Solution of Linear Electrical Circuit Problem using Differential Transformation Method

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Abstract: *In this paper, Differential Transformation Method DTM is introduced to study the singular system of a linear electrical circuit for time invariant and time varying cases. The discrete solutions obtained using DTM are compared with the exact solutions of the electrical circuit problem and are found to be very accurate and compatible. Graphs for inductor currents and capacitor voltages are presented to show the efficiency of DTM.*

Keywords:* *Singular systems, Differential Transformation Method DTM , Runge-Kutta method.

1. Introduction

Singular systems contain a mixture of algebraic and differential equations. In that sense, the algebraic equations represent the constraints to the solution of the differential part. These systems are also known as degenerate, descriptor or semi state and generalized state-space systems. The complex nature of singular system causes many difficulties in the analytical and numerical treatment of such systems, particularly when there is a need for their control. The system arises naturally as a linear approximation of system models or linear system models in many applications such as electrical networks, aircraft dynamics, neural delay systems, chemical, thermal and diffusion processes, large scale systems, robotics, biology, etc.,[1-4].

The mainstreams of electronic circuit theory and neural network theory will in forthcoming decades converge into general methodologies for the optimization of analogue nonlinear dynamic systems. As a demonstration of the viability of such a merger, a new modeling method will be described, which combines and extends ideas borrowed from methods and applications in electronic circuit and device modeling theory and numerical analysis [5-8] the popular error back propagation method (and other methods) for neural networks [9-12] and time domain extensions to neural networks in order to deal with dynamic systems [13-15].

In this paper we study the linear electrical circuit and the time varying linear electrical circuit. Samath et al [16] studied these circuits by applying the neural algorithm and Rang-Kutta method. Murugesan et al [17] studied them and used R-Butcher method. We try to apply the Differential Transformation Method DTM to solve these problems.

2. Linear Electrical Circuit

Consider the physical model of an electrical circuit discussed by Chu et al [6] as shown in figure 1.

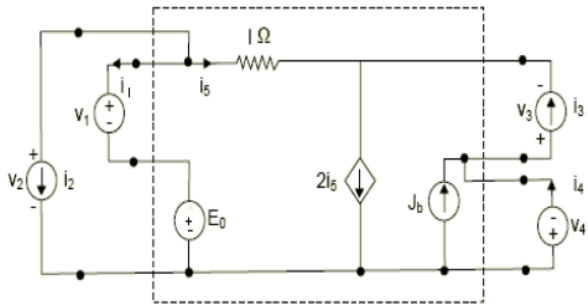

Figure1. Electrical Circuit

This electrical circuit is governed by the following hybrid equations [18]

$$\begin{bmatrix} i_1 \\ i_4 \\ v_2 \\ v_3 \end{bmatrix} = \begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 1 \end{pmatrix} \begin{bmatrix} v_1 \\ v_4 \\ i_2 \\ i_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ 1 & 0 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} E_a \\ J_b \end{bmatrix} \qquad (1)$$

Since $i_c = C\,\dot{v}_c$ and $v_e = L\,i_e$, substituting $i_1 = 2\,\dot{v}_1$, $i_2 = 2\,\dot{v}_2$, $v_3 = 2\,i_3$, and $v_4 = 2\,i_4$ into (1) we then obtain

$$\begin{bmatrix} 2\dot{v}_1 \\ i_4 \\ v_2 \\ 2\dot{i}_3 \end{bmatrix} = \begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ 2\dot{i}_4 \\ 2\dot{v}_2 \\ i_3 \end{pmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ 1 & 0 \\ -1 & 0 \end{bmatrix} \begin{pmatrix} E_a \\ J_b \end{pmatrix} \qquad (2)$$

After re-arranging the terms, we obtain the singular system of equations as

$$\left.\begin{array}{l} K\dot{X}(t)=Ax(t)+Bu(t) \\ \text{With the initial condition } x(0) = x_0 \end{array}\right\} \qquad (3)$$

where

$$K=\begin{pmatrix} 2 & 2 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} A=\begin{pmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix} B=\begin{pmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Where K is an n x n matrix, but singular in nature, therefore it is called singular systems. It is also called "generalized state space systems" or "descriptor systems". A is an n x n matrix, B is an n x r matrix, x(t) is an n-state vector, and u(t) is an r-input vector.

