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Prediction Penalized Analysis of Stroke Based on New Bayesian Hierarchical Priors Model

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Abstract

The penalized or the regularization methods nowadays are the most statistical popular tools used for model selection and variable selection procedure. The quality of the regression parameter estimates depends on the prediction accuracy of the estimated model and the model interpretability. Penalized operator methods usually produced the most parsimonious model (less number of predictor and more explanation). This paper utilized new scale mixture of Rayleigh mixing with normal density to study the relationship between the stroke size and some predictors. New hierarchical priors model has developed as well new Gibbs sampling algorithm. The results demonstrated that the proposed model is comparable to some exists regularization models.

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Introduction

Many problem is real world involve Bayesian estimation, making Bayesian inference about a population using the information from small size of observation and using prior information. Suppose we have the following linear regression model [1]:

$$Y = X\beta + \varepsilon \tag{1}$$

Where Y is the n×1 vector of response observations, X is the n×(p+1) matrix of predictor variables, β is (p+1)×1 vector of coefficients, and ε is (n×1) vector of random errors with E(ε)=0, and Cov(ε) $= \sigma^2 I_n$. So that the regression function of the linear model Eq.(1) is

$$E(Y) = f(X, \beta) \tag{2}$$

The linear regression in Eq.(2) attempts to find the estimate average (mean) of the response variable based on the available information in matrix X. If the Gauss-Markov properties are met, the ordinary least squares (OLS) method that minimize the residual sum of squares (RSS) defined as [1-2]

$$RSS(\beta) = \sum_{i=1}^{n}(y_i - f(X_i, \beta))^2 \quad (3)$$

The OLS method [1-2] gives unbiased and lowest variance estimators. Therefore, OLS method provides best prediction accuracy. But, in many practical situations, the number of the observations for the studied phenomena is berry small with more predictor variables. In these situations the matrix X will be singular matrix (not invertible) because of the correlated predictor variable, which yields biased and inflated variance estimators. Therefore, poor prediction accuracy is based on the non-unique estimates of the parameters. Consequently, many estimation methods have developed to overcome the drawbacks of the OLS method, such as subset selection methods and the penalized ridge method. The penalized or regularized methods produced based estimates but with lowest variance, which is useful in terms of prediction accuracy.

Ridge regression method introduced by [3] to control the variance of the estimated parameters and then gives more prediction accuracy. The ridge estimator is minimize the following optimization problem,

$$\hat{\beta}_{ridge} = argmin\, RSS(\beta) + \lambda\|\beta\|^2 \quad (4)$$

Where $\lambda \geq 0$ is the shrinkage parameter that controls the amount of the shrinkage of regression parameters. When $\lambda = 0$ in Eq.(4) becomes OLS method. $\|\beta\|^2$ is the *l_2-norm* which is the differentiable function at zero, thus ridge method not sparse method. Tibshirani [4] introduced lasso regularization method which is a sparse solution method that removing the irrelevant prediction variables that have no effect on the response variable and include the relevant prediction variables that have effect on the response variable. Removing irrelevant predictor variables means that set its parameters equals to zero. So, lasso provides more prediction accuracy with more interpretable regression model. The lasso estimators defined by the following optimization problem,

$$\hat{\beta}_{lasso} = argmin\, RSS(\beta) + \lambda\|\beta\|_1 \quad (5)$$

Where $\|\beta\|_1$ is the *l_1-norm* function which is non-differentiable function, and $\lambda \geq 0$ is the shrinkage parameter [5]. Also, [4] stated that the parameter β can be estimated in Bayes theorem by assuming that this parameter follows the double exponential distribution as prior density. Since then many studies have done on the Bayesian aspect of lasso. Park and Casella [6] developed the hierarchical priors model based on the normal scale mixture to represent the double exponential distribution of the parameterβ. So, the lasso estimate can be interpreted as the posterior model estimate. The construction of lasso assumed that the design matrix X is standardized and Y is centered in traditional and Bayesian aspects. Mallick and Yi [7] proposed new scale mixture of uniforms mixing with special case of Gamma (2,λ) distribution. This scale mixture represents the prior distribution of double exponential density. Park and Casella [6] use the scale mixture of normal that proposed by [8]. Also, different scale mixtures have developed based on the scale mixture of normal, see [9] and [10]

