Iraqi Journal of Statistical Sciences

www.stats.mosuljournals.com

Improved Mixed Estimator Using Two Auxiliary Variables For Full Extreme Maximum And Minimum Values In Single Phase Sampling

Timothy O. Olatayo Peter N. Madu Peter I. Ogunyinka

Olabisi Onabanjo University, Ago-Iwoye, Department of Mathematical Sciences , Nigeria

Article information

Article history:
Received March 20 ,2024
Accepted April 30 ,2024
Available online June 1, 2024

Keywords:
Auxiliary variables, extreme values, mean square error, ratio estimators, regression estimator, single-phase sampling.

Correspondence:
Peter N. Madu,
saymadupeter@gmail.com

Abstract

The use of multiple auxiliary variables has been established to improve precision in the estimators of ratio, regression and product respectively. However, the presence of extreme values in the distribution could annul such efficiency Olatayo *et al.* (2020). Extreme values could be small or minimum, large or maximum values. This study had developed a ratio-cum-regression estimator with two auxiliary variables, correlation coefficient and coefficient of variation under two types of extreme values in the distribution. This study considers full extreme value cases which assumed that both the study and two auxiliary variables had extreme values present in their distributions. Theoretical, empirical and percentage relative efficiency analyses were carried out for Full High and Maximum Extreme Values (FHMaEV) and Full Low and Minimum Extreme Values cases (FLMiEV). The analysis showed that the developed estimator is efficient over the reviewed estimators.

DOI 10.33899/IQJOSS.2024.183259

1. Introduction

The presence of extreme value in the distribution will lead to either over- estimation or under -estimation of the corresponding population parameter Ogunyinka *et al*. (2021). Consequently, this could result in either wrong decision been taken or drawing of invalid inference. The introduction of auxiliary variables in the estimation process of population parameters have been confirmed to improved precision, efficiency and reliability of estimators Ogunyinka *et al*. (2019). A high correlation coefficient between the study variable and the auxiliary variable is an essential factor that is crucial in maximizing the advantages of auxiliary variables Ogunyinka *et al.* (2019). When the correlation between the study variable and the auxiliary variable is positively high and the line of linear regression between the two passes through the origin, then the ratio estimator shows the highest efficiency; but if the line of linear regression does not pass through the origin, the regression estimator has the highest efficiency. When they are highly negative correlated, the product estimator has the highest efficiency; when the variables show a weak correlation only, then the sample mean is preferred (Agunbiade and Ogunyinka, 2013). According to Ogunyinka *et al* (2019); Cochran (1940) developed the use of ratio estimator in single phase sampling, Raj (1965) introduced the use of more than one auxiliary variable while Mohanty (1967) advanced the use of two estimators in single phase sampling later known as mixed estimation method in in sampling. Ratio-com-regression is an example of mixed estimator and has been confirmed to improve the efficiency of any estimator Ogunyinka *et al* (2019).

The proposed estimator in this study is a combined ratio and regression estimator with full extreme values. The developed estimator used the method developed by Särndal (1972). The combined estimators are the improved ratio and regression estimators of Khan and Shabbir (2013). They were combined in the order of Kadilar and Cingi (2005) while following the procedure of Al-Hossain and Khan (2014). This proposed estimator used one study variable and two auxiliary variables. It assumes that both the study variable and the auxiliary variables have extreme values their distributions and that the population information of all the auxiliary variables were available. This study has made theoretical and empirical comparison of the proposed estimators with the reviewed estimators of Khan and Shabbir (2013) ratio estimator, Al-Hossain and Khan (2014) ratio estimator and Al-Hossain and Khan (2014) regression estimator in single-phase sampling without replacement. The efficiency of the proposed estimators has been established using the variance, the Mean Square Errors (MSEs). Similarly, the biases of the proposed estimators were ascertained in the empirical analysis. Finally, this study also made use of percentage relative efficiency analysis in other to ascertain by what percentage the proposed estimator is efficient over the reviewed.

1. METHODOLOGY

2.1. Review on Särndal's solution to extreme values

Särndal (1972) has developed solution to extreme value by introducing a correction constant c such that if there exists extreme large value in a distribution and $\overline{y}_{max}$ is the sample mean using Simple Random Sampling without Replacement (SRSWOR), then c will be subtracted from $\overline{y}_{max}$ to obtain the corrected mean. This is stated as

$$\overline{y}_1 = \overline{y}_{max} - c \tag{1}$$

Likewise, if there exists extreme low value in a distribution and $\overline{y}_{min}$ is the sample mean with SRSWOR, then c will be added to $\overline{y}_{min}$ to obtain the corrected mean. This is stated as

$$\overline{y}_1 = \overline{y}_{min} + c \tag{2}$$

This can be written in a compressed form as

$$\bar{y}_1 = \begin{cases} \bar{y} + c \; if \; samples \; contains \; y_{min} \; but \; not \; y_{max} \\ \bar{y} - c \; if \; samples \; contains \; y_{max} but \; not y_{min} \\ \bar{y} \qquad\qquad for \; all \; other \; samples \end{cases} \tag{3}$$

c is the correction constant. The minimum variance of $\bar{y}_1$ up to first order of approximation is given as

$$Var(\bar{y}_1)_{min} - \frac{\lambda\Delta^2 y}{2(N-1)} \tag{4}$$

where $\Delta_y = (y_{max-}y_{min})$ and $\Delta x = (x_{max} - x_{min})$. The optimum value of c is given as

$$c_{opt} = \frac{\Delta y}{2(N-1)} \tag{5}$$

2.2. Regression estimator using one auxiliary variable with extreme value

Khan and Shabbir (2013) has proposed an improved regression estimator using one auxiliary variable with extreme value. The estimator is given as

$$\bar{y}_3 = \bar{y}_{c_{11}} + b(\bar{X} - \bar{x}_{c_{21}}) \tag{6}$$

with the corresponding variance as

$$V(\bar{y}_3)_{opt} \cong \mathrm{M}\,(\bar{y}_{lr}) - \frac{\lambda(\Delta y - \beta\Delta x)^2}{2(N-1)} \tag{7}$$

where $M(\bar{y}_{lr}) = \lambda\, S_y^2\,(1 - \rho_{yx}^2)$ and b is the sample regression coefficient.

2.3. Ratio estimator using one auxiliary variable with extreme value

Khan and Shabbir (2013) has proposed an improved ratio estimator using one auxiliary variable with extreme value. The estimator is given as

$$\bar{y}_4 = \frac{\bar{y}_{c_{11}}}{\bar{x}_{c_{21}}}\bar{X}. \tag{8}$$

The corresponding MSE is given as:

$$MSE(\bar{y}_4)_{opt} \cong M(\bar{y}_R) - \frac{\lambda(\Delta y - R\Delta x)^2}{2(N-1)}, \tag{9}$$

where $M(\bar{y}_R) \cong \bar{Y}^2\lambda\left(C_y^2 C_x^2 - 2\rho_{yx}C_y C_x\right)$, is the mean square error of the conventional ratio estimator.