The singular system of equations is:

$$\left.\begin{array}{r} 2\,\dot{v}_1 + 2\,\dot{v}_2 = -\,i_3 \\ 2\,\dot{i}_3 + 2\,\dot{i}_4 = -v_1 + i_3 - E_a \\ v_2 = v_1 + E_a \\ i_4 = i_3 - J_b \end{array}\right\} \qquad (4)$$

In some cases, the variables have some inherent meaning such as voltage, current, position, velocity, or acceleration. or, the coefficient matrices have some special structures that may be lost by manipulating a system of the form in (3) into an ordinary state-space system.

By taking

$$\left.\begin{array}{r} E_a = 1 + t + \frac{t^2}{2} + \frac{t^3}{3} \\ J_b = 1 + t + t^2 \end{array}\right\} \qquad (5)$$

The exact solutions of (4) as [16]:

$$v_1(t) = -\frac{93}{2}\left(1-\sqrt{5}\right)\exp\left[\frac{1+\sqrt{5}}{8}\right]t - \frac{93}{2}\left(1+\sqrt{5}\right)\exp\left[\frac{1-\sqrt{5}}{8}\right]t - 27\,t + \frac{3t^2}{2} - \frac{t^3}{3} + 163$$

$v_2(t) = -\frac{93}{2}\left(1-\sqrt{5}\right)\exp\left[\frac{1+\sqrt{5}}{8}\right]t - \frac{93}{2}\left(1+\sqrt{5}\right)\exp\left[\frac{1-\sqrt{5}}{8}\right]t -$
$26\,t + 2\mathrm{t}^2 + 164$
$i_3(t) = -93\exp\left[\frac{1+\sqrt{5}}{8}\right]t - 93\exp\left[\frac{1-\sqrt{5}}{8}\right]t - 14\,t + 2\mathrm{t}^2 + 106$
$i_4(t) = -93\exp\left[\frac{1+\sqrt{5}}{8}\right]t - 93\exp\left[\frac{1-\sqrt{5}}{8}\right]t - 15\,t + \mathrm{t}^2 + 105$

(6)

With initial conditions

$$(v_1(0), v_2(0), i_3(0), i_4(0))^T = (70, 71, -80, -81)^T \qquad (7)$$

3. Differential Transformation Method DTM:

The differential transformation of the k-th derivatives of function $y(x)$ is defined as follows [19]:

$$Y(k) = \frac{1}{k!}\left[\frac{d^k y}{dx^k}\right]_{x=x_0} \qquad (8)$$

and $y(x)$ is the differential inverse transformation of $Y(k)$ defined as follows:

$$y(x) = \sum_{k=0}^{\infty} Y(k).(x-x_0)^k \qquad (9)$$

for finite series of k = N , Eq.(9) can be written as:

$$y(x) = \sum_{k=0}^{N} Y(k).(x-x_0)^k \qquad (10)$$

The following theorems that can be deduced from Eqs.(8) and (10) are given below:

Theorem 1. If $y(x) = g(x) \pm h(x)$,then $Y(k) = G(k) \pm H(k)$.

Theorem 2. If $y(x) = \alpha.g(x)$,then $Y(k) = \alpha.G(k)$.

Theorem 3.If $y(x) = \frac{dg(x)}{dx}$,then $Y(k) = (k+1).G(k+1)$

Theorem 4. If $y(x) = \frac{d^m g(x)}{dx^m}$, then $Y(k) = ((k+m)!/k!).G(k+m)$.

Theorem 5. If $y(x) = g(x).h(x)$,then $Y(k) = \sum_{l=0}^{k} G(l)H(k-l)$.

Theorem 6. If $y(x) = x^m$, then

$$Y(k) = \delta(k-m) = \begin{cases} 1 & if \quad k = m \\ 0 & if \quad k \neq m \end{cases}$$

Theorem 7. If $y(x) = \exp(\alpha.x)$,then $Y(k) = \alpha^k / k!$.

Theorem 8. If $y(x) = \sin(\alpha.x + \lambda)$,then $Y(k) = (\alpha^k / k!)\sin(k\pi/2 + \lambda)$.

Theorem 9. If $y(x) = \cos(\alpha.x + \lambda)$,then

$Y(k) = (\alpha^k / k!)\cos(k\pi/2 + \lambda)$.

