Solution of an Extraordinary differential equation by variational iteration method and homotopy perturbation method

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Abstract
In this paper, variational iteration method (VIM) and homotopy perturbation method (HPM) are implemented to solve extraordinary differential equation. The comparison between VIM and HPM with Adomian decomposition method (ADM) shows that VIM and HPM are more effective and convenient to use and overcomes the difficulty arising in calculating Adomian polynomials.

1. Introduction

The fractional calculus has found diverse applications in various scientific and technological fields, such as thermal engineering, acoustics, electromagnetism, control, robotics, viscoelasticity, diffusion, edge detection, turbulence, signal processing, and many other physical processes. Fractional differential equations have also been applied in modeling many physical, engineering problems, and fractional differential equations in nonlinear dynamics. Extraordinary differential equation is one of types from a differential equation containing a fractional derivative of order half along with an ordinary first-order derivative, consider the following extraordinary differential equation:

$$\frac{dy}{dx}+\frac{d^{1/2}y}{dx^{1/2}}-2y=0, \quad (1.1)$$

with an initial condition $y_0 = c$ where c constants. We first of all give definitions of fractional integral and fractional derivative introduced by Riemann-Liouville.

Definition 1.1 [12,13] (fractional integral). Let $q > 0$ denote a real number. Assuming $f(x)$ to be a function of class $C^{(n)}$ (the class of functions with continuous nth derivatives), the fractional integral of a function f of order $-q$ is given by

$$\frac{d^{-q}f(x)}{dx^{-q}}=\frac{1}{\Gamma(q)}\int_0^x \frac{f(t)dt}{(x-t)^{1-q}}.$$

Definition 1.2 [12,13] (fractional derivative). Let $q > 0$ denote a real number and n the smallest integer exceeding q such that $n-q > 0$ ($n = 0$ if $q < 0$). Assuming $f(x)$ to be a function of class $C^{(n)}$ (the class of functions with continuous nth derivatives), the fractional derivative of a function f of order q is given by

$$\frac{d^{q}f(x)}{dx^{q}}=\frac{1}{\Gamma(n-q)}\frac{d^n}{dx^n}\int_0^x \frac{f(t)dt}{(x-t)^{1-n+q}}.$$

Definition 1.3 [12] (Mittag-Leffler function). A two-parameter function of the Mittag-Leffler type is defined by the series expansion

$$E_{\alpha,\beta}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)} \qquad (\alpha>0,\beta>0) \quad .$$

Extraordinary differential equation was solved by Adomian decomposition method in [13]. Adomian decomposition method depends on decomposing the nonlinear differential equation $F(x, y(x)) = 0$ into the two components [1]:

$$L(y(x)) + N(y(x)) = 0 \,,$$

where L and N are the linear and the nonlinear parts of F respectively. The operator L is assumed to be an invertible operator. Solving for $L(y)$ leads to:

$$L(y) = -N(y). \qquad (1.2)$$

Applying the inverse operator L^{-1} on both sides of (1.2) yields

$$y = -L^{-1}(N(y)) + \varphi(x). \qquad (1.3)$$

where $\varphi(x)$ is the constant of integration satisfies the condition $L(\varphi) = 0$. Now assuming that the solution y can be represented as infinite series of the form

$$y = \sum_{n=0}^{\infty} y_n \, . \qquad (1.4)$$

Furthermore, suppose that the nonlinear term $N(y)$ can be written as infinite series in terms of the Adomian polynomials A_n of the form

$$N(y) = \sum_{n=0}^{\infty} A_n \, , \qquad (1.5)$$

where the Adomian polynomials A_n of $N(y)$ are evaluated using the formula:

$$A_n(x) = \frac{1}{n!}\frac{d^n}{d\lambda^n} N\left(\sum_{n=0}^{\infty} [(\lambda^n y_n)]\Big|_{\lambda=0}\right) .$$

Then substituting (1.4) and (1.5) in (1.3) gives

$$\sum_{n=0}^{\infty} y_n = \varphi(x) - L^{-1}\left(\sum_{n=0}^{\infty} [A_n]\right) . \qquad (1.6)$$

Then equating the terms in the linear system of (1.6) gives the recurrent relation

$$y_0 = \varphi(x),$$

$$y_{n+1} = -L^{-1}(A_n) \, , \qquad n \geq 0 \, .$$

The solution of extraordinary differential equation (1.1) by Adomian decomposition method is [13]:

$$y(x) = \frac{c}{3}(2e^{4x} erfc(2\sqrt{x}) + e^{x} erfc(-\sqrt{x})) \, .$$

(1.7)

In this paper, we solve extraordinary differential equation (1.1) by two methods the first variational iteration method and the second homotopy perturbation method.