2.4. Ratio estimator using two auxiliary variables with extreme value

Al-Hossain and Khan (2014) has proposed an improved ratio estimator using two auxiliary variables with extreme value. The estimator is given as

$$\bar{y}_5 = \bar{y}_{c_{11}}\left(\frac{\bar{X}_1}{\bar{x}_{1_{c_{21}}}}\right)\left(\frac{\bar{X}_2}{\bar{x}_{2_{c_{31}}}}\right). \tag{10}$$

The corresponding MSE is presented as

$$MSE(\bar{y}_5)_{opt} \cong M(\bar{y}_{R2}) - \frac{\lambda(\Delta y - R_1\Delta x_1 - R_2\Delta x_2)^2}{2(N-1)}, \tag{11}$$

where $M(\bar{y}_{R2}) = \lambda\left(S_y^2 + R_1^2S_{x_1}^2 + R_2^2S_{x_2}^2 + 2R_1R_2S_{x_1x_2} - 2R_2S_{yx_2} - 2R_1S_{yx_1}\right)$.

2.5. Regression estimator using two auxiliary variables with extreme value

Al-Hossain and Khan (2014) has proposed an improved regression estimator using two auxiliary variables with extreme value. The improved regression estimator of Al-Hossain and Khan (2014) is given as

$$\bar{y}_6 = \bar{y}_{c_{11}} + b_1\left(\bar{X} - \bar{x}_{1_{c_{21}}}\right) + b_2\left(\bar{X} - \bar{x}_{2_{c_{31}}}\right). \tag{12}$$

The corresponding MSE given as

$$MSE(\bar{y}_6)_{opt} \cong M(\bar{y}_{lr}) - \frac{\lambda(\Delta y - \beta_1\Delta x_1 - \beta_2\Delta x_2)^2}{2(N-1)} \tag{13}$$

where $M(\bar{y}_{lr}) = \lambda S_y^2\left(1 - \rho_{yx_1}^2 - \rho_{yx_2}^2 + 2\rho_{yx_1}\rho_{yx_2}\rho_{x_1x_2}\right)$. Similarly, $\beta_1 = \rho_{yx_1}\frac{S_y}{S_{x_1}}$ *and* $\beta_2 = \rho_{yx_2}\frac{S_y}{S_{x_1}}$ are the population regression coefficient between y and x_1 and between y and x_2.

2. Proposed Estimator

This study has proposed an improved mixed estimator derived by combing the improved ratio and regression estimators of Khan and Shabbir (2013) in single phase sampling without replacement. The proposed estimator has been termed Full Extreme Value in both Study and Auxiliary variable (FEVSA) and is denoted by:

$$\bar{y}_7 = \left(\frac{\bar{y}_{c_1}}{\bar{x}_{c_{11}}}\right)\bar{X}_1 + b\left(\bar{X}_2 - \bar{x}_{c_{22}}\right) \tag{14}$$

The relative error terms are defined as

$$\left.\begin{aligned} \bar{y}_{c_1} &= \bar{Y}(1+\varepsilon_0) \\ \bar{x}_{c_{11}} &= \bar{X}_1(1+\varepsilon_1) \\ \bar{x}_{c_{22}} &= \bar{X}_2(1+\varepsilon_2) \end{aligned}\right\} \tag{15}$$

such that

$E(\varepsilon_0) = E(\varepsilon_1) = E(\varepsilon_2) = 0$

The estimators $\bar{y}_{c_1} = (\bar{y} + c_1)$, $\bar{x}_{c_{11}} = (\bar{x}_1 + c_1)$, and $\bar{x}_{c_{22}} = (\bar{x}_2 + c_2)$ provided the sample contains y_{min} , x_{1min} and x_{2min}. Similarly, the estimators $\bar{y}_{c_1} = (\bar{y} - c_1)$, $\bar{x}_{c_{11}} = (\bar{x}_1 - c_1)$, $\bar{x}_{c_{22}} = (\bar{x}_2 - c_2)$. If the sample contains y_{max} , x_{1max} and x_{2max}. Finally, the estimator $\bar{y}_{c_1} = \bar{y}$, $\bar{x}_{c_{11}} = \bar{x}_1$, $\bar{x}_{c_{22}} = x_2$ for all other combinations of samples. The following expectation would be used.

$$\left.\begin{aligned} E(\varepsilon_0^2) &= \frac{\lambda}{\bar{Y}^2}\left[s_y^2 - \frac{2nc_0}{N-1}(\Delta y - nc_0)\right] \\ E(\varepsilon_1^2) &= \frac{\lambda}{\bar{X}_1^2}\left[s_{x_1}^2 - \frac{2nc_1}{N-1}(\Delta x_1 - nc_1)\right] \\ E(\varepsilon_2^2) &= \frac{\lambda}{\bar{X}_2^2}\left[s_{x_2}^2 - \frac{2nc_2}{N-1}(\Delta x_2 - nc_2)\right], \end{aligned}\right\} \tag{16}$$

and

$$\left.\begin{aligned} E(\varepsilon_0\varepsilon_1) &= \frac{\lambda}{\bar{Y}\bar{X}_1}\left[s_{yx_1} - \frac{n}{N-1}(c_1\Delta \mathrm{y} + c_0\Delta x_1 - 2nc_0c_1)\right] \\ E(\varepsilon_0\varepsilon_2) &= \frac{\lambda}{\bar{Y}\bar{X}_2}\left[s_{yx_2} - \frac{n}{N-1}(c_2\Delta \mathrm{y} + c_0\Delta x_2 - 2nc_0c_2)\right] \\ E(\varepsilon_1\varepsilon_2) &= \frac{\lambda}{\bar{X}_1\bar{X}_2}\left[s_{x_1x_2} - \frac{n}{N-1}(c_2\Delta x_1 + c_1\Delta x_2 2nc_1c_2)\right] \end{aligned}\right\}. \tag{17}$$

Substituting equation (15) into equation (14) will give

$$\bar{y}_8 = \bar{Y}(1+\varepsilon_0)(1+\varepsilon_1)^{-1} - \mathrm{b}\varepsilon_2\bar{X}_2$$

Applying Taylor series of expanding $(1+\varepsilon_1)^{-1}$ up to second order of degree, will give

$$\bar{y}_7 = \bar{Y}(1 - \varepsilon_1 + \varepsilon_1^2 + \varepsilon_0 - \varepsilon_0\varepsilon_1) - \mathrm{b}\varepsilon_2\bar{X}_2$$

$$Bias(\bar{y}_7) = E(\bar{y}_8 - \bar{Y}) \tag{18}$$

but

$$\bar{y}_7 - \bar{Y} = \bar{Y}(1 - \varepsilon_1 + \varepsilon_1^2 + \varepsilon_0 - \varepsilon_0\varepsilon_1) - b\varepsilon_2\bar{X}_2 - \bar{Y}$$

$$Bias(\bar{y}_7) = E[\bar{Y}(1 - \varepsilon_1 + \varepsilon_1^2 + \varepsilon_0 - \varepsilon_0\varepsilon_1) - \mathrm{b}\varepsilon_2\bar{X}_2]$$

Applying expectation,

$$Bias(\bar{y}_7) = \bar{Y}[E(\varepsilon_1^2) - E(\varepsilon_0\varepsilon_1)]$$