Theorem 10. If $y(x) = \exp(u(x))$, then

$$Y(k) = \sum_{k=0}^{\infty}\sum_{n=0}^{r} \frac{1}{n!} \sum_{k_{n-1}=0}^{k} \sum_{k_{n-2}=0}^{k_{n-1}} \ddot{\cdots} \sum_{k2=0}^{k3} \sum_{k1=0}^{k2} U(k_1)U(k_2-k_1)\ldots\times$$

$$U(k_{n-1}-k_{n-2})U(k-k_{n-1})(x-k_0)^k$$

4. Solving the linear circuit by DTM

To solve system (4) for the definitions in (5), the system of recurrences equations is:

$$2\,(k+1)V_1(k+1) + 2(K+1)V_2\,(k+1) = -\,I_3(k)$$

$$2\,(k+1)I_3(k+1) + 2(K+1)I_4\,(k+1) = -V_1(k) + I_3(k\,) - \left[\delta(k) + \delta(k-1) + \frac{\delta(k-2)}{2} + \frac{\delta(k-3)}{3}\right]$$

$$I_4\,(k) = I_3\,(k) - [\delta(k) + \delta(k-1) + \delta(k-2)]$$

$$V_2\,(k) = V_1\,(k) + \left[\delta(k) + \delta(k-1) + \frac{\delta(k-2)}{2} + \frac{\delta(k-3)}{3}\right]$$

(11)

Where the initial values are:

$$I_4\,(0) = -81 \quad , I_3\,(0) = -80$$

$$V_2\,(0) = 71 \quad , V_1\,(0) = 70$$

Solutions by DTM for system (4) by the recurrences equations in system (11) are:

$$v_1(t) = v_1(0) + v_1(1)t + v_1(2)t^2 + v_1(3)t^3 + \ \dots\dots..$$

$$v_2(t) = v_2(0) + v_2(1)t + v_2(2)t^2 + v_2(3)t^3 + \ \dots\dots..$$

$$i_3(t) = i_3(0) + i_3(1)t + i_3(2)t^2 + i_3(3)t^3 + \ \dots\dots..$$

$$i_4(t) = i_4(0) + i_4(1)t + i_4(2)t^2 + i_4(3)t^3 + \ \dots\dots..$$

For $k = 0$, system (11) becomes:

$$2\,V_1(1) + 2V_2\,(1) = -\ I_3(0)$$

$$2\,I_3 + 2I_4 \ = \ -V_1(0) \ + \ I_3(0\,) - 1$$

$$I_4\,(0) = I_3\,(0) - 1$$

$$V_2\,(0) = V_1\,(0) + 1$$

and for $k = 1$, then

$$4V_1(2) + 4V_2\,(2) = -\ I_3(1)$$

$$4I_3(2) + 4I_4\,(2) = \ -V_1(1) \ + \ I_3(1\,) - 1$$

$$I_4\,(1) = I_3\,(1) - 1$$

$$V_2\,(1) = V_1\,(1) + 1$$

When $k = 2$, then

$$6V_1(3) + 6V_2\,(3) = -\ I_3(2)$$

$$6I_3(3) + 6I_4\,(3) = -V_1(2) + I_3(2\,) - \frac{1}{2}$$

$$I_4\,(2) = I_3\,(2) - 1$$

$$V_2\,(2) = V_1\,(2) + \frac{1}{2}$$

While for $k = 3$, then

$$8V_1(4) + 8V_2\,(4) + I_3(3) = 0$$

$$8I_3(4) + 8I_4\,(4) + V_1(3) - I_3(3) = -\frac{1}{3}$$

$$I_4\,(3) = I_3\,(3) - 1$$

$$V_2\,(3) = V_1\,(3) + \frac{1}{3}$$

Now a system of linear equations can be written as:

$[A]\ [X] = [B]$ (12)

Where

$$[A] = \begin{bmatrix} 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 4 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 4 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 6 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 6 & 6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 8 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 8 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \end{bmatrix}$$

and

$$[X] = [v_1(j), v_2(j), i_3(j), i_4(j)]^T \text{ for j=1,2,3,4}$$

and

$$[B] = [\,80\,, -151, 0, -1, -1, 1, 0, -\tfrac{1}{2}\,, -1, \tfrac{1}{2}, 0, \tfrac{1}{3}\,, 0, -\tfrac{1}{3}, 0, 0]^T$$

Solving system (12) by Gauss-Jordon elimination method then:

$$v_1(t) = 70 + 19.5\ t\ + 4.40625\ \ t^2 - 1/6\ \ t^3 + 0.02034505\ \ t^4 + \ldots\ldots$$

$$v_2(t) = 71 + 20.5\, t + 4.90625\, t^{\,2} + 1/6\ \ t^3 + 0.02034505\, t^4 + \ \ldots\ldots$$

$$i_3(t) = -80 - 37.25\, t - 6.65625\, t^2 - 0.3255208\, t^3 - 0.041178381\, t^4 + \ \ldots\ldots$$

$$i_4(t) = -81 - 38.25\, t - 7.65625\, t^2 - 0.3255208\, t^3 - 0.041178381\, t^4 + \ \ldots\ldots \quad (13)$$

Solutions in (13) by DTM for the variables v1(t), v2(t), i3(t) and i4(t) are represented by graphs in Figures 2 to 5 at various time intervals.

