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Some Properties and Applications of the Gompertz Topp-Leone Inverse Exponential Distribution

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Abstract

The Gompertz Topp Leone Inverse Exponential (GoTLIE) distribution, which has three parameters, is a novel distribution that we introduce in this article. Inverse exponential (IE) distributions with various parameter values can be combined to form the new distribution. The moments, moment generating function, quantile function, reliability function, and hazard function are only a few of the crucial structural aspects of the new model that are derived. We utilize the approach of greatest likelihood to estimate the new distribution's parameters (MLE). To show how flexible the new distribution is, we utilized two real data used to demonstrate the GoTLIE distribution.

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Introduction

Nowadays, in probability theory, both the reliability function and the survival function have the same property, which is the measurement of the life span of a particular system or living organism. The flexible distribution is considered to fit different lifetime data. These distributions have more flexibility than the base line distribution because they add one or more shape parameters. Furthermore, new distribution can take many different shapes. Many families and distributions have been established and studied by researchers. In 1993, Mudholkar and Srivastava proposed a modification of the Weibull family named Exponentiated Weibull distribution (EW) by adding one shape parameter [1], [2], and [3]. Marshall and Olkin (1997) developed a technique for adding one parameter to the baseline distribution, and the new distribution is very flexible for modeling real time data [4-8]. In 2002, Eugene et al. introduced a new technique by adding two shape parameters named Beta-G [9-11]. In 2012, Ristić and Balakrishnan introduced a new family of distributions generated by gamma-G [12-13]. In 2013, Cordeiro et al. investigated a new family by adding two parameters to the baseline distribution named exponentiated generalized-G [14]. Bourguignon et al. (2014) introduced a new family using an odd formula to define Weibull-G [15-16]. Abid et al. proposed a new method in 2017 to discover a new family called the [0,1] truncated-G family [17-19]. In 2017, Alizadeh et al. defined a new family named Gompertz-G by adding two shape parameters to any base line distribution [20]. Many researchers follow the work by Alizadeh et al. to define new distributions like [21-22]. In 2021, Khaleel et al. defined and studied another new family called Marshall Olkin Topp-Leone-G a days in probability theory, the flexible distribution consider to fit different the lifetime data. These distribution will be more flexibility then the base line distribution because we add one or more shape parameters as we can proof that in experimental application. Furthermore new distribution can gives many different shapes [23]. Finally, The Gompertz Tope Leone-G (GoTL-G) family of distributions was presented by Rannona in 2022 [24].Therefore the cumulative density function (cdf) and the probability density function (pdf) of the GoTL-G family of distributions is given by.

$$F_{GOTL-G}(x;\gamma,b,\zeta) = \left[1 - e^{\left\{\frac{1}{\gamma}(1-(1-[1-\bar{G}^2(x,\zeta)]^b)^{-\gamma}\right\}}\right] \quad x > 0 \tag{1}$$

Where $\bar{G}(x;\zeta) = 1 - G(x;\zeta) \Longrightarrow \bar{G}^2(x;\zeta) = (1 - G(x;\zeta))^2$ and respectively, for γ, b > 0 and ζ is the parameter vector.

$$x;\zeta)\ (1 - \bar{G}^2(x;\zeta))^{b-1}[1 - (1 - \bar{G}^2(x;\zeta)^b]^{-\gamma-1} * \left(e^{\left\{\frac{1}{\gamma}(1-(1-[1-\bar{G}^2(x,\zeta)]^b)^{-\gamma}\right\}}\right) \tag{2}$$

We are motivated to propose a distribution because:

- It can be considered as an appropriate distribution to model the skewed data available in the literature may not be adequately fitted.
- It may also be used to represent a wide range of real-world data sets in the disciplines of survival and industrial reliability.