Substituting and collecting some like terms, gives

$$Bias(\bar{y}_7) = \frac{\bar{Y}\lambda}{\bar{X}_1^2}\left[s_{x_1}^2 - \frac{2nc_1}{N-1}(\Delta x_1 - nc_1)\right] - \frac{\bar{\mathrm{Y}}\lambda}{\bar{\mathrm{Y}}\bar{\mathrm{X}}_1}\left[\mathrm{s}_{\mathrm{yx}_1}\frac{\mathrm{n}}{\mathrm{N}-1}(\mathrm{c}_1\Delta y + \mathrm{c}_0\Delta\mathrm{x}_1 - 2\mathrm{nc}_0 c_1)\right]$$

$$Bias(\bar{y}_7) = \frac{R_1^2\lambda}{\bar{Y}}\left[s_{x_1}^2 - \frac{2nc_1}{N-1}(\Delta x_1 - nc_1)\right] - \frac{R_1\lambda}{\bar{Y}}\left[\mathrm{s}_{\mathrm{yx}_1}\frac{\mathrm{n}}{\mathrm{N}-1}(\mathrm{c}_1\Delta y + \mathrm{c}_0\Delta\mathrm{x}_1 - 2\mathrm{nc}_0 c_1)\right]$$

$$= \frac{R_1\lambda}{\bar{Y}}\left[\left(R_1 s_{x_1}^2 - s_{yx_1}\right)\right] - \frac{R_1^2\lambda}{\bar{Y}}\left[\frac{2nc_1(\Delta x_1 - nc_1)}{N-1}\right] - \frac{R_1\lambda n}{\bar{Y}(N-1)}\left[(\mathrm{c}_1\Delta\mathrm{y} + \mathrm{c}_0\Delta\mathrm{x}_1 - 2\mathrm{nc}_0\mathrm{c}_1)\right]$$

$$Bias(\bar{y}_7) = \frac{R_1\lambda}{\bar{Y}}\left\{\left[\left(R_1 s_{x_1}^2 - s_{yx_1}\right)\right] - \frac{n}{(N-1)}\left[\left(2c_1R_1(\Delta x_1 - nc_1)\right)\right.\right.$$
$$\left.\left. + (c_1\Delta y + c_0\Delta x_1 - 2nc_0c_1)\right]\right\} \tag{19}$$

Substituting the optimum values of c_0, $c_1 and c_2$ into (3.30), where

$c_0 = \frac{\Delta y}{2n}$, $c_1 = \frac{\Delta x_1}{2n}$, $c_2 = \frac{\Delta x_2}{2n}$, gives

$$Bias(\bar{y}_7) = \frac{R_1\lambda}{\bar{Y}}\left[\left(R_1 s_{x_1}^2 - s_{yx_1}\right) - \frac{1}{2(N-1)}(R_1\Delta^2 x_1 + \Delta x_1\Delta y)\right] \tag{20}$$

Similarly, $MSE(\bar{y}_7) = E(\bar{y}_8 - \bar{Y})^2$ (21)

$$= E[\bar{Y}(1+\varepsilon_0)(1+\varepsilon_1)^{-1} - \mathrm{b}\varepsilon_2\bar{X}_2]^2 \tag{22}$$

Appling expectation after expanding (20) gives

$$\begin{aligned} MSE(\bar{y}_7) = {} & \bar{Y}^2\left\{\frac{\lambda}{\bar{Y}^2}\left[S_y^2 - \frac{2nc_0}{N-1}(\Delta y - nc_0)\right]\right\} + \bar{Y}^2\left\{\frac{\lambda}{\bar{X}_1^2}\left[S_{x_1}^2 - \frac{2nc_1}{N-1}(\Delta x_1 - nc_1)\right]\right\} \\ & - 2\bar{Y}^2\left\{\frac{\lambda}{\bar{Y}\bar{X}_1}\left[S_{yx_1} - \frac{n}{N-1}(c_1\Delta \mathrm{y} + c_0\Delta x_1 - 2nc_0c_1)\right]\right\} + b^2\bar{X}_2^2\left\{\frac{\lambda}{\bar{X}_2^2}\left[S_{x_2}^2 - \frac{2nc_2}{N-1}(\Delta x_2 - nc_2)\right]\right\} \\ & - 2b\bar{X}_2\bar{Y}\left[S_{yx_2} - \frac{n}{N-1}(c_2\Delta y + c_0\Delta x_2 - 2nc_0c_2)\right] \\ & + 2b\bar{X}_2\bar{Y}\left\{\frac{\lambda}{\bar{X}_2\bar{Y}}\left[S_{x_1x_2} - \frac{n}{N-1}(c_2\Delta x_1 + c_1\Delta x_2 - 2nc_1c_2\right]\right\} \end{aligned}$$

$$MSE(\bar{y}_7) = \left[\lambda s_y^2 + \lambda R_1^2 s_{x_1}^2 - 2R_1\lambda s_{yx_1} + b^2\lambda s_{x_2}^2 - 2b\lambda s_{yx_2} + 2bR_1\lambda s_{x_1x_2}\right]$$
$$- \left\{\lambda\left[\frac{2nc_0}{N-1}(\Delta y - nc_0)\right] + \lambda R_1^2\left[-\frac{2nc_1}{N-1}(\Delta x_1 - nc_1)\right] - 2R_1\lambda\left[-\frac{n}{N-1}(c_1\Delta y + c_0\Delta x_1 - 2nc_0c_1)\right]\right.$$
$$+ b^2\lambda\left[-\frac{2nc_2}{N-1}(\Delta x_2 - nc_2)\right] - 2b\lambda\left[-\frac{n}{N-1}(c_2\Delta y + c_0\Delta x_2 - 2nc_0c_2)\right]$$
$$\left. + 2bR_1\lambda\left[-\frac{n}{N-1}(c_2\Delta x_1 + c_1\Delta x_2 - 2nc_1c_2\right]\right\}$$
$$MSE(\bar{y}_7) = \lambda\left[s_y^2 + R_1^2 s_{x_1}^2 - 2R_1 s_{yx_1} + b^2 s_{x_2}^2 - 2b s_{yx_2} + 2bR_1 s_{x_1x_2}\right]$$
$$- \frac{2\lambda n}{N-1}[\Delta y c_0 - c_0^2 n + R_1^2 c_1\Delta x_1 - R_1^2 c_1^2 n - R_1 c_1\Delta y - R_1 c_0\Delta x_1 + 2R_1 c_0 c_1 n + b^2 c_2\Delta x_2 - b^2 c_2^2 n$$
$$- bc_2\Delta y - bc_0\Delta x_2 + 2bnc_0c_2 + bR_1c_2\Delta x_1 + bR_1c_1\Delta x_2$$
$$- 2bR_1c_1c_2n] \qquad (23)$$