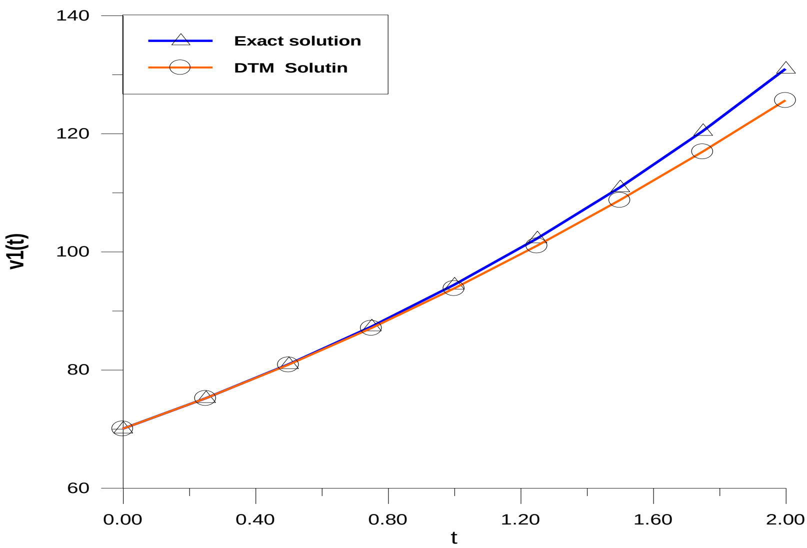

Figure (2) represents Exact and DTM solutions for $V_1(t)$.

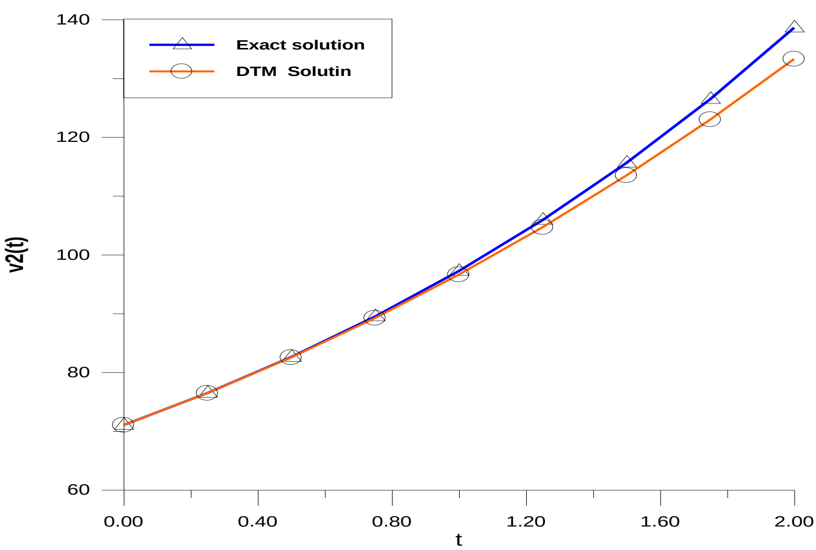

Figure (3) represents Exact and DTM solutions for V2(t).

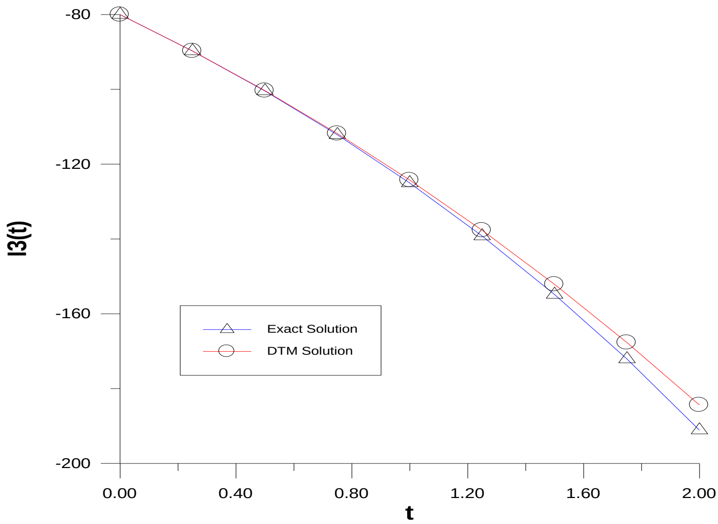

Figure (4) represents Exact and DTM solutions for $I_3(t)$.