It is highly prevalent in survival analysis to search for alternative distributions with a high degree of adaptability in the hazard rate function (hrf). The one parameter Inverse Exponential distribution otherwise known as the inverted Exponential distribution was introduced by Keller and Kamath [25]. It has an inverted bathtub failure rate and it is an important competitive model for the Exponential distribution. It has been identified and discussed by Lin et al., [26] as a lifetime model If X is a nonnegative Exponential random variable, then the distribution of a random variable $Y = \frac{1}{X}$ follows an inverse Exponential distribution. Hence, if X denotes a random variable, the cumulative density function (cdf) and the probability density function (pdf) of the inverse Exponential distribution with a scale parameter are respectively given by:

$$G(x;\alpha) = e^{\left(-\frac{\alpha}{x}\right)}, \quad x > 0\ , \alpha > 0 \tag{3}$$

$$g(x;\alpha) = \frac{\alpha}{x^2} e^{(-\frac{\alpha}{x})} \tag{4}$$

The following are the major themes covered in the sections: The Gompertz Topp Leone inverse Exponential (GoTLIE) distribution is defined in Section 2. Some of its features are obtained in Section 3. Section 4 investigates the precision of maximum likelihood estimators (MLEs). Section 5 uses two real data sets to demonstrate the superiority of the GoTLIE distribution over five known distributions. These competitors were picked based on their past real data management efforts (the focus of the applications of this work). The paper is concluded in Section 6.

Gompertz Topp - Leone Inverse Exponential Distribution

Inverse Exponential (IE) distribution has received appreciable attention in recent times. Since then, a number of authors have been applying and extending the distribution. By inserting Eq. (3) in Eq. (1) we have a cdf of new distribution as follows:

$$F(x;\gamma,b,\alpha) = \left[1 - e^{\left\{\frac{1}{\gamma}(1-(1-\left[1-\left[(1-e^{(-\frac{\alpha}{x})})^2\right])\right]^b)^{-\gamma}\right\}}\right] \quad x > 0, \quad \gamma, b, \alpha > 0 \tag{5}$$

Substituting Eq. (3) and Eq. (4) into Eq. (2) yields the appropriate pdf:

$$f(x;b,\gamma,\alpha)=2b\frac{\alpha}{x^2}e^{(-\frac{\alpha}{x})}(1-e^{(-\frac{\alpha}{x})})^2(1-\left[(1-e^{\left(-\frac{\alpha}{x}\right)})^2\right])^{b-1}$$
$$*\left[1-\left[1-\left[(1-e^{(-\frac{\alpha}{x})})^2\right]\right]^b\right]^{-\gamma-1}\left(e^{\left\{\frac{1}{\gamma}(1-(1-\left[1-\left[(1-e^{(-\frac{\alpha}{x})})^2\right]\right]^b)^{-\gamma}\right\}}\right) \tag{6}$$

The survival function and hazard function of GoTLIE distribution, respectively,

$$S(x;\gamma,b,\alpha)=e^{\left\{\frac{1}{\gamma}(1-(1-\left[1-\left[(1-e^{(-\frac{\alpha}{x})})^2\right])\right]^b)^{-\gamma}\right\}} \tag{7}$$

$$H(x;\gamma,b,\alpha)=\frac{2b\alpha e^{(-\frac{\alpha}{x})}(1-e^{(-\frac{\alpha}{x})})^2}{x^2}\left(1-\left[(1-e^{\left(-\frac{\alpha}{x}\right)})^2\right]\right)^{b-1}\left[1-\left[1-\left[(1-e^{(-\frac{\alpha}{x})})^2\right]\right]^b\right]^{-\gamma-1} \tag{8}$$

Figures 1, and 2, show plots of the pdf, and hazard rate function for the GoTLIE distribution. Figure 1 shows decreasing curves and unimodal curves right skewed and revise J shapes. Hence, the hazard rate shape of the GoTLIE distribution could decreasing or bathtub, increasing decreasing increasing depending on the parameter values.