To obtain, $\hat{b}_{opt}$ differentiating (21) and equating to zero

$$\frac{\partial(\mathrm{M}(\bar{y}_8)\,)}{\partial(b)} = 2\lambda b s_{x_2}^2 - 2\lambda s_{yx_2} + 2\lambda R_1 s_{x_1x_2}$$
$$- \frac{2\lambda n}{N-1}\{2bc_2\Delta x_2 - 2bc_2^2 n - c_2\Delta y - c_0\Delta x_2 + 2nc_0c_2 + R_1c_2\Delta x_1 + R_1c_1\Delta x_2 - 2R_1c_1c_2n\} = 0$$
$$\Rightarrow 2\lambda\left[b s_{x_2}^2 - s_{yx_2} + R_1 s_{x_1x_2}\right] = \frac{2\lambda n}{N-1}\{2bc_2\Delta x_2 - 2bc_2^2 n - c_2\Delta y - c_0\Delta x_2 + 2nc_0c_2 + R_1c_2\Delta x_1 + R_1c_1\Delta x_2 - 2R_1c_1c_2n\}$$
$$\Rightarrow b s_{x_2}^2 - \frac{bn}{N-1}(2c_2\Delta x_2 - 2c_2^2 n)$$
$$= s_{yx_2} - R_1 s_{x_1x_2} + \frac{n}{N-1}[-c_2\Delta y - c_0\Delta x_2 + 2nc_0c_2] + \frac{n}{N-1}[R_1c_2\Delta x_1 + R_1c_1\Delta x_2 - 2R_1c_1c_2n]$$

Hence,

$$\hat{b} = \frac{s_{yx_2} - R_1 s_{x_1x_2} + \frac{n}{N-1}[-c_2\Delta y - c_0\Delta x_2 + 2nc_0c_2 + R_1c_2\Delta x_1 + R_1c_1\Delta x_2 - 2R_1c_1c_2n]}{s_{x_2}^2 - \frac{n}{N-1}[2c_2\Delta x_2 - 2c_2^2 n]} \qquad (24)$$

To obtain the optimum values of b, substituting $c_{0,}\ c_{1,} and c_2$ into equation (22), it gives

$$\hat{b}_{opt} = \frac{2[s_{yx_2} - R_1 s_{x_1x_2}](N-1) + \Delta x_2[R_1\Delta x_1 - \Delta y]}{2s_{x_2}^2(N-1) - \Delta^2 x_2}$$

or $\hat{b}_{opt} = \frac{[s_{yx_2} - R_1 s_{x_1x_2}](N-1) + \frac{\Delta x_2}{2}[R_1\Delta x_1 - \Delta y]}{s_{x_2}^2(N-1) - \frac{\Delta^2 x_2}{2}}$ (25)

The $MSE(\bar{y}_8)_{min}$ is obtained by substituting the optimum values of $c_{0,}\ c_{1,}\ c_2$ and $\hat{b}_{opt}$ into (21) as follows:

First, substitute the optimum values of c_0, c_1, c_2 into equation (21), it gives

$$MSE(\bar{y}_8)_{min} = \lambda\left[s_y^2 + R_1^2 s_{x_1}^2 - 2R_1 s_{yx_1} + b^2 s_{x_2}^2 - 2b s_{yx_2} + 2bR_1 s_{x_1x_2}\right]$$
$$- \frac{2\lambda\, n}{N-1}\left[\frac{\Delta y}{2n}\Delta y - \left(\frac{\Delta y}{2n}\right)^2 n + R_1^2\left(\frac{\Delta x_1}{2n}\right)\Delta x_1 - R_1^2\left(\frac{\Delta x_1}{2n}\right)^2 n\right]$$
$$- \frac{2\lambda\, n}{N-1}\left[-R_1\left(\frac{\Delta x_1}{2n}\right)\Delta y - R_1\left(\frac{\Delta x_1}{2n}\right)\Delta y + 2R_1\left(\frac{\Delta x_1}{2n}\frac{\Delta y}{2n}\right)n\right]$$
$$- \frac{2\lambda\, n}{N-1}\left[+b^2\left(\frac{\Delta x_2}{2n}\right)\Delta x_2 - b^2\left(\frac{\Delta x_2}{2n}\right)^2 n - b\left(\frac{\Delta x_2}{2n}\right)\Delta y - b\left(\frac{\Delta x_2}{2n}\right)\Delta y\right]$$
$$- \frac{2\lambda\, n}{N-1}\left[2bn\left(\frac{\Delta x_2}{2n}\frac{\Delta y}{2n}\right) + bR_1\left(\frac{\Delta x_2}{2n}\right)\Delta x_1 + bR_1\left(\frac{\Delta x_1}{2n}\right)\Delta x_2 - 2bnR_1\left(\frac{\Delta x_1}{2n}\frac{\Delta x_2}{2n}\right)\right]$$

$$MSE(\bar{y}_7)_{min} = \lambda\left[s_y^2 + R_1^2 s_{x_1}^2 - 2R_1 s_{yx_1} + b^2 s_{x_2}^2 - 2b s_{yx_2} + 2bR_1 s_{x_1x_2}\right]$$
$$- \frac{2\lambda\, n}{N-1}\left[\frac{\Delta^2 y}{4n} + \frac{R_1^2\Delta^2 x_1}{4n} - \frac{R_1\Delta x_1\Delta_y}{2n} + \frac{b^2\Delta_x^2}{4n} - \frac{b\Delta x_2\Delta y}{2n} + \frac{bR_1\Delta x_2\Delta x_1}{2n}\right]$$
$$- \frac{\lambda}{N-1}\left[\frac{\Delta^2 y}{2} + \frac{R^2\Delta^2 x_1}{2} - R_1\Delta x_1\Delta y + b^2\Delta^2 x_2 - b\Delta x_2\Delta y + bR_1\Delta x_2\Delta x_1\right]$$

$$MSE(\bar{y}_7)_{min} = \lambda\left[s_y^2 + R_1^2 s_{x_1}^2 - 2R_1 s_{yx_1} + b^2 s_{x_2}^2 - 2b s_{yx_2} + 2bR_1 s_{x_1x_2}\right] - \frac{\lambda}{2(N-1)}[\Delta^2 y + R_1^2\Delta^2 x_1 - 2R_1\Delta x_1\Delta y + b^2\Delta^2 x_2 - 2b\Delta x_2\Delta y + 2bR_1\Delta x_2\Delta x_1] \quad (26)$$

Hence,

$$MSE(\bar{y}_7)_{min} = \lambda\left[s_y^2 + R_1^2 s_{x_1}^2 - 2R_1 s_{yx_1}\right] - \frac{\lambda}{2(N-1)}[\Delta y - R_1\Delta x_1]^2 + \lambda b^2\left[\frac{2s_{x_2}^2(N-1) - \Delta^2 x_2}{2(N-1)}\right] + \lambda\left[2bR_1 s_{x_1x_2} - 2b s_{yx_2}\right] - \frac{\lambda}{2(N-1)}[2bR_1\Delta x_1\Delta x_2 - 2b\Delta x_2\Delta y]$$

Next, substituting $\hat{b}_{opt}$ into equation (24),

$$MSE(\bar{y}_7)_{min} = \lambda\left[s_y^2 + R_1^2 s_{x_1}^2 - 2R_1 s_{yx_1} + b^2 s_{x_2}^2 - 2b(s_{yx_2} - R_1 s_{x_1x_2})\right] - \frac{\lambda}{2(N-1)}[\left(\Delta_y^2 + R_1^2\Delta x_1^2 - 2R_1\Delta_y\Delta x_1 + b^2\Delta x_1^2 - 2b(\Delta y\Delta x_2 - R_1\Delta x_2\Delta x_1\right) \quad (27)$$