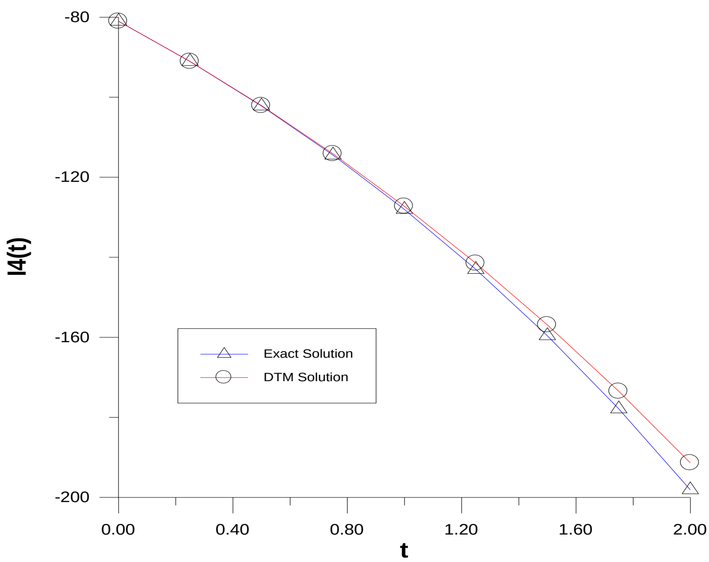

Figure (5) represents Exact and DTM solutions for I4(t).

5. Study of Time-Varying Linear Electrical Circuit

We applied DTM for the time–invariant electrical circuit problem and the results are compared with exact solution for studying the time-varying electrical circuit, which is represented by a singular system. Consider the electrical circuit depicted in Fig. 1 . The following hybrid equation is obtained[18].

$$\begin{pmatrix} 2 & 2 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i_3 \\ i_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} E_a \\ J_b \end{bmatrix} \tag{14}$$

This is of the form:

$$K\dot{X}(t) = Ax(t) + Bu(t) \tag{15}$$

In order to study the effectiveness of the time varying singular system in electrical circuits, a hypothetical system is formed by transforming the matrices K, A, and B, which are basically time independent in (14) with time-varying components.
Hence, the singular system of the time-varying electrical circuit is of the form

$$\begin{pmatrix} 2 & 2 & 0 & 0 \\ 0 & 0 & 2t & 2t \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & t & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & t & -t \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ i_3 \\ i_4 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -t & 0 \\ t & 0 \\ 0 & -t \end{pmatrix} \begin{pmatrix} t \\ \cos(t) \end{pmatrix} \tag{16}$$

This is of the form $K(t)\,\dot{x}(t) = A(t)x(t) + B(t)u(t)$.
The singular system of equations is:

$$2\,\dot{v}_1 + 2\,\dot{v}_2 = -\,i_3$$

$$2\,\dot{i}_3 + 2\,\dot{i}_4 = i_3 - t$$

$$v_2 = v_1 + t^2$$

$$i_4 = i_3 - cost \tag{17}$$

The exact solution of (15) is[16]

$$v_1(t) = -\frac{t^3}{12} - t^2 - \frac{(4-t)\,cost + (4t+1)\,sint}{34} + \frac{59(t-4)}{17}\, e^{t/4} + \frac{255}{17}$$

$$v_2(t) = v_1(t) + t^2$$

$$i_3(t) = t + 4 + \frac{2}{17}[sint + 4\,cost] - \frac{59}{17}\, e^{t/4}$$

$$i_4(t) = i_3(t) - cost$$

(18)

With initial conditions

$$(v_1(0), v_2(0), i_3(0), i_4(0))^T = (1,1,1,0)^T$$

To solve system (17) the system of recurrences equations is:

$$2\,(k+1)V_1(k+1) + 2(K+1)V_2\,(k+1) = -\,I_3(k)$$

$$2\,(k+1)I_3(k+1) + 2I_4\,(k+1) = I_3(k) - \delta(k-1)$$

$$I_4\,(k) = I_3\,(k) - \frac{1}{k!}\cos(k\pi/2)$$

$$V_2\,(k) = V_1\,(k) + \delta(k-2)$$

(19)