The prior distribution of double exponential that proposed by [6] used the following form:

$$\pi(\sigma^2) = (\frac{\lambda}{2\sqrt{\sigma^2}})^p\, exp\left\{-\sum_{j=1}^{p}\frac{\lambda|\beta_j|}{\sqrt{\sigma^2}}\right\} \quad (6)$$

The Eq. (6) is conditional on σ^2 to guarantee the unimodal. Kyug et al. [11] state the Bayesian Gibbs sampler of [6] provide a good measure of standard error for the parameter estimates [7] state that MCMC of Bayesian estimation gives a flexible way for estimate by the shrinkage parameter.

Scale Mixture of Rayleigh-Normal (SMRN)

In the prior we will use the scale mixture of Rayleigh mixing with normal distribution that proposed by [12], where the conditional double exponential distribution have the following form

$$\pi(\sigma^2) = \prod_{j=1}^{p} \frac{1}{2\sqrt{\sigma^2}}\, exp\left\{-\frac{\lambda|\beta_j|}{\sqrt{\sigma^2}}\right\} = \prod_{j=0}^{p} \int_0^{\infty} \frac{1}{\sqrt{2\pi\sigma^2\tau}} e^{-\frac{\beta_j^2}{2\sigma^2\tau}} \frac{\lambda}{2}\, e^{-\frac{\lambda\tau}{2}} d\tau \qquad (7)$$

and the full joint density was of the form :

$$y|X,\beta,\sigma \sim Normal(X\beta,\sigma^2 I_n)$$
$$\beta|\sigma^2,\tau_1,\dots,\tau_p \sim Normal(0,\sigma^2 E_\tau)$$
$$E_\tau = diag(\tau_1,\dots,\tau_p)$$
$$\sigma^2,\tau_1,\dots,\tau_p \sim \pi(\sigma^2)d(\sigma^2)\prod_{j=1}^{p}\frac{\lambda}{2}exp\left\{-\frac{\lambda\tau_j}{2}\right\}d\tau_j \qquad (8)$$

Hierarchical Priors Model

Using the prior distribution in Eq. (7) and the linear regression mode Eq.(1), we formulate the following representation of hierarchical prior model. Also, we modified the likelihood function in in the Bayesian rule through raising the likelihood function to new parameter (θ) which called safe parameter, see [13] for more details. The new method is called generalized Bayesian penalized lasso. Now the full joint density is defined by :

$$f(y|\beta,\sigma^2)\pi(\sigma^2)\prod_{j=1}^{p}\pi(\tau_j,\sigma^2)\,\pi(\tau_j)$$
$$= \left[\left(\frac{1}{2\pi\sigma^2}\right)^{\frac{n}{2}} exp\left\{-\frac{1}{2\sigma^2}(y-X\beta)'(y - X\beta)\right\}\right]^{\theta} \frac{\gamma^a}{\Gamma a}\,(\sigma^2)^{-a-1}e^{-\frac{\gamma}{\sigma^2}}\prod_{j=1}^{p}\frac{1}{\sqrt{2\pi\sigma^2\tau_j}}e^{-\frac{\beta_j^2}{2\sigma^2\tau}}\,\frac{\lambda}{2}\,e^{-\frac{\lambda\tau_j}{2}} \qquad (9)$$

The parameter (θ) controls the amount of data (likelihood) effect on prior density. Now based on Eq.(7), Eq.(1) and Eq.(8) the full conditional distribution distributions are as follows:

1. The posterior distribution of β is multivariate normal distribution with mean $(\theta X'X - E_\tau^{-1})^{-1}X'y$ and variance $\sigma^2(X'X - E_\tau^{-1})^{-1}$.
2. The posterior distribution of σ^2 is inverse Gamma with the following shape parameter $\left[\theta\left(\frac{n+1}{2}\right)+\frac{p}{2}+a\right]$ and $\frac{\theta}{2}(y-X\beta)'(y - X\beta)+\frac{\beta' E_\tau^{-1}\beta}{2}+\gamma$.
3. The posterior distribution of τ is Inverse Gaussian $\left(\frac{1}{2},\lambda,\sqrt{\frac{\lambda\sigma^2}{\beta_j^2}}\right)$.
4. The posterior distribution of λ is Gamma $(p+a,\frac{1}{2}\sum_{j=1}^{p}\tau_j+b)$.

