=(y^2,s), gϵH then

$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{8p}{|C_H(g)|}.1=\frac{8p}{8}.1=\Phi_{l+1}(g)$

(v)If g=(y,s) then

$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2p}xD_3}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{4}{|C_H(g)|}.(1+1)=\frac{4}{8}.2=1$

since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

otherwise $\Phi_{(l+1,3)}(g)=0$ since $H\cap CL(g)=\phi$

Example (5.2): To find Artine's character table of the group ($Q_{14}xD_3$) when p=7 is a prime number .

$Ar(Q_{14}xD_3)$=

Γ-classes	[1,I]	$[x^2,I]$	$[x^7,I]$	[x,I]	[y,I]	[1,r]	$[x^2,r]$	$[x^7,r]$	[x,r]	[y,r]	[1,s]	$[x^2,s]$	$[x^7,s]$	[x,s]	[y,s]
$\lvert cL_\alpha\rvert$	1	2	1	2	2p	2	2	2	2	2p	3	3	3	3	6p
$\lvert c_{Q_{2p}xD_3}(cL_\alpha)\rvert$	168	84	168	84	12	84	84	84	84	12	56	56	56	56	4
$\Phi_{(1,1)}$	168	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	24	24	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	84	0	84	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	12	12	12	12	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,1)}$	42	0	42	0	6	0	0	0	0	0	0	0	0	0	0
$\Phi_{(1,2)}$	56	0	0	0	0	56	0	0	0	0	0	0	0	0	0
$\Phi_{(2,2)}$	8	8	0	0	0	8	8	0	0	0	0	0	0	0	0
$\Phi_{(3,2)}$	28	0	28	0	0	28	0	28	0	0	0	0	0	0	0
$\Phi_{(4,2)}$	4	4	4	4	0	4	4	4	4	0	0	0	0	0	0
$\Phi_{(5,2)}$	14	0	14	0	2	14	0	14	0	2	0	0	0	0	0
$\Phi_{(1,3)}$	84	0	0	0	0	0	0	0	0	0	28	0	0	0	0
$\Phi_{(2,3)}$	12	12	0	0	0	0	0	0	0	0	4	4	0	0	0
$\Phi_{(3,3)}$	42	0	42	0	0	0	0	0	0	0	14	0	14	0	0
$\Phi_{(4,3)}$	6	6	6	6	0	0	0	0	0	0	2	2	2	2	0
$\Phi_{(5,3)}$	21	0	21	0	3	0	0	0	0	0	7	0	7	0	1

Table(9)

6.To find Artin's cokernel of the group ($Q_{2p}xD_3$) when p is a prime number denoted by $AC(Q_{2p}xD_3)$

Definition (6.1):[1]

Let T(G) be the subgroup of $\bar{R}(G)$ gererated by Artin's characters .T(G) is normal subgroup of $\bar{R}(G)$,then the finite factor an a blain group $\frac{\bar{R}(G)}{T(G)}$ is called Artin cokernel of G,denoted by AC(G).

Definition (6.2):[2]

Let M be a matrix with entries in a principle ideal domain R.A K-minor of M is the determinate of KxK sub-matrix preserving row and column order.

Proposition (6.3)[1]

AC(G) is a finitely generated Z-modul.Let m be the number of all distinct Γ-classes then Ar(G) and ≡*(G) are of the rank l.There exists an invertible matrix M(G) with entries in rational number such that :

$\equiv^*(G)=M^{-1}(G).Ar(G)$ and this implies $M(G)=Ar(G).(\equiv^*(G))^{-1}$

Proposition (6.4)

By proposition(6.3) then $M(Q_{2p}xD_3)=Ar(Q_{2p}xD_3).(\equiv^*(Q_{2p}xD_3))^{-1}=$

$$\begin{pmatrix} 4&2&2&2&1&1&4&2&2&2&1&1&2&1&1\\ 0&2&2&0&1&1&0&2&2&0&1&1&0&1&1\\ 2&2&0&1&1&0&2&2&0&1&1&0&1&1&0\\ 0&0&0&2&1&1&0&0&0&2&1&1&2&1&1\\ 0&0&0&0&1&1&0&0&0&0&1&1&0&1&1\\ 0&0&0&1&1&0&0&0&0&1&1&0&1&1&0\\ 4&2&2&2&1&1&0&0&0&2&1&1&0&0&0\\ 0&2&2&0&1&1&0&0&0&0&1&1&0&0&0\\ 2&2&0&1&1&0&0&0&0&1&1&0&0&0&0\\ 0&0&0&2&1&1&0&0&0&2&1&1&0&0&0\\ 0&0&0&0&1&1&0&0&0&0&1&1&0&0&0\\ 0&0&0&1&1&0&0&0&0&1&1&0&0&0&0 \end{pmatrix}$$

Definition (6.5):[2]

A k-th determinat divisor of M is the greatest common divisor (g.c.d)for all the k-minor ,this is denoted by $D_k(M)$.