Where the initial values are:

$$I_4\,(0) = 0 \quad, I_3\,(0) = 1, V_2\,(0) = 1 \quad, V_1\,(0) = 1$$

Solutions by DTM for system (17) by the recurrences equations in system (19) are:

$$v_1(t) = v_1(0) + v_1(1)t + v_1(2)t^2 + v_1(3)t^3 + \dots\dots$$

$$v_2(t) = v_2(0) + v_2(1)t + v_2(2)t^2 + v_2(3)t^3 + \dots\dots$$

$$i_3(t) = i_3(0) + i_3(1)t + i_3(2)t^2 + i_3(3)t^3 + \dots\dots$$

$$i_4(t) = i_4(0) + i_4(1)t + i_4(2)t^2 + i_4(3)t^3 + \dots\dots$$

(20)

For $k = 0$, system (19) becomes:

$$2\,V_1(1) + 2V_2\,(1) = -1$$

$$2\,I_3(1) + 2I_4(1) = 1$$

and for $k = 1$, then

$$4V_1(2) + 4V_2\,(2) = -\,I_3(1)$$

$$4I_3(2) + 4I_4\,(2) = I_3(1) - 1$$

$$I_4\,(1) = I_3\,(1)$$

$$V_2\,(1) = V_1\,(1)$$

When $k = 2$, then

$$6V_1(3) + 6V_2\,(3) = -\,I_3(2)$$

$$6I_3(3) + 6I_4\,(3) = I_3(2)$$

$$I_4\,(2) = I_3\,(2) + 1/2$$

$$V_2\,(2) = V_1\,(2) + 1$$

While for $k = 3$, then

$$8V_1(4) + 8V_2\,(4) + I_3(3) = 0$$

$$8I_3(4) + 8I_4\,(4) - I_3(3) = 0$$

$$I_4\,(3) = I_3\,(3)$$

$$V_2\,(3) = V_1\,(3)$$

Now a system of linear equations can be written as:

$$[A]\ [X] = [B] \tag{21}$$

Where

$$[A] = \begin{bmatrix}
2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 4 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 4 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 6 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 6 & 6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 8 & 8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 8 & 8 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1
\end{bmatrix}$$

and

$$[X] = [v_1(j), v_2(j), i_3(j), i_4(j)]^T \text{ for j=1,2,3,4}$$

and

$$[B] = [\,-1\,,1,-1,0,0,0,0,0,\tfrac{1}{2}\,,1,0,0,0,0,0,-\tfrac{1}{24}]^T$$

Solving system (21) by Gauss-Jordon elimination method then

$$v_1(t) = 1 - \frac{1}{4}\,t - \frac{17}{32}\,t^2 + 11/384\ t^3 + 11/6144\ t^4 + \ \dots\dots$$

$$v_2(t) = 1 - \frac{1}{4}\,t + \frac{15}{32}\,t^2 + 11/384\ t^3 + 11/6144\ t^4 + \ \dots\dots$$

$$i_3(t) = 1 + \tfrac{1}{4}\,t - \tfrac{11}{32}\,t^2 - 11/384\ t^3 + 117/6144\ t^4 + \ \dots\dots$$

$$i_4(t) = \frac{1}{4}\,t + \frac{5}{32}\,t^2 - \frac{11}{384}\,t^3 - 139/6144\ t^4 + \ \dots\dots$$

(22)

Solutions in (22) by DTM for the variables v1(t), v2(t), i3(t) and i4(t) are represented by graphs in Figures 6 to 9 at various time intervals.

6. Conclusion

The solutions obtained using the DTM gives more accurate values when compared to the exact solutions of the electrical circuit problem irrespective of whether they are time-invariant or time varying cases,