Finally, the parameter θ can be estimated by implementing the k-fold cross-validation method, see [14] for more information.

Simulation study

To evaluation the performance of the proposed method that based on modified the likelihood function in the Bayesian rule through raising the likelihood function to safe parameter (θ), simulation study is considered. In this study the estimation accuracy is conducted to the proposed method NBLS compared to the other existing method Ridge regression Method (RRM), Lasso Regularized Method (LRM), and Bayesian Lasso Method (BLM), the bias of parameters estimators are computed by using the following formula,

$$Bias = \hat{\beta} - \beta^{true}$$

where β^{true} is the true parameter vector. As well as, the performance of the methods are evaluated based on the median of mean absolute deviations denoted as MMAD; where, $MMAD = median\left(mean\left(\left|x_i^T\hat{\beta} - x_i^T\beta^{true}\right|\right)\right)$.

In our simulation study, we have simulated eight predictor variables from the normal distribution $N_8(0,\Sigma)$, where the variance- covariance matrix Σ is $e^{|i-j|}$ for each (i,j)th element. From the following model we have simulated two samples size (30 and 100):

$$y = X\beta + e$$

Where e is the error term with normal distribution $e \sim N(0,3)$, β is the coefficients vector and we consider two cases:

Case one $\beta = (0.85, 0.85,0.85,0.85,0.85,0.85,0.85,0.85)$
Case two $\beta = (3, 2.5,0,0,2,0,0,0)$

We have constructed an R code to implement MCMC algorithm. K-fold cross validation method have been used to compute the value of the safe Bayes parameter, we found the best values are ($\theta = 0.4$ and $\theta = 0.5$). MCMC algorithm is run 20000 iteration and remove the first 5000 as burn in. We have shown the standard division of the parameters estimates in Table 1 for all methods proposed NBLS and existing methods RRM, LRM and BLM at the samples size 30 and 100. From this table we can see clearly that the proposed method get the smallest values of the SD among other methods at both cases and also at both samples. Compare to RRM and LRM methods, we can note that BLM method get small values in most cases and samples size. On the contrary, RRM method has the largest values of SD in both cases and also in both samples size.

Table 1: The average SD of the parameter estimates of NBLS , RRM, LRM and BLM methods

Sample size	Cases	Methods	$SD.\beta_1$	$SD.\beta_2$	$SD.\beta_3$	$SD.\beta_4$	$SD.\beta_5$	$SD.\beta_6$	$SD.\beta_7$	$SD.\beta_8$
N=30	Case one	RRM	1.4360	0.7321	1.5695	1.6209	0.9034	0.4059	0.7037	0.8488
		LRM	1.7742	0.6878	1.0184	1.4226	0.8358	0.1437	0.5911	0.7653
		BLM	1.0008	0.5328	0.7285	0.7683	0.5274	0.2846	0.4607	0.3798
		NBLS	0.7866	0.3457	0.1022	0.5557	0.4273	0.0472	0.3506	0.1550
	Case two	RRM	1.8973	0.7757	1.6323	1.3544	1.7719	1.1834	1.5282	1.2173
		LRM	1.6741	0.6845	1.4403	1.1950	1.5634	1.0442	1.3484	1.0741
		BLM	0.5116	0.5873	0.3120	0.4896	0.7835	0.4886	0.7731	0.4619
		NBLS	0.4644	0.3326	0.2439	0.4383	0.5535	0.3434	0.6317	0.3375
N=100	Case one	RRM	1.0959	0.9680	0.8243	1.7038	0.8435	1.0513	0.6250	1.1051
		LRM	0.9980	0.8533	0.6692	0.9303	0.2623	0.8122	0.5967	0.7126
		BLM	0.7721	0.3218	0.3843	0.7034	0.2563	0.4101	0.5853	0.6256
		NBLS	0.5975	0.2617	0.2934	0.4480	0.2572	0.2133	0.3720	0.3069
	Case two	RRM	1.1661	0.6955	1.3488	0.8576	1.0777	1.2370	0.8345	1.1088
		LRM	0.9020	0.6737	1.2003	0.5224	0.7681	1.1196	0.7460	0.8650
		BLM	0.4174	0.2995	0. 9503	0.3331	0.6261	0.7629	0.6216	0.7930
		NBLS	0.1502	0.1356	0.6099	0.3156	0.4396	0.5189	0.4309	0.6133