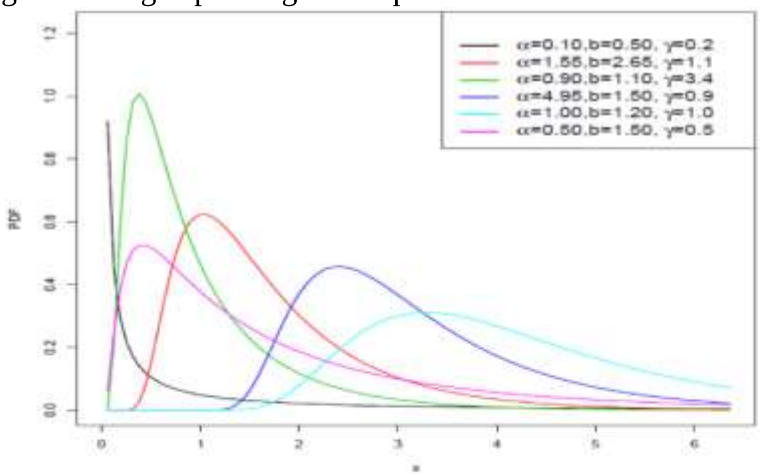

Figure 1: Different shapes of the pdf, with different value of parameters.

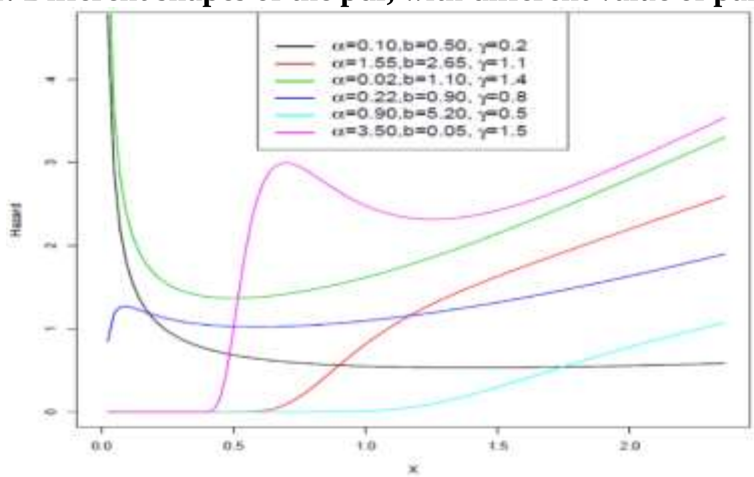

Figure 2: Different shapes of the h, function with different value of parameters.

Mathematical Properties of GoTLIE

In this section, many important properties of new distribution will find. It can help us to understand the used of distribution in real life phenomena. These properties are such as follows:

Useful Expansion

The idea of expansion distribution may be used to obtain expansions for equations (6). The expansion for the pdf of the GoTLIE distribution are provided as:

Let us use the exponential Taylor series $e^{-x} = \sum_{i=0}^{\infty} \frac{(-1)^i \, x^i}{i!}$ for expansion for the pdf of new distribution on Eq. (6):

$$f(x; b, \gamma, \alpha) = \sum_{k,L,M,p,q=0}^{\infty} \frac{(-1)^{L+M+p+q}}{k! \, \gamma^k (q+1)} \binom{k}{L} \binom{-\gamma(L+1)-1}{M} \binom{b(M+1)-1}{p} \binom{1+2p}{q} * (q+1) \frac{\alpha \left[e^{(-\frac{\alpha}{x})}\right]^{q+1}}{x^2} \tag{9}$$

Equation (9) can be written as a short form and it help us to find many properties than need many simplified before find the integration

$$f(x; b, \gamma, \alpha) = \mho \, (q+1) \, \alpha x^{-2} e^{-\alpha(1+q)x^{-1}} \tag{10}$$

Where

$$\mho = \sum_{k,L,M,p,q=0}^{\infty} \frac{(-1)^{L+M+p+q}}{k! \, \gamma^k (q+1)} \binom{k}{L} \binom{-\gamma(L+1)-1}{M} \binom{b(M+1)-1}{p} \binom{1+2p}{q}$$

Equation (10) is very important to study and find many properties that need integration and it is like (exponential-G) family distribution with different parameters.