2. Variational iteration method

In this section, we introduce the basic idea underlying the variational iteration method for solving nonlinear equations. Consider the general nonlinear differential equation [3,6,15,16]:

$$Lu + Nu = g(t),$$

where L is a linear differential operator, N is a nonlinear operator, and g is a given analytical function. The essence of the method is to construct a correction functional of the form

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda(t,s)(Lu_n(s) + N\tilde{u}_n(s) - g(s))ds,$$

where λ is a Lagrange multiplier which can be identified optimally via the variational theory, u_n is the approximate solution and $\tilde{u}_n$ denotes the restricted variation, i.e. $\delta\tilde{u}_n = 0$. After determining the Lagrange multiplier λ and selecting an appropriate initial function u_0, the successive approximations u_n of the solution u can be readily obtained. Consequently, the solution of (1.1) is given by $u = \lim_{n\to\infty} u_n$.

3. Solution an Extraordinary by variational iteration method

To solve (1.1) by means of the variational iteration method, we construct a correction functional as:

$$y_{n+1}(x) = y_n(x) + \int_0^x \lambda(x,s)\left(\frac{dy_n(s)}{ds} + \frac{d^{1/2}\tilde{y}_n(s)}{ds^{1/2}} - 2\tilde{y}_n(s)\right)ds,$$

$$\delta y_{n+1}(x) = \delta y_n(x) + \delta\int_0^x \lambda(x,s)\left(\frac{dy_n(s)}{ds} + \frac{d^{1/2}\tilde{y}_n(s)}{ds^{1/2}} - 2\tilde{y}_n(s)\right)ds,$$

integrating by part, we have:

$$\delta y_{n+1}(x) = \delta y_n(x) + \lambda(x,x)\delta y_n(x) - \int_0^x \frac{\partial\lambda(x,s)}{\partial s}\delta y_n(s)ds,$$

where $\delta\tilde{y}_n$ is considered as a restricted variation. Its stationary conditions can be obtained as follows:

$$1 + \lambda(x,x) = 0, \qquad -\frac{\partial\lambda(x,s)}{\partial s} = 0,$$

where λ is a Lagrange multiplier, therefore, can be identified as $\lambda = -1$, and the following variational iteration formula can be obtained:

$$y_{n+1}(x) = y_n(x) - \int_0^x \left(\frac{dy_n(s)}{ds} + \frac{d^{1/2}y_n(s)}{ds^{1/2}} - 2y_n(s)\right)ds.$$

Or equivalently;

$$y_{n+1}(x) = y_n(x) - \int_0^x \left(\frac{d^{1/2}y_n(s)}{ds^{1/2}} - 2y_n(s)\right)ds.$$

We start with an initial condition $y_0(x) = c$ and use the definition (1.1), we can obtain the following successive approximations:

$$y_1(x) = y_0(x) - \int_0^x \left[\frac{dy_0(x)}{ds} + \frac{d^{1/2}y_0(x)}{ds^{1/2}} - 2y_0(x)\right]ds$$

$$= c - \int_0^x \left[0 + \frac{d^{1/2}c}{ds^{1/2}} + -2c\right]ds$$

$$= c - \int_0^x \left[\frac{1}{\sqrt{\pi}}\frac{d}{ds}\int_0^s c(s-t)^{-1/2}\,dt - 2c\right]ds$$

$$= c - \left[\frac{2c}{\sqrt{\pi}}s^{1/2} - 2cs\right]_0^x$$

$$= (1 + 2x - \frac{2}{\sqrt{\pi}}x^{1/2})c \quad .$$

$$y_2(x) = y_1(x) - \int_0^x \left[\frac{dy_1(x)}{ds} + \frac{d^{1/2}y_1(x)}{ds^{1/2}} - 2y_1(x)\right]ds$$