3. Results

4.1 Theoretical Comparison of FEVSA-Case with the reviewed estimators.

The condition will be if $MSE(\bar{y}_1)_{min}$ minus the MSE of any of the reviewed is less than zero is satisfied, then $\bar{y}_1$ is more efficient, otherwise, decision will be reverse.

a. Comparing FEVSA-Case with MSE of Khan and Shabbir (2013) improved ratio estimator

$$MSE(\bar{y}_7)_{min} - MSE(\bar{y}_4)_{min} < 0$$

$$\lambda\left[s_y^2 + R_1^2 s_{x_1}^2 - 2R_1 s_{yx_1} + b^2 s_{x_2}^2 - 2b\left(s_{yx_2} - R_1 s_{x_1x_2}\right)\right] - \frac{\lambda}{2(N-1)}\left[\left(\Delta_y^2 + R_1^2\Delta x_1^2 - 2R_1\Delta_y\Delta x_1 + b^2\Delta x_2^2 - 2b(R_1\Delta x_2\Delta x_1 - \Delta y\Delta x_2\right) - \lambda\left[S_y^2 + R^2 S_y^2 - 2\frac{RS_{xy}}{\bar{Y}^2}\right] - \frac{\lambda(\Delta y - R\Delta x)^2}{2(N-1)} < 0$$

$$\lambda\left[s_y^2 + R_1^2 s_{x_1}^2 - 2R_1 s_{yx_1}\right] + \lambda b^2\left[s_{x_2}^2 - \frac{\Delta x_2^2}{2(N-1)}\right] + 2b\lambda\left[R_1 s_{x_1x_2} - s_{yx_2}\right] - \frac{\lambda}{2(N-1)}\left[\Delta_y^2 + R_1^2\Delta x_1^2 - 2R_1\Delta_y\Delta x_1\right] - \frac{\lambda}{2(N-1)}\left[2b(R_1\Delta x_2\Delta x_1 - \Delta y\Delta x_2)\right] - \left\{\lambda\left[s_y^2 + R^2 s_y^2 - 2\frac{RS_{xy}}{\bar{Y}^2}\right] - \frac{\lambda(\Delta y - R\Delta x)^2}{2(N-1)}\right. < 0 \qquad (28)$$

This implies that $\bar{y}_7$ is more efficient than $\bar{y}_4$

b. Comparing FEV-Case with AL-Hossain and Khan (2014) ratio Estimator

$$\boldsymbol{MSE(\bar{y}_7)_{min} - MSE(\bar{y}_5)_{min} < 0}$$

This implies that

$$\lambda\left[s_y^2 + R_1^2 s_{x_1}^2 - 2R_1 s_{yx_1} + b^2 s_{x_2}^2 - 2b(s_{yx_2} - R_1 s_{x_1x_2})\right] - \frac{\lambda}{2(N-1)}\left[\left(\Delta_y^2 + R_1^2\Delta x_1^2 - 2R_1\Delta_y\Delta x_1 + b^2\Delta x_1^2 - 2b(\Delta y\Delta x_2 - R_1\Delta x_2\Delta x_1\right) - \left\{\lambda\left(S_y^2 + R_1^2 S_{x_1}^2 + R_2^2 S_{x_2}^2 + 2R_1R_2 S_{x_1x_2} - 2R_2 S_{yx_2} - 2R_1 S_{yx_1}\right) - \frac{\lambda(\Delta y - R_1\Delta x_1 - R_2\Delta x_2)^2}{2(N-1)}\right\}$$

$$\lambda\left[b^2\left(s_{x_2}^2 - \frac{\Delta x_2^2}{2(N-1)} + 2b(R_1 s_{x_1x_2} - s_{yx_2}\right)\right] - \lambda\left[R_2^2 S_{x_2}^2 + 2R_1R_2 S_{x_1x_2} - 2R_1 S_{yx_2}\right] - \frac{\lambda}{2(N-1)}\left[2b(R_1\Delta x_2\Delta x_1 - \Delta y\Delta x_2)\right] - \frac{\lambda}{2(N-1)}\left[\Delta_y^2 + R_1^2\Delta x_1^2 - 2R_1\Delta_y\Delta x_1\right] + \frac{\lambda(\Delta y - R_1\Delta x_1 - R_2\Delta x_2)^2}{2(N-1)} < 0 \qquad (29)$$

This implies that $\bar{y}_7$ is superior to $\bar{y}_5$

c. Comparing FEV-Case with AL-Hossain and Khan (2014) of regression estimator

$$\boldsymbol{MSE(\bar{y}_7)_{min} - MSE(\bar{y}_6)_{min} < 0}$$

$$\lambda\left[s_y^2 + R_1^2 s_{x_1}^2 - 2R_1 s_{yx_1} + b^2 s_{x_2}^2 - 2b\left(s_{yx_2} - R_1 s_{x_1x_2}\right)\right] - \frac{\lambda}{2(N-1)}\left[\left(\Delta_y^2 + R_1^2\Delta x_1^2 - 2R_1\Delta_y\Delta x_1 + b^2\Delta x_2^2 - 2b(R_1\Delta x_2\Delta x_1 - \Delta y\Delta x_2)\right] - \left\{S_y^2\left[1 - b_1^2\frac{S_{x_1}^2}{S_y^2} - b_2^2\frac{S_{x_2}^2}{S_y^2}\right] + \lambda S_y^2\left[2b_1\frac{S_{x_1}}{S_y}\, b_2\frac{S_{x_2}}{S_y}\frac{S_{x_1x_2}}{S_{x_1}S_{x_2}}\right] - \frac{\lambda(\Delta y - b_1\Delta x_1 - b_2\Delta x_2)^2}{2(N-1)}\right\} < 0 \qquad (30)$$

The efficiency of $\bar{y}_7$ over $\bar{y}_6$ will be obtained empirically using equation (30).