Quantile Function

By inverting Eq. (6), we get the GoTLIE quantile function, say, x = Q(u), as shown below:[27-28]

$$F^{-1}(x) = u \tag{11}$$

Which yield

$$Q(x)_{GoTLIE} = \left(\alpha^{-1} log\, 1 - \left[1 - \left(1 - \left((1 - \gamma \, log(1-u)) \right)^{\frac{1}{-\gamma}} \right)^{\frac{1}{b}} \right]^{\frac{1}{2}} \right)^{-1} \tag{12}$$

Equation (12) is very important to find the Kurtosis and Skewness based on quartiles of the GoTLIE distribution. With this understanding, we can generate random sample for the GoTLIE distribution using:

$$x = \left(\alpha^{-1} log\, 1 - \left[1 - \left(1 - \left((1 - \gamma \, log(1-U)) \right)^{\frac{1}{-\gamma}} \right)^{\frac{1}{b}} \right]^{\frac{1}{2}} \right)^{-1} \tag{13}$$

Equation (13) very important to study simulation of the GoTLIE distribution where U is $Uniform$ (0,1).

Moments

Moments are significant in determining and quantifying several statistical features, such as smoothing and dispersion, coefficient of variation, standard deviation, and others [29-30]. The following connection may be used to calculate the rth moments of the GoTLIE distribution are expressed as:

$$E(X^r) = \int_0^{\infty} x^r f(x; b, \gamma, \alpha) dx$$

Using the PDF of GoTLIE in Eq. (10), we get

$$E(X^r) = \mho \int_0^{\infty} x^r \ (q+1)\ \alpha x^{-2} e^{-\alpha(1+q)x^{-1}} dx \tag{14}$$

Let $y = \alpha(1+q)x^{-1} \Longrightarrow x = \alpha(1+q)y^{-1} \Longrightarrow dy = -\ \alpha(1+q)x^{-2}dx$: then after some mathematical steps we can reduce Eq. (14) to

$$E(X^r) = \mu_r = \mho \int_0^{\infty} (\alpha(1+q)y^{-1})^r \, e^{-y} dy \Longrightarrow \mho\big(\alpha(1+q)\big)^r \int_0^{\infty} y^{-r} e^{-y} dy$$

The end result as

$$\mu_r = E(X^r) = \mho \left(\alpha(1+q)\right)^r \Gamma(1-r) \tag{15}$$

Equation (15) very important to find many statistical concepts like mean, the 4th moments, variance, CV, Moment generating function (MGF) and so on.

Moments Generating Function

By using Eq.(15) and Taylor expanstion $e^{tx} = \sum_{l=0}^{\infty} \frac{(\mathrm{t})^l x^l}{l!}$ we can get the MGF of new distribution as follows:[31]

$$M_x(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} f(x; b, \gamma, \alpha) dx$$

$$M_x(t) = \sum_{l=0}^{\infty} \frac{t^l}{l!} \int_0^{\infty} x^l f(x) dx = \sum_{l=0}^{\infty} \frac{t^l}{l!} E(X^l) = \sum_{l=0}^{\infty} \frac{t^l}{l!} [E(X^l)]$$

By using Eq. (58), it is same moment's function we have

$$M_x(t) = \sum_{l=0}^{\infty} \frac{t^l}{l!} \left[\mho \left(\alpha(1+q)\right)^r \Gamma(1-r)\right] \tag{16}$$

Order Statistics

Let $X_1, X_2, X_3, \ldots, X_n$ denote a random sample of size n drawn from the GoTLIE distribution, and $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(\mathrm{n})}$ denote the order statistics. If $X_{s:n}$ denotes statistics of the ith order, then the probability density function of $X_{s:n}$ is given by[31-32]

$$f_{s:n}(x) = \frac{n!}{(s-1)!\,(n-s)!} f(x). [F(x)]^{s-1}. [1-F(x)]^{n-s} \tag{17}$$