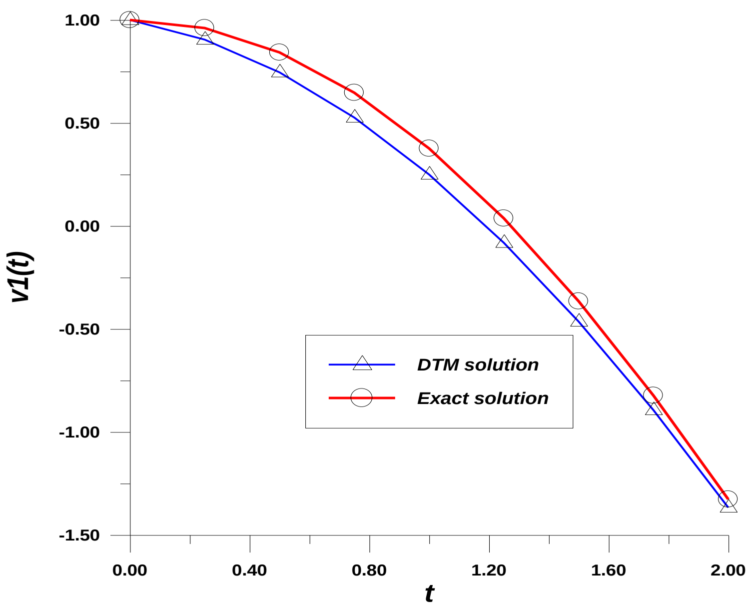

Figure (6) represents Exact and DTM solutions for $V_1(t)$.

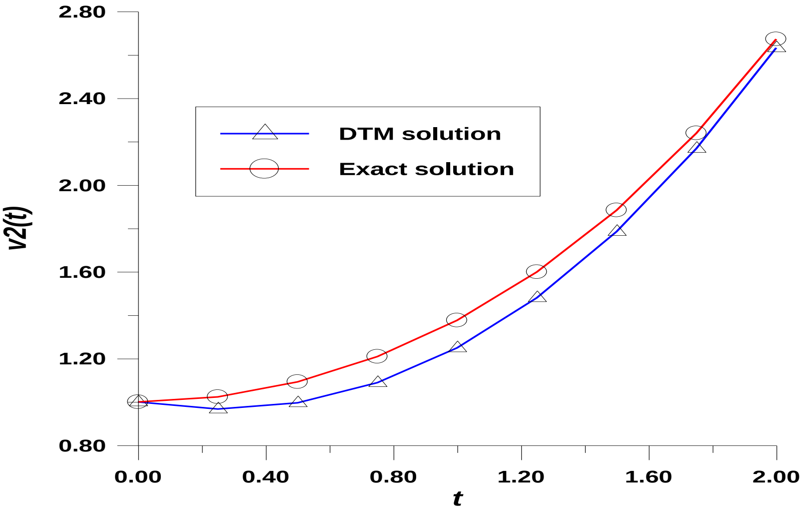

Figure (7) represents Exact and DTM solutions for $V_2(t)$.

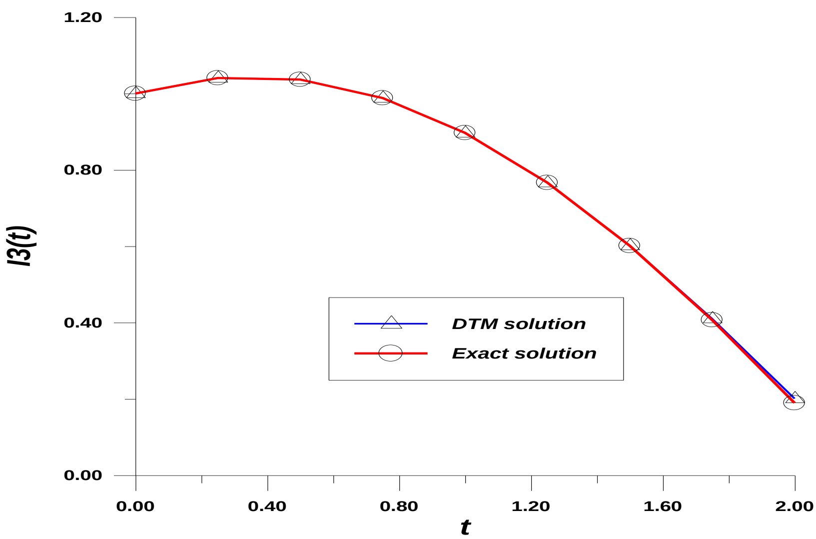

Figure (8) represents Exact and DTM solutions for $I_3(t)$.