$$= c + 2cx - \frac{2c}{\sqrt{\pi}}x^{1/2} - \int_0^x \left[2c - \frac{c}{\sqrt{\pi}}s^{-1/2} + \frac{d^{1/2}c}{ds^{1/2}} + 2c\frac{d^{1/2}s}{ds^{1/2}} - \frac{2c}{\sqrt{\pi}}\frac{d^{1/2}s^{1/2}}{ds^{1/2}} - 2c - 4cs + \frac{4c}{\sqrt{\pi}}s^{1/2}\right]ds,$$

where:

$$\frac{d^{1/2}s}{ds^{1/2}} = \frac{1}{\Gamma(1/2)}\frac{d}{ds}\int_0^s t(s-t)^{-1/2}dt$$
$$= \frac{1}{\sqrt{\pi}}\frac{d}{ds}\left[\left[-2t(s-t)^{1/2}\right]_0^s + 2\int_0^s (s-t)^{1/2}dt\right]$$
$$= \frac{2}{\sqrt{\pi}}s^{1/2}, \qquad (3.1)$$

and

$$\frac{d^{1/2}s^{1/2}}{ds^{1/2}} = \frac{1}{\Gamma(1/2)}\frac{d}{ds}\int_0^s t^{1/2}(s-t)^{-1/2}dt$$
$$= \frac{1}{\sqrt{\pi}}\frac{d}{ds}\left[sB\left(\frac{3}{2},\frac{1}{2}\right)\right] = \frac{1}{\sqrt{\pi}}\frac{d}{ds}\left[s\frac{\Gamma\left(\frac{3}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma(2)}\right] = \frac{\sqrt{\pi}}{2}. \qquad (3.2)$$

where $B(m,n), \Gamma(n)$ are beta and gamma functions respectively such that:

$$B(m,n) = \int_0^1 x^{(m-1)}(1-x)^{(n-1)}dx,$$
$$\Gamma(n) = \int_0^\infty e^{-x}x^{(n-1)}\,dx,$$

so;

$$y_2(x) = c + 2cx - \frac{2c}{\sqrt{\pi}}x^{1/2} - \int_0^x \left[\left[\frac{8c}{\sqrt{\pi}}s^{1/2}\right] - c - 4cs\right]ds,$$

then;

$$y_2(x) = (1 + 3x + 2x^2 - \frac{2}{\sqrt{\pi}}x^{\frac{1}{2}} - \frac{16}{3\sqrt{\pi}}x^{3/2})c \quad .$$

to find $y_3(x)$ we need to calculate $\frac{d^{1/2}s^2}{ds^{1/2}}$ and $\frac{d^{1/2}s^{3/2}}{ds^{1/2}}$:

$$\frac{d^{1/2}s^2}{ds^{1/2}} = \frac{1}{\Gamma(1/2)}\frac{d}{ds}\int_0^s t^2(s-t)^{-1/2}dt$$
$$= \frac{1}{\sqrt{\pi}}\frac{d}{ds}s^{5/2}\left[B\left(3,\frac{1}{2}\right)\right] = \frac{1}{\sqrt{\pi}}\frac{d}{ds}\left[s^{5/2}\frac{\Gamma(3)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{7}{2}\right)}\right] = \frac{8}{3\sqrt{\pi}}x^{3/2}. \qquad (3.3)$$

$$\frac{d^{1/2}s^{3/2}}{ds^{1/2}} = \frac{1}{\Gamma(1/2)}\frac{d}{ds}\int_0^s t^{3/2}(s-t)^{-1/2}dt$$
$$= \frac{1}{\sqrt{\pi}}\frac{d}{ds}s^2\left[B\left(\frac{5}{2},\frac{1}{2}\right)\right] = \frac{1}{\sqrt{\pi}}\frac{d}{ds}\left[s^2\frac{\Gamma\left(\frac{5}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma(3)}\right] = \frac{3\sqrt{\pi}}{4}x. \qquad (3.4)$$

now;

$$y_3(x) = y_2(x) - \int_0^x \left[\frac{dy_2(x)}{ds} + \frac{d^{1/2}y_2(x)}{ds^{1/2}} - 2y_2(x) \right] d\square$$

then we have:

$$y_3(x) = (1 + 3x + 5x^2 + \frac{4}{3}x^3 - \frac{2}{\sqrt{\pi}}x^{\frac{1}{2}} - \frac{20}{3\sqrt{\pi}}x^{3/2} - \frac{32}{5\sqrt{\pi}}x^{5/2})c \quad .$$