By using (5) and (6) we have

$$f_{s:n}(x) = \sum_{j=0}^{n-s} W(-1)^j \binom{n-s}{j} \left[1 - e^{\left\{\frac{1}{\gamma}(1-(1-\left[1-((1-e^{-\alpha(x)^{-1}})^2)\right]^b)^{-\gamma})\right\}}\right]^{j+s-1}$$
$$* 2\alpha b e^{-\alpha(x)^{-1}}(x)^{-2}\left[1 - e^{-\alpha(x)^{-1}}\right]\left[1 - \left(1 - e^{-\alpha(x)^{-1}}\right)^2\right]^{b-1} \tag{18}$$
$$* \left[1 - (1 - \left(1 - e^{-\alpha(x)^{-1}}\right)^2)^b\right]^{-\gamma-1} \left(e^{\left\{\frac{1}{\gamma}(1-(1-\left[1-\left(1-e^{-\alpha(x)^{-1}}\right)^2\right]^b)^{-\gamma})\right\}}\right)$$

Equation (18) very useful to find the 1st order when $s = 1$ and the large order when $s = n$.

Renyi Entropy

The amount of uncertainty for a random variable and the oscillation of any researched event is defined as entropy. It is employed in the field of statistics, for example, in statistical inference [33-34]. The Renyi Entropy is defined by $I_R(s) = \frac{1}{1-s}\log\int_0^\infty f(x)^s dx$. For the GoTLIE distribution given distribution, we have

$$I_R(S) = \frac{1}{1-s}\log\int_0^\infty [f(x;\alpha,b,\gamma)]^s dx \tag{19}$$

Which yield

$$(S) = \frac{1}{1-s}\log\int_0^\infty \left[2\alpha b e^{-\alpha(x)^{-1}}(x)^{-2}\left[1 - e^{-\alpha(x)^{-1}}\right]\left[1 - \left(1 - e^{-\alpha(x)^{-1}}\right)^2\right]^{b-1}\right]^s$$
$$* \left[\left[1 - (1 - \left(1 - e^{-\alpha(x)^{-1}}\right)^2)^b\right]^{-\gamma-1} \left(e^{\left\{\frac{1}{\gamma}(1-(1-\left[1-\left(1-e^{-\alpha(x)^{-1}}\right)^2\right]^b)^{-\gamma})\right\}}\right)\right]^s dx \tag{20}$$

After some algebra steps for reduce the Renyi Entropy we have

$$I_R(S) = \frac{(a^s)[(\alpha(s+p)]^{-2s+1}\log(\psi) + Log(\Gamma(2s-1)}{1-s} \tag{21}$$

where

$$\psi = \sum_{i,j,k,,L,p=0}^{\infty} \frac{s^i(2b)^s(-1)^{j+k+L+p}}{i!\,\gamma^i}\binom{i}{j}\binom{-\gamma(s+j)-s}{k}\binom{b(s+k)-s}{L}\binom{2L+s}{p}$$

Equation (21) very important in this section when $S \Longrightarrow 0$ we have Shannon Entropy.

Parameter Estimation of GoTLIE distribution

In this section, we continue by demonstrating the maximum likelihood (ML) estimation method for estimating parameters of the GOTLIE class of distributions. Many methods for estimating distribution parameters have been studied in the literature, but the ML estimation method is the most widely used. Let $X_1, X_2, \dots, X_n$ represent a random sample of observed values $x_1, x_2, \dots, x_n$ from the GOTLIE distribution. Consider $\Omega = (\alpha, b, \gamma)^T$. The log-likelihood (LL) function is given as:

$$l = n\log(2) + n\log(b) + \sum_{i=0}^{n}\log\left(\frac{\alpha}{x^2}Exp\left(-\frac{\alpha}{x}\right)\right)$$
$$+ \sum_{i=0}^{n}\log\left(1 - Exp\left(-\frac{\alpha}{x}\right)\right)^2 + (b-1)\sum_{i=0}^{n}\log\left(1 - \left(1 - Exp\left(-\frac{\alpha}{x}\right)\right)^2\right) \tag{22}$$
$$-(\gamma+1)\sum_{i=0}^{n}\ln\left(1 - \left(1 - \left(1 - Exp\left(-\frac{\alpha}{x}\right)\right)^2\right)^b\right) + \sum_{i=0}^{n}\frac{1}{\gamma}\left(1 - \left(1 - \left(1 - \left(1 - Exp\left(-\frac{\alpha}{x}\right)\right)^2\right)^b\right)^{-\gamma}\right)$$