Table 2: The average bias of the parameter estimates of NBLS , RRM, LRM and BLM methods

Sample size	Cases	Methods	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
N=30	Case one	RRM	3.1588	2.4656	1.9383	2.2446	2.5297	1.9792	2.4242	2.1697
		LRM	1.6494	0.5111	1.4018	0.2586	0.0682	0.3063	0.4530	0.6354
		BLM	1.2759	0.1928	0.8050	0.9989	0.5865	0.1547	0.6683	0.9635
		NBLS	0.5921	0.6464	0.6823	0.6413	0.6712	0.7047	0.6509	0.6405
	Case two	RRM	2.1843	1.5420	1.8562	2.0804	2.2005	1.4699	1.8490	2.0787
		LRM	1.8448	0.8455	2.0546	1.3811	1.1822	0.9628	0.9125	0.8604
		BLM	1.4271	0.7303	1.1800	1.1720	1.0675	0.4861	0.9037	0.8499
		NBLS	0.6721	0.7168	0.6913	0.6947	0.6856	0.7209	0.7100	0.6948
N=100	Case one	RRM	1.8417	1.1523	13.0973	18.7547	13.0973	18.7547	1.5826	2.0296
		LRM	1.0047	0.0578	12.3043	3.9292	12.3043	3.9292	1.0032	0.4996
		BLM	0.2858	0.0338	7.4384	1.2172	7.4384	1.2172	1.6317	0.1657
		NBLS	0.0623	0.0133	3.0035	0.9685	3.0035	0.9685	0.5016	0.1445
	Case two	RRM	2.8112	1.8661	0.6402	0.4930	0.6402	0.4930	0.5486	0.6281
		LRM	1.4006	0.2304	0.3151	0.4804	0.5151	0.4804	0.3756	0.4715
		BLM	0.0169	0.0136	0.4154	0.4101	0.4154	0.4101	0.2166	0.2752
		NBLS	0.0070	0.0106	0.1456	0.2673	0.1456	0.2673	0.1050	0.1875

We have summarized the bias of all methods under study the existing methods RRM, LRM and BLM and the proposed method NBLS in Table 2. From the table we can see that clearly the proposed method NBLS get the smallest bias at both cases and samples, that indicated the proposed method is more accurate than the other methods. Whereas, RRM method get the largest values of bias in most cases and sample. Also we can notice that the bias of the BLM method was smaller than the LRM method. Table 3 show MMAD values for all methods in our study. The NBLS method has the lowest values of MMAD compare to the other method in this study. As same as that in Table 1 and Tale 2 the RRM method gets the largest values of MMAD. In addition, in this table the MMAD values for BLM method was smaller than that for LRM method.

Table 3: The values of MMAD of NBLS , RRM, LRM and BLM methods

Cases	Methods	MMAD	
		N=30	N=100
Case one	RRM	3.5208	3.4760
	LRM	2.0445	2.9983
	BLM	1.9210	1.8792
	NBLS	1.7172	1.5740
Case two	RRM	8.7384	6.0994
	LRM	7.3233	5.5467
	BLM	4.7506	3.8485
	NBLS	2.5801	1.6014

To check the convergence performance for the coefficient that estimated by the proposed method trace plots have been used in Figures 1-2. In Figure 1, we show the trace plots for the estimated parameters by the proposed method for case one and sample size n=30, whereas in Figure 2 we show the case two with sample size 100 for 20000 iterations. From these figures we can see that the chain of the estimated parameters for our proposed method there is low fluctuation in the chain and it became more stationary after the first 5000 iterations.