similarly, we obtain:

$$y_4(x) = (1 + 3x + \frac{11}{2}x^2 + \frac{16}{3}x^3 + \frac{2}{3}x^4 - \frac{2}{\sqrt{\pi}}x^{\frac{1}{2}} - \frac{20}{3\sqrt{\pi}}x^{\frac{3}{2}} - \frac{32}{3\sqrt{\pi}}x^{5/2} - \frac{512}{105\sqrt{\pi}}x^{7/2})c \quad ,$$

$$y_5(x) = (1 + 3x + \frac{11}{2}x^2 + 7x^3 + 4x^4 + \frac{4}{15}x^5 - \frac{2}{\sqrt{\pi}}x^{\frac{1}{2}} - \frac{20}{3\sqrt{\pi}}x^{\frac{3}{2}} - \frac{56}{5\sqrt{\pi}}x^{5/2} - \frac{384}{35\sqrt{\pi}}x^{7/2}$$

$$- \frac{512}{189\sqrt{\pi}}x^{9/2})c \quad ,$$

and so on. therefore, the solution of extraordinary differential equation (1.1) is :

$$y(x) = c(1 + 3x + \frac{11}{2}x^2 - \frac{2}{\sqrt{\pi}}x^{1/2} - \frac{20}{3\sqrt{\pi}}x^{3/2} - \frac{56}{5\sqrt{\pi}}x^{5/2} + \ldots). \tag{3.5}$$

4. Homotopy perturbation method

To explain this method , let us consider the following function [2,8,14]:

$$A(u) - f(r) = 0 \quad ; \qquad r \in \Omega \quad , \tag{4.1}$$

with the boundary condition of

$$B(u, \frac{\partial u}{\partial n}) = 0 \quad ; \quad r \in \Gamma \quad . \tag{4.2}$$

where A is a general differential operator, B a boundary operator, $f(r)$ a known analytical function and Γ is the boundary of the domain Ω. The operator A can be divided into two : parts, which are L and N , where L is a linear, but N is nonlinear. Equation (4.1) can be rewritten as follows:

$$L(u) + N(u) - f(r) = 0, \tag{4.3}$$

which satisfies:

$v(r, p) : \Omega \times [0,1] \to R$.

By the homotopy technique, we construct a homotopy

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0 , \tag{4.4}$$

or

$$H(v, p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0 \quad . \tag{4.5}$$

Is an embedding parameter, while u_0 is an initial approximation of (4.1) and $p \in [0,1]$. Where (4.1), which satisfies the boundary conditions. Obviously, from (4.4) and (4.5) we will have;

$$H(v,0) = L(v) - L(u_0) = 0 \quad , \tag{4.6}$$

$$H(v,1) = A(v) - f(r) = 0 \quad . \tag{4.7}$$

The changing process of " p " from zero to unity is just that of $v(r,p)$ from u_0 to $u(r)$.

In topology, this is called deformation ,while $L(v)-L(u_0)$ and $A(v)-f(r)$ are called homotopy . According to the HPM, we can first use the embedding parameter " p " as a asmall parameter ,and assume that the solutions of (4.4) and (4.5) can be written as a power series " p " in setting " $p=1$ " a result in the approximate solution of (4.3) is;

$$u=\lim_{p\to 1} v = v_0 + v_1 + v_3 + \ldots \quad (4.8)$$

The combination of the perturbation method and the homotopy method is called HPM , which eliminates the drawbacks of the traditional perturbation methods while keeping all its advantages. The series (4.8) is convergent for most cases. However ,the rate convergent depends on nonlinear term $A(v)$.