4. Empirical Analysis

5.1 Comparison of FEVSA with the reviewed estimators for the case of High and Maximum Extreme Values (HMAEV) table 1-3.

Table1. MSE comparison of the proposed estimator with the reviewed estimators for the twenty stimulated populations for HMaEV

S/N Populations	1	2	3	4	5	6	7
1. MSE ($\bar{y}_7$)	9039.007	3585.255	9454.076	8186.263	7190.929	8555.007	10067.8
2. MSE ($\bar{y}_4$)	253180.2	92865.49	271747.6	226558.7	193342.2	254878.2	313252.8
3. MSE ($\bar{y}_5$)	13928.04	7307.471	16462.41	14393.79	12197.14	16976.95	22577.38
4. MSE ($\bar{y}_6$)	9572.582	4243.1	10553.71	8893.569	7672.536	10508.48	12693.74
Rank MSE($\bar{y}_1$)	1	1	1	1	1	1	1
Rank MSE($\bar{y}_4$)	4	4	4	4	4	4	4
Rank MSE($\bar{y}_5$)	3	3	3	3	3	3	3
Rank MSE($\bar{y}_6$)	2	2	2	2	2	2	2

Table 2: MSE Comparison of the proposed estimator with the reviewed estimators for the twenty stimulated populations for HMaEV cases (continue)

S/N	Populations	8	9	10	11	12	13	14
1.	MSE ($\bar{y}_7$)	13153.84	21964.61	25480.39	30147.24	38698.05	15043.49	16986.67
2.	MSE ($\bar{y}_4$)	419045.3	754112	923106.7	1177701	1612307	573734.1	667053.4
3.	MSE ($\bar{y}_5$)	27112.14	34497.47	50521.47	62759.85	88084.77	45497.15	50967.38

4. MSE ($\bar{y}_6$)	**15921.55**	**23275.36**	**29689.32**	**38058.76**	**48689.05**	**23201.94**	**26482.01**
Rank MSE($\bar{y}_7$)	**1**	**1**	**1**	**1**	**1**	**1**	**1**
Rank MSE($\bar{y}_4$)	**4**	**4**	**4**	**4**	**4**	**4**	**4**
Rank MSE($\bar{y}_5$)	**3**	**3**	**3**	**3**	**3**	**3**	**3**
Rank MSE($\bar{y}_6$)	**2**	**2**	**2**	**2**	**2**	**2**	**2**

Table 3: MSE Comparison of the proposed estimator with the reviewed estimators for the twenty stimulated populations for HMaEV cases

S/N	Populations	15	16	17	18	19	20	Overall Ranking
1.	MSE ($\bar{y}_7$)	104641.2	39447.89	33902.91	111080.3	309540.7	649428.8	
2.	MSE ($\bar{y}_4$)	6258080	1956279	2026384	7303558	31577741	90611571	
3.	MSE ($\bar{y}_5$)	174441.8	139203.4	184225.9	452657.2	1432352	3191902	
4.	MSE ($\bar{y}_6$)	111874.1	68002.77	79395.13	192497	629051	1290081	
Rank MSE($\bar{y}_7$)		1	1	1	1	1	1	1
Rank MSE($\bar{y}_4$)		4	4	4	4	4	4	4
Rank MSE($\bar{y}_5$)		3	3	3	3	3	3	3
Rank MSE($\bar{y}_6$)		2	2	2	2	2	2	2

5.2 Comparison of FEVSA with the reviewed estimators for the case of Low and Minimum Extreme Values (LMiEV) table 4-6.

Table 4. Comparison of the proposed estimators with the reviewed estimators for the twenty stimulated populations for LMiEV

S/N **Populations**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
1. MSE ($\bar{y}_7$)	4639.3836	9647.557	13363.6194	9734.5354	13803.6141	13382.7563	9367.2453
2. MSE ($\bar{y}_4$)	62526.0429	80547.27837	95128.42097	91465.53736	108745.356	94488.16596	109258.909
3. MSE ($\bar{y}_5$)	7530.87287	11565.93007	15222.01492	12934.9066	17750.6313	15748.2352	14597.4654
4. MSE ($\bar{y}_6$)	5095.52272	9645.987777	13355.4278	9921.678177	13977.5592	13385.63772	10373.735
Rank MSE($\bar{y}_7$)	1	2	2	1	1	1	1
Rank MSE($\bar{y}_4$)	4	4	4	4	4	4	3
Rank MSE($\bar{y}_5$)	3	3	3	3	3	3	3
Rank MSE($\bar{y}_6$)	2	1	1	2	2	2	2

Table 5: Comparison of the proposed estimator with the reviewed estimators for the twenty stimulated populations for LMiEV cases

S/N Populations	**8**	**9**	**10**	**11**	**12**	**13**	**14**
1. MSE ($\bar{y}_7$)	28789.6481	27995.2403	33715.8422	29915.4547	25844.5775	67089.5804	59188.4223
2. MSE ($\bar{y}_4$)	105446.317	117322.785	86960.1988	91591.5829	144668.6619	99446.11086	62115.4812
3. MSE ($\bar{y}_5$)	31630.5049	32116.9394	36588.7598	33135.9329	33323.84253	70175.05245	61616.8991
4. MSE ($\bar{y}_6$)	28836.7787	27993.4493	34131.6335	30228.739	26280.09414	72475.27807	61926.4792
Rank MSE($\bar{y}_7$)	1	2	1	1	1	1	1
Rank MSE($\bar{y}_4$)	4	4	4	4	4	4	4
Rank MSE($\bar{y}_5$)	3	3	3	3	3	2	2
Rank MSE($\bar{y}_6$)	2	1	2	2	2	3	3

Table 6: Comparison of the proposed estimator with the reviewed estimators for the twenty stimulated populations for LMiEV

S/N	Populations	15	16	17	18	19	20	Overall Ranking
1.	MSE ($\bar{y}_7$)	69350.7307	70078.14	76624.8971	724155.3257	23664387.66	1906665.695	
2.	MSE ($\bar{y}_4$)	111018.1903	120989.8886	500293.3261	223833578.3	3130175596	367888798.1	
3.	MSE ($\bar{y}_5$)	72291.75637	71855.23721	86482.73769	1150713.922	32801061.58	22311430.98	
4.	MSE ($\bar{y}_6$)	74927.23977	76224.78422	95954.58546	515671.6613	920584.0752	2969628.188	
Rank MSE($\bar{y}_7$)		1	1	1	2	2	1	1
Rank MSE($\bar{y}_4$)		4	4	4	4	4	4	4
Rank MSE($\bar{y}_5$)		2	2	2	3	3	3	3
Rank MSE($\bar{y}_6$)		3	3	3	1	1	2	2

5. PERCENTAGE RELATIVE EFFICIENCY ANALYSIS FOR (HMaEV) CASE

6.1 Relative efficiency comparison of the proposed estimator with the reviewed estimators for High and Maximum Extreme Value Cases is given in table 7 to 9.

Table 7: The Relative Efficiency (RE) of the proposed estimator to the reviewed estimators for the twenty simulated populations (measured in percentage) (HMaEV) case.