The ML equations of the GOTLIE are obtained as follows:

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} - \sum_{i=0}^{n} \frac{1}{x} - \sum_{i=0}^{n} \frac{e^{-\alpha(x)^{-1}}}{x\left(1-e^{-\alpha(x)^{-1}}\right)} + \sum_{1}^{n} \frac{2\left(e^{-\alpha(x)^{-1}}\right)\left(1-e^{-\alpha x^{-1}}\right)}{x\left(1-\left(1-e^{-\alpha(x)^{-1}}\right)^{2}\right)}$$
$$+ \sum_{i=0}^{n} \frac{2(b-1)\left(e^{-\alpha(x)^{-1}}\right)\left(1-e^{-\alpha x^{-1}}\right)}{x\left(1-\left(1-e^{-\alpha(x)^{-1}}\right)^{2}\right)}$$
$$+ \sum_{i=0}^{n} \frac{2b(\lambda+1)\, e^{-\alpha(x)^{-1}}\left(1-e^{-\alpha x^{-1}}\right)\left(1-\left(1-e^{-\alpha(x)^{-1}}\right)^{2}\right)^{b-1}}{x\left(1-\left(1-\left(1-e^{-\alpha(x)^{-1}}\right)^{2}\right)^{b}\right)} \qquad (23)$$
$$- \sum_{i=0}^{n} \frac{2b\, e^{-\alpha(x)^{-1}}\left(1-e^{-\alpha x^{-1}}\right)\left(1-\left(1-e^{-\alpha(x)^{-1}}\right)^{2}\right)^{b-1}\left[1-\left(1-(1-e^{-\alpha(x)^{-1}})^{2}\right)^{b-1}\right]^{-\gamma-1}}{x}$$

$$\frac{\partial l}{\partial b} = -\sum_{i=0}^{n} ln\left(1-\left(1-e^{-\alpha(x)^{-1}}\right)^{2}\right) - \sum_{i=0}^{n} \frac{(\gamma+1)\, ln\left[1-\left(1-e^{-\alpha(x)^{-1}}\right)^{2}\right]\left[1-\left(1-e^{-\alpha(x)^{-1}}\right)^{2}\right]^{b}}{\left(1-\left[1-\left(1-e^{-\alpha(x)^{-1}}\right)^{2}\right]^{b}\right)} - \qquad (24)$$
$$-\sum_{i=0}^{n} \left(\left[1-(1-\left(1-e^{-\alpha(x)^{-1}}\right)^{2}\right]^{b}\right)^{-\gamma-1} ln\left(\left(1-e^{-\alpha(x)^{-1}}\right)^{2}\right)\left[(1-\left(1-e^{-\alpha(x)^{-1}}\right)^{2}\right]^{b}$$

$$\frac{\partial l}{\partial \gamma} = \sum_{i=0}^{n} \frac{\ln\left(1-\left(1-\left(1-e^{-\alpha x^{-1}}\right)^{2}\right)^{b}\right)\gamma - \left(1-\left(1-\left(1-e^{-\alpha x^{-1}}\right)^{2}\right)^{b}\right)^{\gamma} + 1}{\left(1-\left(1-\left(1-e^{-\alpha x^{-1}}\right)^{2}\right)^{b}\right)^{\gamma}\gamma^{2}} \qquad (25)$$
$$- \sum_{i=0}^{n} \ln\left(1-\left(1-\left(1-e^{-\alpha x^{-1}}\right)^{2}\right)^{b}\right)$$

By solving the nonlinear equations $\frac{\partial l}{\partial \alpha} = \frac{\partial l}{\partial b} = \frac{\partial l}{\partial \gamma} = 0$, using a numerical method such as the Newton-Raphson procedure. Because it is difficult to solve manually, we used here the package in R.