5. Solution an Extraordinary by homotopy perturbation method

To solve (1.1) by homotopy perturbation method, we construct the following homotopy:

$$(1-p)\left(\frac{dy}{dx}-0\right)+p\left(\frac{dy}{dx}+\frac{d^{1/2}y}{dx^{1/2}}-2y\right)=0,$$

Or equivalently;

$$\frac{dy}{dx}+p\frac{d^{1/2}y}{d\ x^{1/2}}-2py=0. \quad (5.1)$$

Suppose that the solution of (5.1) has the form:

$$y=y_0+py_1+p^2y_2+\ldots \quad (5.2)$$

Substituting(5.2) into (5.1)we get:

$$\frac{dy_0}{dx}+p\frac{dy_1}{dx}+p^2\frac{dy_2}{dx}+\ldots+p\frac{d^{1/2}y_0}{dx^{1/2}}+p^2\frac{d^{1/2}y_1}{dx^{1/2}}+\ldots-2py_0-2p^2y_1-2p^3y_3-\ldots=0$$

$$\frac{dy_0}{dx}+p\left(\frac{dy_1}{dx}+\frac{d^{1/2}y_0}{dx^{1/2}}-2y_0\right)+p^2\left(\frac{dy_2}{dx}+\frac{d^{1/2}y_1}{dx^{1/2}}-2y_1\right)+p^3\left(\frac{dy_3}{dx}+\frac{d^{1/2}y_2}{dx^{1/2}}-2y_2\right)+\ldots=0$$

By comparing the coefficients of terms with identical powers of p, we have:

$$y_{n+1}(x)=2\frac{d^{-1}y_n}{dx^{-1}}-\frac{d^{-1/2}y_n}{dx^{-1/2}}, \qquad n=0,1,\ldots$$

Then we have:

$$p^0: y_0(x)=c \ ,$$

$$P^1: y_1(x)=2\frac{d^{-1}y_0}{dx^{-1}}-\frac{d^{-1/2}y_0}{dx^{-1/2}} \Rightarrow y_1(x)=\left(2x-2\frac{\sqrt{x}}{\sqrt{\pi}}\right)c \ ,$$

$$p^2: y_2(x)=2\frac{d^{-1}y_1}{dx^{-1}}-\frac{d^{-1/2}y_1}{dx^{-1/2}} \ ,$$

by (3.1) and (3.2) we have:

$$y_2(x)=\left(2x^2-\frac{16}{3}\frac{x^{3/2}}{\sqrt{\pi}}+x\right)c \ ,$$

$$P^3: y_3(x)=2\frac{d^{-1}y_2}{dx^{-1}}-\frac{d^{-1/2}y_2}{dx^{-1/2}} \ ,$$

by (3.1), (3.2), (3.3) and (3.4) we obtain:

$$y_3(x)=(\frac{4}{3}x^3-\frac{32}{5}\frac{x^{5/2}}{\sqrt{\pi}}+3x^2-\frac{4}{3}\frac{x^3}{\sqrt{\pi}})c\quad ,$$

similarly:

$$p^4: y_4(x)=(\frac{1}{2}x^2+4x^3+\frac{2}{3}x^4-\frac{64}{15\sqrt{\pi}}x^{5/2}-\frac{512}{105\sqrt{\pi}}x^{7/2})c\quad ,$$

$$p^5: y_5(x)=(\frac{4}{15}x^5-\frac{512}{189}\frac{x^{9/2}}{\sqrt{\pi}}+\frac{5}{3}x^3-\frac{128}{21}\frac{x^{7/2}}{\sqrt{\pi}}-\frac{8}{15}\frac{x^{5/2}}{\sqrt{\pi}}+\frac{10}{3}x^4)c\quad ,$$

and so on. Setting $p=1$ in (5,2) we can obtain the solution :

$$y(x)=c(1+3x+\frac{11}{2}x^2-\frac{2}{\sqrt{\pi}}x^{1/2}-\frac{20}{3\sqrt{\pi}}x^{3/2}-\frac{56}{5\sqrt{\pi}}x^{5/2}+\ldots)\quad . \tag{5.3}$$

6. Computational aspects

The solutions (3.5) and (5.3) of (1.1) obtained by VIM and HPM respectively are equivalent, these solutions can be written as:

$$y(x)=\frac{c}{3}(2\sum_{k=0}^{\infty}\frac{(-1)^k 2^k x^{k/2}}{\Gamma(k/2+1)}+\sum_{k=0}^{\infty}\frac{x^{k/2}}{\Gamma(k/2+1)}),$$

by definition (1.3), the explicit formula for $E_{1/2,1}(z)$ is

$$E_{1/2,1}(z)=\sum_{k=o}^{\infty}\frac{z^k}{\Gamma(k/2+1)}=e^{z^2}(1+erf(z)),$$

that is,

$$E_{1/2,1}(z)=e^{z^2}erfc(-z),$$

therefore , the solution of (1.1) becomes:

$$y(x)=\frac{c}{3}(2e^{4x}erfc(2\sqrt{x})+e^{x}erfc(-\sqrt{x})), \tag{6.1}$$

solution (6.1) completely matches solution (1.7) obtained by solve (1.1) by Adomian decomposition method.