S/N	Populations	1	2	3	4	5	6	7
2.	$\boldsymbol{RE}(\bar{y}_7/\ \bar{y}_4)$	2800.974	2590.206	2874.397	2767.547	2688.695	2979.287	3111.432
3.	$\boldsymbol{RE}(\bar{y}_7/\ \bar{y}_5)$	154.0881	203.8201	174.1303	175.8285	169.6184	198.4446	224.2533
4.	$\boldsymbol{RE}(\bar{y}_7/\ \bar{y}_6)$	105.903	118.3486	111.6314	108.6402	106.6974	122.8343	126.0826
8.	$\boldsymbol{RE}(\bar{y}_5/\ \bar{y}_4)$	1817.774	1270.829	1650.716	1574.003	1585.144	1501.32	1387.463
9.	$\boldsymbol{RE}(\bar{y}_6/\ \bar{y}_4)$	2644.848	2188.624	2574.9	2547.444	2519.925	2425.453	2467.773
10.	$\boldsymbol{RE}(\bar{y}_6/\ \bar{y}_5)$	145.4993	172.2201	155.9869	161.8449	158.9714	161.5547	177.8623

Table 8: The Relative Efficiency (RE) of estimators developed by Khan and Shabbir (2013) ratio, AL- Hossain and Khan (2014) regression and AL-Hossain and Khan (2014) ratio with the proposed estimator for the twenty simulated populations (percentage)

S/N	Populations	8	9	10	11	12	13	14
2.	$\boldsymbol{RE}(\bar{y}_7/\ \bar{y}_4)$	3185.725	3433.304	3622.812	3906.497	4166.378	3813.837	3926.922

3.	$\boldsymbol{RE}(\bar{\boldsymbol{y}}_{7}/\ \bar{\boldsymbol{y}}_{5})$	206.1158	157.0593	198.2759	208.1778	227.6207	302.4375	300.0433
4.	$\boldsymbol{RE}(\bar{\boldsymbol{y}}_{7}/\ \bar{\boldsymbol{y}}_{6})$	121.041	105.9675	116.5183	126.2429	125.8178	154.2325	155.8988
8.	$\boldsymbol{RE}(\bar{\boldsymbol{y}}_{5}/\ \bar{\boldsymbol{y}}_{4})$	1545.6	2185.992	1827.157	1876.52	1830.404	1261.033	1308.785
9.	$\boldsymbol{RE}(\bar{\boldsymbol{y}}_{6}/\ \bar{\boldsymbol{y}}_{4})$	2631.938	3239.958	3109.221	3094.428	3311.437	2472.785	2518.892
10.	$\boldsymbol{RE}(\bar{\boldsymbol{y}}_{6}/\ \bar{\boldsymbol{y}}_{5})$	170.2858	148.2145	170.1672	164.9025	180.9129	196.092	192.4603

Table 9: The Relative Efficiency (RE) of estimators developed by Khan and Shabbir (2013) ratio, AL- Hossain and Khan (2014) regression and AL-Hossain and Khan (2014) ratio with the proposed estimator for the twenty-one simulated populations (percentage) for High and Maximum Cases

S/N	Populations	15	16	17	18	19	20	Average
2.	$\boldsymbol{RE(\bar{y}_7/\ \bar{y}_4)}$	5980.514	4959.148	5977.023	6575.026	10201.48	13952.5	**4675.686**
3.	$\boldsymbol{RE(\bar{y}_7/\ \bar{y}_5)}$	166.7048	352.8791	543.3927	407.5045	462.7348	491.4937	**266.2312**
4.	$\boldsymbol{RE(\bar{y}_7/\ \bar{y}_6)}$	106.9121	172.3863	234.1838	173.2953	203.2208	198.6486	**139.7252**
8.	$\boldsymbol{RE(\bar{y}_5/\ \bar{y}_4)}$	3587.488	1405.339	1099.945	1613.485	2204.607	2838.796	**1768.62**
9.	$\boldsymbol{RE(\bar{y}_6/\ \bar{y}_4)}$	5593.858	2876.764	2552.278	3794.116	5019.901	7023.709	**3230.413**
10.	$\boldsymbol{RE(\bar{y}_6/\ \bar{y}_5)}$	155.9269	204.7025	232.0368	235.1503	227.7005	247.4186	**182.9955**

7. PERCENTAGE RELATIVE EFFICIENCY ANALYSIS for Low and Minimum Extreme Values (LMiEV) Cases

7. 1. Relative efficiency comparison of the proposed estimator with the reviewed estimators of Khan and Shabbir (2013) ratio, AL-Hossain and Khan (2014) regression and AL-Hossain and Khan (2014) ratio for LMiEV Cases is shown in table 10 to 12.

Table 10: The Relative Efficiency (RE) for the twenty simulated populations (measured in percentage) for (LMiEV) Cases

S/N	Populations	1	2	3	4	5	6	7
2.	$RE(\bar{y}_7/\ \bar{y}_4)$	1347.723	834.8982	711.8462	939.5984	787.8035	706.0441	1166.393
3.	$RE(\bar{y}_7/\ \bar{y}_5)$	162.3249	119.8845	113.9064	132.8765	128.5941	117.6756	155.8352
4.	$RE(\bar{y}_7/\ \bar{y}_6)$	109.8319	99.98373	99.9387	101.9225	101.2601	100.0215	110.7448
8.	$RE(\bar{y}_5/\ \bar{y}_4)$	830.2629	696.4185	624.9397	707.1217	612.6281	599.9921	748.4786
9.	$RE(\bar{y}_6/\ \bar{y}_4)$	1227.078	835.034	712.2828	921.8757	777.9996	705.8922	1053.226
10.	$RE(\bar{y}_6/\ \bar{y}_5)$	147.7939	119.9041	113.9762	130.3701	126.9938	117.6502	140.7156

Table11: The Relative Efficiency (RE) for the twenty simulated populations (measured in percentage) for (LMiEV) Cases

S/N	Populations	14	15	16	17	18	19	20	Average
2.	$\boldsymbol{RE(\bar{y}_7/\ \bar{y}_4)}$	104.9453	160.0822	172.65	652.9122	30909.61	13227.37	19294.88	**3653.709**

S/N	Populations	8	9	10	11	12	13	14
2.	$\boldsymbol{RE(\bar{y}_7/\ \bar{y}_4)}$	366.2647	419.0812	257.9209	306.1681	559.7641	148.2288	104.9453
3.	$\boldsymbol{RE(\bar{y}_7/\ \bar{y}_5)}$	109.8676	114.7229	108.521	110.7653	128.9394	104.599	104.103
4.	$\boldsymbol{RE(\bar{y}_7/\ \bar{y}_6)}$	100.1637	99.9936	101.2332	101.0472	101.6851	108.0276	104.626
8.	$\boldsymbol{RE(\bar{y}_5/\ \bar{y}_4)}$	333.3691	365.2988	237.6692	276.4117	434.1296	141.7115	100.8092
9.	$\boldsymbol{RE(\bar{y}_6/\ \bar{y}_4)}$	365.6661	419.108	254.7789	302.995	550.4876	137.2138	100.3052
10.	$\boldsymbol{RE(\bar{y}_6/\ \bar{y}_5)}$	109.6881	114.7302	107.199	109.6173	126.8026	96.82619	99.50008

Table12: The Relative Efficiency (RE) for the twenty simulated populations (measured in percentage) for LMiEV Cases

3.	$\boldsymbol{RE(\bar{y}_7/\ \bar{y}_5)}$	104.103	104.2408	102.5359	112.8651	158.9043	138.6094	1170.181	**174.9976**
4.	$\boldsymbol{RE(\bar{y}_7/\ \bar{y}_6)}$	104.626	108.041	108.7711	125.2264	71.21009	3.890166	155.7498	**100.6684**
8.	$\boldsymbol{RE(\bar{y}_5/\ \bar{y}_4)}$	100.8092	153.5696	168.3801	578.4892	19451.71	9542.909	1648.88	**1912.659**
9.	$\boldsymbol{RE(\bar{y}_6/\ \bar{y}_4)}$	100.3052	148.168	158.7278	521.3855	43406.22	340020.6	12388.38	**20250.37**
10.	$\boldsymbol{RE(\bar{y}_6/\ \bar{y}_5)}$	99.50008	96.48261	94.26755	90.12882	223.1486	3563.071	751.3207	**324.0093**

DISCUSSION AND CONCLUSIONS

This study has proposed improved mixed estimator in the presence of extreme values using two auxiliary variables in single-phase sampling. The theoretical analysis shows that comparing the proposed estimator FEVSA ($\bar{y}_7$) with the ratio estimator of Khan and Shabbir (2013), ($\bar{y}_4$) and the ratio estimator of Al-Hossain and Khan (2014), ($\bar{y}_5$)., using their mean square errors, equations 28 and 29 both makes it obvious that FEVSA ($\bar{y}_7$) is superior to these estimators. Lastly, comparing FEVSA ($\bar{y}_7$) with the regression estimator of Al-Hossain and Khan (2014), ($\bar{y}_6$) using equation 30, could not be concluded; but later resolved using empirical analysis.