Application

In this part, we provide a real-world phenomenon for the GOTLIE distribution, which also fits better than other distributions. Their (NLL) negative log-likelihood is included in the comparison, (HQIC) Hanan and Quinn Information Criteria, (BIC) Bayesian Information Criteria, (CAIC) Consistent Akaike Information Criteria, (AIC) Akaike Information Criteria values, Kolmogorov-Smirnov (KS), P-Value and Anderson-Darling (A) statistics. The data fitting comparison between the GoTLIE distribution and other distributions such as, (KuIE) Kumaraswamy Inverse Exponential, (EGIE) Exponentiated Generalized Inverse Exponential, (BeIE) Beta Inverse Exponential, and Gompertz Inverse Exponential (GoIE), Weibull Inverse Exponential (WeIE) and Inverse Exponential (GoIE) distributions.

The first data set

The failure times of 40 lifting engines for heavy equipment are reported in the dataset. [35], [36], and [37] have already used the dataset to forecast the failure times of various engines. The data is (1.6, 2.0, 2.6, 3.0, 3.5, 3.9, 4.5, 4.6, 4.8, 5.0, 5.1, 5.3, 5.4, 5.6, 5.8, 6.0, 6.0, 6.1, 6.3, 6.5, 6.5, 6.7, 7.0, 7.1, 7.3, 7.3, 7.3, 7.7, 7.7, 7.8, 7.9, 8.0, 8.1, 8.3, 8.4, 8.4, 8.5, 8.7, 8.8, 9.0).

The second data set wireless mobile data

The second data set represents a wireless mobile data set were collected from 1432 participants in several metro cities across China [38]. (4.88, 5.16, 5.03, 4.87, 4.61, 4.79, 4.72, 4.97, 4.83, 4.82, 4.68, 4.97, 4.94, 4.12, 4.45, 4.40, 4.49, 4.49, 4.10, 4.28, 4.36, 4.10, 4.15, 4.28, 5.00, 4.90, 4.86, 4.63, 4.75, 4.81, 4.79, 4.68, 4.78, 4.81, 4.59, 4.66, 4.66, 4.51, 4.58, 4.88, 4.22, 4.93).

Table (1) include the MLE of the parameters for the models. Table (2) demonstrates that our recommended distribution fits the data substantially better than the other distributions. Under consideration these Statistics show that the **GoTLIE** distribution outperforms all fitted models. It has the lowest W, A, KS, HQIC, BIC, AIC, and CAIC values using real dataset 1,

Table (1): MLEs of GOTLIE for the failure times of 40 lifting engines.

Distribution	Parameters estimation		
	$\hat{\alpha}$	$\hat{b}$	$\hat{\gamma}$
GoTLIE	**6.37**	**0.34**	**31.52**
BeIE	0.47	32.7	29.6
KuIE	7.47	15.9	2.84
EGIE	47.2	0.42	31.7
WeIE	18.0	4.89	0.05
GoIE	0.06	3.51	2.64
IE			5.3

Table (2): -LL, AIC, CAIC, BIC, KS, P-Value, W, A. for the failure times of 40 lifting engines.

Distribution	-LL	AIC	CAIC	BIC	HQIC	KS	P-value	W	A
GoTLIE	**81.3**	168.9	169.4	173.8	170.6	**0.108**	**0.738**	**0.052**	**0.417**
BeIE	87.9	181.9	182.6	187.03	183.7	0.146	0.353	0.223	1.470
KuIE	90.9	187.8	188.5	192.9	189.6	0.150	0.323	0.291	1.84
EGIE	87.4	180.7	181.4	185.8	182.6	0.146	0.360	0.209	1.388
WeIE	83.7	173.4	174.1	178.5	175.2	0.115	0.656	0.150	0.758
GOIE	84.0	174.1	174.8	179.2	175.9	0.128	0.523	0.086	0.636
IE	114.6	231.2	231.3	232.8	231.8	0.445	0.0007	0.443	2654

Table (3) include the MLE of the parameters for the models. Table (4) demonstrates that our recommended distribution fits the data substantially better than the other distributions. Under consideration these Statistics show that the **GoTLIE** distribution outperforms all fitted models. It has the lowest W, A, KS, HQIC, BIC, AIC, and CAIC values using real dataset 2,