It is clear that, y_0+y_1 in HPM is equivalent to y_1 in VIM, similarly $y_0+y_1+y_2$ is equivalent to y_2, generally we find $y_0+y_1+\cdots+y_n$ in HPM is equivalent to y_n in VIM, this results can be show in Fig.1.

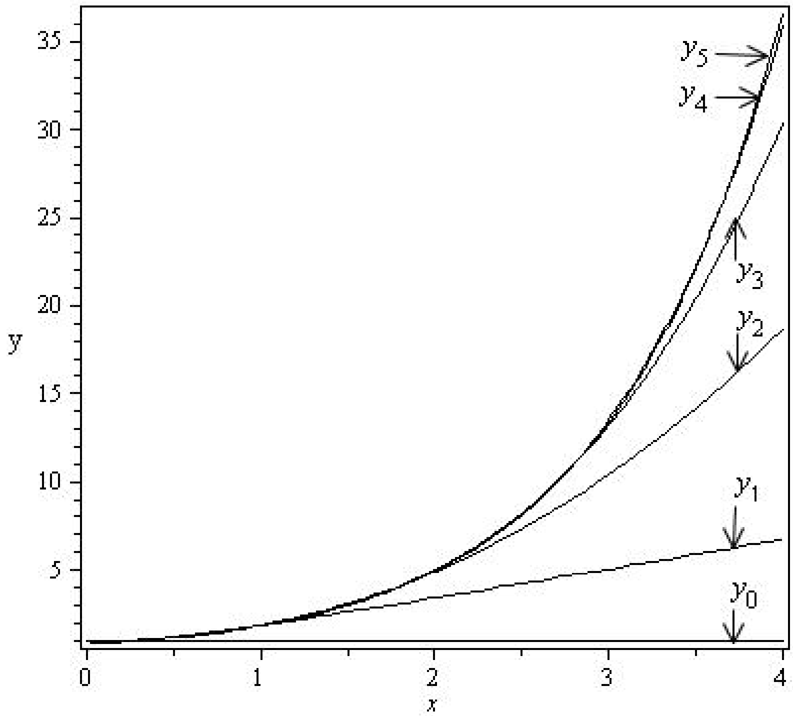

Fig. 1. The approximate solutions of extraordinary differential equation (1.1) at c=1 by VIM

7. Conclusions

In this paper, variational iteration method and homotopy perturbation method have been successfully used to solve extraordinary differential equation. The extraordinary differential equation can be solved by Adomian decomposition method [13].

The obtained solution in this paper using VIM and HPM have got many merit and much more advantage than the ADM, these methods are to overcome the difficulties arising in calculation of Adomian polynomials. From Fig.1 we show that the solutions of extraordinary differential equation (1.1) at $c = 1$ by using VIM and HPM have a good approximation.

The calculations as a results of HPM and VIM are not be difficult but the calculations of integrations by using fractional derivative introduced by Riemann-Liouville are difficult because there are needing to transformation to beta functions and then to gamma functions for obtain the solutions of these integrations, so the computations can't be found by a development programming as maple (13-14) therefore we compute it by our self.

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حل المعادلة التفاضلية فوق الأعتيادية باستعمال طريقتي التغايرالتكراري والهوموتوبي المثارة

ايلاف جعفر علي   عبير مجيد جاسم

قسم الرياضيات

كلية العلوم /جامعة البصرة

المستخلص:

في هذا البحث، استعملنا طريقة التغاير التكراري (VIM) وطريقة الاضطراب الهوموتوبي (HPM) لحل المعادلة التفاضلية فوق الأعتيادية (Extraordinary). حيث وجدنا ان تطبيق هاتين الطريقتين أكثر فاعلية مقارنة مع تطبيق طريقة تحليل أدومين (ADM) بسبب التغلب على الصعوبة التي تنشأ عند حساب متعددات الحدود في طريقة أدومين (Adomian polynomials).