In the empirical analysis, the R statistical programming language was used to performed exploratory Data Analysis (EDA) for each of the twenty- populations and to summarize the main characteristics (not with visual statistical tools) of the distributions. The function code asymptotically computed the bias, Mean Square Error (MSE) and variance for the proposed and reviewed estimators in each of the twenty stimulated population.

Table 1 through 3 show that the Mean Square Error (MSE) obtained for the proposed estimator is smaller compared to that of the revealed estimators for the cases of HMaEV; this implies that the proposed estimator is more efficient than the reviewed. Likewise, table 4 through 6, agreed with table 1 to 3, that the proposed estimator is superior to the reviewed estimators. Using the ranks for the HMaEV cases, table 3 (the overall rank table for the HMaEV) shows that the proposed estimator FEVSA ($\bar{y}_7$), is ranked first and hence more effective than the reviewed estimators. This is also supported by table 6 (the overall rank table for the LMiEV cases). The empirical analysis revealed that the proposed FEVSA ($\bar{y}_7$), outperformed all the considered estimators for HMaEV and LMiEV cases.

Finally, the function code computed the Relative Efficiency (RE) of the proposed estimator to the reviewed estimators. This answer the question that says by what percentage is the proposed estimator efficient over the reviewed estimators. The percentage relative efficiency Table 9, reveals that the proposed estimator FEVSA, $\bar{y}_7$ is 6.3803%, 4375.686%, 66.2312% and 39.7252% relatively efficient over $\bar{y}_4, \bar{y}_5$ and $\bar{y}_6$ respectively for the HMaEV cases. Likewise, table 12 reveals that $\bar{y}_7$ is 12.361%, 3553.709%, 74.9976% and 0.6684% relatively efficient over $\bar{y}_4, \bar{y}_5$ and $\bar{y}_6$ respectively for the LMiEV cases. This implies that the proposed estimator $\bar{y}_7$ is asymptotically more efficient over all the estimators considered in this study irrespective of the type of extreme value case. Therefore, the proposed estimator is recommended subject to the validation of the condition of usage.

REFERENCES

1. Agunbiade, D. A. and Ogunyinka, P. I. (2013). Effect of Correction Level on the use of AuxiliaryVariable in Double Sampling for Regression Estimation. Open Journals of Statistics, 3(5): ISSN:2161-718,312-318.
2. Al – Hossain, A. and Khan, M. (2014). Efficiency of ratio, product and regression estimators under maximum and minimum values using two auxiliary variables, The Scientific World Journal, Article ID: 693783, 1-6.
3. Cochran, W. G. (1940). The estimation of the yields of cereal experiments by sampling for the ratio of grain to total produce. The Journal of Agricultural Science, 30(2), 262-275.
4. Kadilar, C. and Cingi, H. (2005). A new estimator using two auxiliary variables. Applied Mathematics and Computation, 162 (2): 901–908.
5. Khan, M. and Shabbir, J. (2013). Some improved ratio, product and regression estimators of finite population mean when using minimum and maximum values. The Scientific World Journal, 013: Article ID 431868, 7 pages.
6. Mohanty, S. (1967). Combination of Regression and Ratio Estimate. Journal of Indian Statistical Association, 5, 16-19.
7. Ogunyinka, P. I., Emmanuel, F. O. and Sodipo, A. A. (2019). General Class of ratio-cum- product estimators in two- phase sampling using multi-auxiliary variables. Anale.Seria Informatica. XVII fasc.2

8. Olatayo, T.O., Onibudo, M. M. and Ogunyinka, P. I. (2020). Extended Ratio Type Estimators Using Correlation Coefficient and Coefficient of Variation for Full Extreme Maximum and Minimum Values in Single Phase Sampling. Nigerian Journal of Physics, 29(2), 172-181
9. Ogunyinka, P. I., Olatayo, T. O. and Onibudo, M. M. (2021). Improved Ratio Type Estimators for Extreme Maximum and Minimum Value Cases in Single-Phase Sampling Scheme. Futo Journal Series (FUTOJNLS), 7(1), 263-277.
10. Raj, D. (1965). On a Method of Using Multi-auxiliary Information in Sampling Surveys. J. Amer. Statist. Assoc., 60, 270-277.
11. Särndal, C. E. (1972). Sample survey theory vs general statistical theory: Estimation of the population mean. *International Statistical Review/Revue Internationale de Statistique*, 40(1), 1-12

مقدر مختلط محسن باستخدام متغيرين مساعدين للقيم القصوى والدنيا القصوى الكاملة في أخذ العينات أحادية الطور

تيموثي أولاتايو بيتر ن. مادو بيتر الأول أوغونيينكا

جامعة أولابيسي أونابانجو ، آغو–إيوي ، قسم العلوم الرياضية ، نيجيريا

الخلاصة: تم إنشاء استخدام متغيرات مساعدة متعددة لتحسين الدقة في مقدرات النسبة والانحدار والمنتج على التوالي. ومع ذلك ، فإن وجود قيم متطرفة في التوزيع يمكن أن يلغي هذه الكفاءة أولاتايو وآخرون. (2020). يمكن أن تكون القيم القصوى قيم صغيرة أو دنيا أو كبيرة أو قصوى. طورت هذه الدراسة مقدر نسبة الانحدار مع متغيرين مساعدين ، معامل الارتباط ومعامل التباين تحت نوعين من القيم القصوى في التوزيع. تتناول هذه الدراسة حالات القيمة القصوى الكاملة التي افترضت أن كل من الدراسة ومتغيرين مساعدين لهما قيم متطرفة موجودة في توزيعاتهما. تم إجراء تحليلات الكفاءة النظرية والتجريبية والنسبة المئوية النسبية للقيم القصوى العالية والقصوى الكاملة (فهمييف) وحالات القيم القصوى المنخفضة والدنيا الكاملة (فلمييف). أظهر التحليل أن المقدر المطور فعال على المقدرات التي تمت مراجعتها.

الكلمات المفتاحية: المتغيرات المساعدة ، القيم القصوى ، متوسط الخطأ المربع ، مقدرات النسبة ، مقدر الانحدار ، أخذ العينات أحادي الطور .