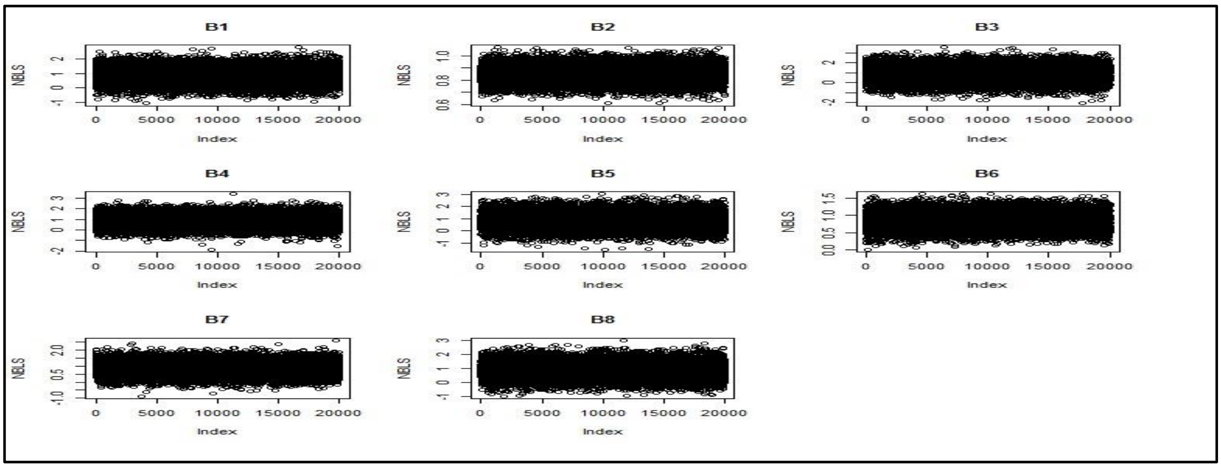

Figure 1: Shows the trace plots for the estimated parameters by the proposed method for case one and sample size n=30.

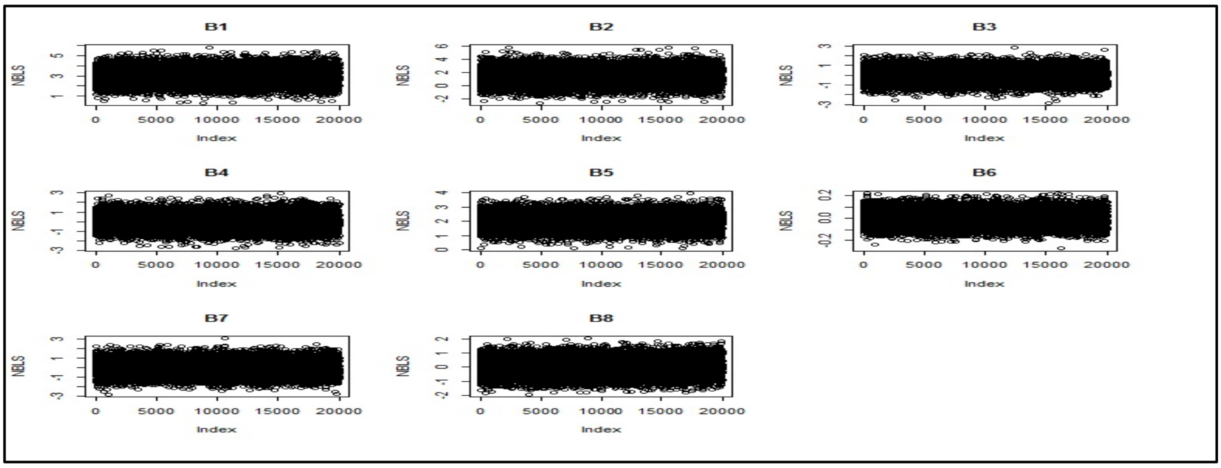

Figure 2: Shows the trace plots for the estimated parameters by the proposed method for case two and sample size n=100.

Real Application

A stroke size is one of the factors that impacting functional outcome for human. There are many different variables may be effect on the stroke size and then the human life. In this study, 150 patients are drawing from Al-Diwaniyah Hospital during the period from 1/10/2022 to 15/12/2022. We study the effect of the some independent variables on the response variable (Y: stroke size), whereas the independent variables is (X1: Age, X2: Smoking, X3: the weight of patient, X4: Blood pressure, X5: Diabetes, X6: Cholesterol, X7: Have a heart disease, X8: Gender, X9: Alcohol, X10: Exercise). Proposed method is adapted to relation between the response variable Y and covariates variables.

Table 4: Shows the parameters estimates by RRM, LRM, BLM and NBLM with$(\boldsymbol{\theta} = \mathbf{0.4})$.