Lemma(6.6):[2]

Let M,P,W be matrices with entries in the principal ideal domain R.Let P and W be invertible matrices then $D_k(P,M,W)=D_K(M)$ modulo the group of units of R.

Proposition (6.7):[8]

$$M(Q_{2p})=\begin{bmatrix} 2&1&2&1&1\\ 0&1&0&1&1\\ 2&1&0&1&0\\ 0&1&0&1&0\\ 1&1&0&0&0 \end{bmatrix}$$

Proposition (6.8):[7]

$$M(D_3)=\begin{bmatrix} 2&1&1\\ 0&1&1\\ 1&1&0 \end{bmatrix}$$

Proposition (6.9) : $M(Q_{2p}xD_3)=M(Q_{2p})\otimes M(D_3)=$

$$\begin{pmatrix}
4 & 2 & 2 & 2 & 1 & 1 & 4 & 2 & 2 & 2 & 1 & 1 & 2 & 1 & 1 \\
0 & 2 & 2 & 0 & 1 & 1 & 0 & 2 & 2 & 0 & 1 & 1 & 0 & 1 & 1 \\
2 & 2 & 0 & 1 & 1 & 0 & 2 & 2 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 2 & 1 & 1 & 0 & 0 & 0 & 2 & 1 & 1 & 2 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\
4 & 2 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 2 & 1 & 1 & 0 & 0 & 0 \\
0 & 2 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
2 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 1 & 1 & 0 & 0 & 0 & 2 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
2 & 1 & 1 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Proposition (6.10)[8]: $p(Q_{2p})=$

$$\begin{pmatrix}
1 & -1 & -1 & 1 & 0 \\
0 & 1 & 0 & -1 & 0 \\
0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

Proposition (6.11)[7] : $p(D_3)=$

$$\begin{bmatrix}
1 & -1 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}$$

Proposition (6.12) : $p(Q_{2p}xD_3)=p(Q_{2p})\otimes p(D_3)=$

$$\begin{pmatrix}
1 & -1 & 0 & -1 & 1 & 0 & -1 & 1 & 0 & 1 & -1 & 0 & 1 & -1 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

Proposition (6.13):[8]

$$W(Q_{2p})=\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Proposition (6.14):[7]

$$W(D_3)=\begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

Proposition (6.15):

$W(Q_{2p}xD_3)=W(Q_{2p})\otimes W(D_3)=$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & -1 & 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Definition (6.16):[2]

Let M be a matrix with entries in a principal domain R, be equivalent $D=diag\{d_1,d_2,\ldots,d_m,0,0,\ldots,0\}$ such that d_j/ d_{j+1} for $1\leq j\leq m$. We call D **the invariant factor matrix of M** and $d_1,d_2,\ldots, d_m$ the invariant factor of M.

Proposition (6.17) : $P(Q_{2p}{}^{x}D_3)*M(Q_{2p}{}^{x}D_3)*W(Q_{2p}{}^{x}D_3)=$

$$\begin{pmatrix}
4&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&2&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&-2&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&2&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&4&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&2&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&-2&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&2&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&2&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1
\end{pmatrix}$$

$=\text{diag}\{4,4,2,2,2\ ,2,2,1,1,1,-2,-2,-1,-1,-1\}=D\ (Q_{2p}{}^{x}D_3)$

The following theorem gives the cyclic decomposition of the factor group $AC(D\ (Q_{2p}{}^{x}D_3))$ when p is $D\ (Q_{2p}{}^{x}D_3)$ prime number.

References:

[1] M.J.Hall "The Theory of Group ",Macmillan,Neyork,1959.

[2] M.S.Kirdar,"The factor Group of the Z-valued class function Modulo the group of the Generalized characters"University of Birmingham1980

[3] C.Curits and I.Reiner,"Methods of Representation Theory with Application to finite Groups and order",John Wily and sons,NewYork,1981.

[4] H.H.Abass,"On the factor Group of class function over the Group of Generalized characters of D_n",M.S.C.thesis,Technology University,1994.

[5] N.R.Mahamood"The cyclic Decomposition of the factor group of $\frac{(Q_{2m})}{\bar{R}(Q_{2m})}$ ",M.SC. thesis University of Technology,1995.

[6] A.S.Abid,"Artin's characters table of Dihedral group for odd number",M.S.C.thesis,University of Kufa,2006.

[7] R.N.Mirza,"On Artin cokernel of Dihedral Group D_n when n is an odd number",2007.

[8] A.H.Abdul-Mun'em,"On Artin Cokernel of the Quaternion Group Q_{2m} when m is odd number ",2008.