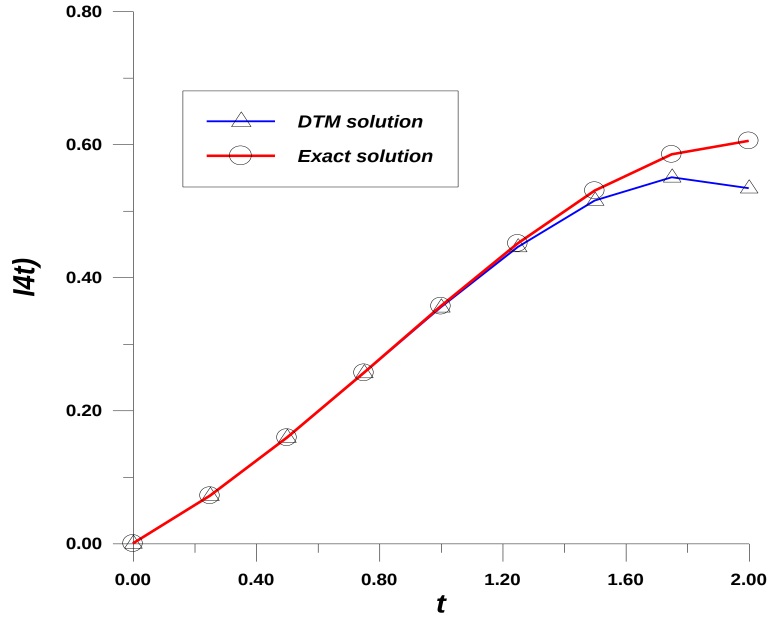

Figure (9) represents Exact and DTM solutions for $I_4(t)$.

References

[1] S.L.Campbell, Singular Systems of Differential Equations, Pitman, Marshfield, MA, (1980).

[2] S.L.Campbell, Singular Systems of Differential EquationsII, Pitman, Marshfield, MA, (1982).

[3] L.Dai, Singula r Control Systems, Lecture Notes in Control and Information Sciences, Springer Verlag, NewYork, (1989).

[4] F.L. Lewis, A Survey of linear singular systems, Circ. Syst. Signal Process. 5 , 3–36, (1986).

[5] S.I.Amari "Mathematical Foundations of neurocomputing" Proc. IEEE, Vol. 78, pp. 1443-1463.(1990).

[6] L.O. Chua and P.M. Lin, Computer-Aided Analysis of Electronic Circuits, Prentice-Hall, New Jersey, USA, (1975).

[7] L.O.Chua, C. A. Desoer and E. S. Kuh, Linear and Nonlinear Circuits. McGraw-Hill, (1987).

[8] D.R.Kincaid and E.W.Cheney, Numerical Analysis: Mathematics of Scientific Computing. Books/Cole Publishing Company, (1991).

[9] J. A. Anderson and E. Rosenfeld, Eds., Neurocomputing: Foundations of Research.Cambridge, MA: MIT Press, (1988).

[10] G.K.Boray and M.D.Srinath, ”Conjugate Gradient Techniques for Adaptive Filtering," IEEE Trans. CircuitsSyst.-I, Vol. 39, pp. 1-10, (1992).

[11] P. Friedel and D.Zwierski, Introduction to Neural networks. (Introduction aux Reseaux de Neurones.) LEP Technical Report C 91 503, (1991).

[12] D.Hammerstrom, “Neural networks at work," IEEE Spectrum, pp. 26-32, (1993).

[13] R. Hecht-Nielsen, “Nearest matched filter classification of spatio- temporal patterns ", Applied Optics, Vol. 26,pp. 1892-1899. , (1987).

[14] D. R. Hush and B. G. Horne, “Progress in Supervised Neural Networks," IEEE Sign.Proc. Mag., pp. 8-39, (1993).

[15] J. S. R. Jang, “Self-Learning Fuzzy Controllers Based on Temporal Back Propagation," IEEE Trans. Neural Networks, Vol. 3, pp. 714-723, (1992).

[16] J.A.Samath, P.S.Kumar, A.Begum, Solution of Linear Electrical Circuit Problem using Neural Networks, International Journal of Computer Applications, Volume 2, No.1, (2010).

[17] K. Murugesan, N.P. Gopalan, and Devarajan Gopal, Error Free Butcher Algorithms for Linear Electrical Circuits, ETRI Journal, Volume 27, Number 2, April (2005).

[18] P. De Wilde, Neural Network Models, second ed., Springer Verlag, London, (1997).

[19] Vedat Suat , Application of differential transformation method to linear sixth-order boundary value problems, applied mathematical Sciences, vol. 1, No. 2 , pp 51-58, (2007).

حل مسألة الدائرة الكهربائية الخطية باستخدام طريقة التحويل التفاضلي

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المستخلص:

تم في هذا البحث استخدام طريقة التحويل التفاضلي في دراسة النظام الفردي للدائرة الكهربائية الخطية الثابتة والمتغيرة مع الزمن. تم مقارنة نتائج الطريقة المقترحة مع الحلول التامة من خلال المخططات البيانية للفولتيات والتيارات لملاحظة كفاءة الطريقة المقترحة.