Parameters	Variables	RRM	LRM	BLM	NBLS
$\boldsymbol{\beta_1}$	**Age**	1.6937	1.1547	1.1548	3.3754
$\boldsymbol{\beta_2}$	**Smoking**	-0.1403	1.7504	2.1750	1.5553
$\boldsymbol{\beta_3}$	**the weight of patient**	-0.0515	3.0511	0.0511	7.6430
$\boldsymbol{\beta_4}$	**Blood pressure**	1.0712	1.4700	3.6997	3.2848
$\boldsymbol{\beta_5}$	**Diabetes**	0.1462	1.5625	1.6254	4.1775
$\boldsymbol{\beta_6}$	**Cholesterol**	0.2266	-0.4045	1.0404	1.8183
$\boldsymbol{\beta_7}$	**Have a heart disease**	0.1082	7.5755	0.0058	0.0032
$\boldsymbol{\beta_8}$	**Gender**	0.1236	4.3548	1.3548	0.0008
$\boldsymbol{\beta_9}$	**Alcohol**	-0.5850	-1.1857	1.1857	-0.0124
$\boldsymbol{\beta_{10}}$	**Exercise**	-0.1108	4.9749	0.9749	-3.5498

In Table 4, result of the real data application is reported. This result show the coefficients estimated by the proposed NBLS and existing methods RRM, LRM and BLM. It's clearly, that the estimated coefficients (Have a heart disease, Gender and Alcohol) close to zero that meaning these variables do not have important effect on the size of stroke. The proposed method also showing that the variables (the weight of patient, Diabetes, Exercise, Age and Blood pressure) have large effect on the response variables, this method can selected the important variables. Also, we can see that the perform of the BLM method is better than the other existing method where this method can choose the important and unimportant variables.

Table 5: Shows the RMSE and MAE for the different methods

Methods	RMSE	MAE
RRM	9.7376	8.2063
LRM	8.1475	7.7808
BLM	5.0463	5.3358
NBLS	3.0291	2.7089

Table 5 shows the RMSE and the MAE for the proposed method, NBLS and the other three existing methods RRM, LRM and BLM. The performance of the method was better than that of the other methods, since it showed the smallest RMSE and MAE. From this table we can also see that the BLM method does better than the RRM, LRM methods.

Conclusion

The new formulation of the scale mixture of Rayleigh distribution mixing with exponential distribution has employed with the safe Bayesian parameter to the likelihood function. The proposed formula applied to the linear regression for the sake of variable selection procedure. This paper proposed new hierarchical priors model representation based on the developed formulation and then new posterior distribution have derived. Gibbs sampler algorithms have used to compute the mode of the posterior distribution of the interested parameters. Simulation experiments analysis have been conducted to find the solution of the shrinkage method and the results yields the minimum bias for the proposed method compared with the other models, also we have obtained the minimum value of the prediction accuracy criterion MMAD. Furthermore we performed the variable selection for real data analysis and the results showed that the proposed model is sparse model with the minimum MMAD and MAE.

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التنبؤ بالتحليل المعاقب للسكتة الدماغية بناءً على نموذج بايزي الهرمي الجديد

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المستخلص

تعد طرق العقاب أو التنظيم في الوقت الحاضر من أكثر الأدوات الإحصائية المستخدمة في اختيار النموذج وإجراءات اختيار المتغير شيوعًا. تعتمد جودة تقديرات معلمة الانحدار على دقة التنبؤ للنموذج المقدر وقابلية تفسير النموذج. عادة ما تنتج أساليب المشغل المعاقب النموذج الأكثر بخلا (عدد أقل من المتنبئين ومزيد من التفسير). تم في هذا البحث استخدام خليط مقياس جديد من خلط رايلي مع الكثافة الطبيعية لدراسة العلاقة بين حجم الضربة وبعض المتنبئات. لقد تم تطوير نموذج هرمي جديد للأقدمية بالإضافة إلى خوارزمية أخذ عينات Gibbs الجديدة. أظهرت النتائج أن النموذج المقترح يمكن مقارنته ببعض نماذج التنظيم الموجودة.

معلومات البحث

تواريخ البحث:
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الكلمات المفتاحية:
تحليل بايزي، النموذج الهرمي السابق، أخذ عينات جيبس، البخل.

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