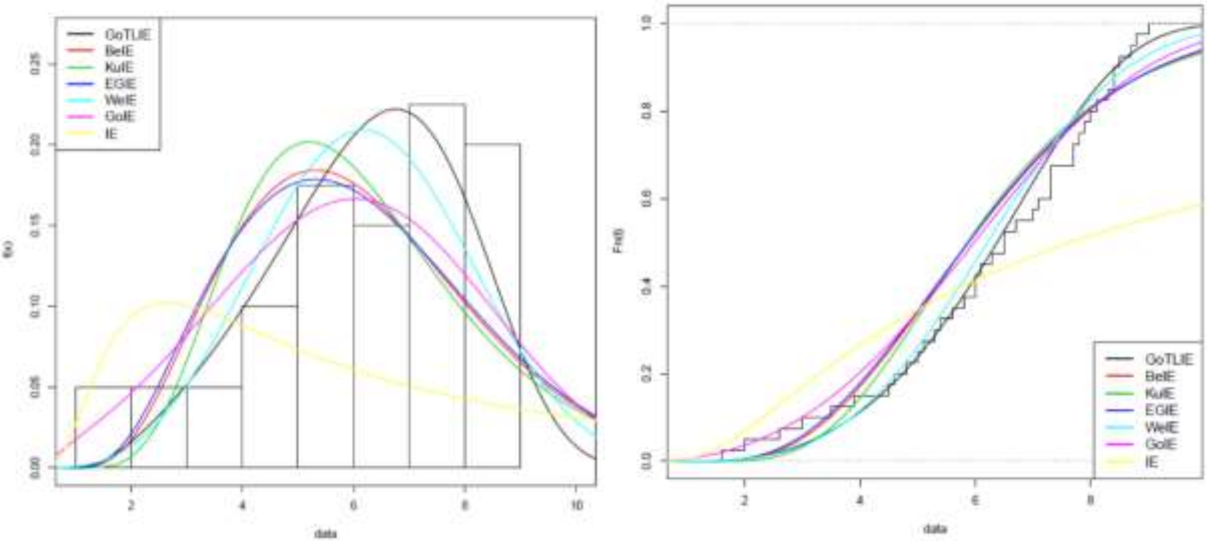

Figure 3: The estimated PDF, CDF of GoTLIE for Data1

Figures (3) shows the histogram plot for failure times of 40 lifting engines data set. It indicates that the GoTLIE distribution has the greatest peak and best matches the dataset's histogram. As a consequence, our distribution is a superior fit for the above real-world data.

Table (3): MLEs for wireless mobile data set.

Distribution	Parameters estimation		
	$\hat{\alpha}$	$\hat{b}$	$\hat{\gamma}$
GoTLIE	**68.52**	**20.84**	**2.10**
BeIE	103.6	318.2	6.52
KuIE	5.60	646.4	5.52
EGIE	200.6	53.86	17.77
WeIE	6.55	.45	4.78
GoIE	0.004	30.7	6.63
IE			4.63

Table (4): -LL, AIC, CAIC, BIC, KS, P-Value, W, A. for wireless mobile data set.

Distribution	-LL	AIC	CAIC	BIC	HQIC	KS	P-value	W	A
GoTLIE	**4.41**	**14.8**	**15.4**	**20.0**	**16.7**	**0.069**	**0.98**	**0.024**	**0.234**
BeIE	6.44	18.8	19.5	24.1	20.8	0.124	0.44	0.158	0.989
KuIE	20.3	60.7	61.5	65.5	62.4	0.268	0.004	0.11	0.785
EGIE	8.72	23.4	24.5	28.6	25.6	0.132	0.432	0.223	1.40
WEIE	21.5	49.5	49.8	54.3	51.8	0.251	0.009	0.100	0.643
GOIE	8.15	22.3	22.9	27.5	24.2	0.146	0.325	0.047	0.338
IE	106.5	215.1	215.2	216.9	215.8	0.592	0.0001	0.161	1.001

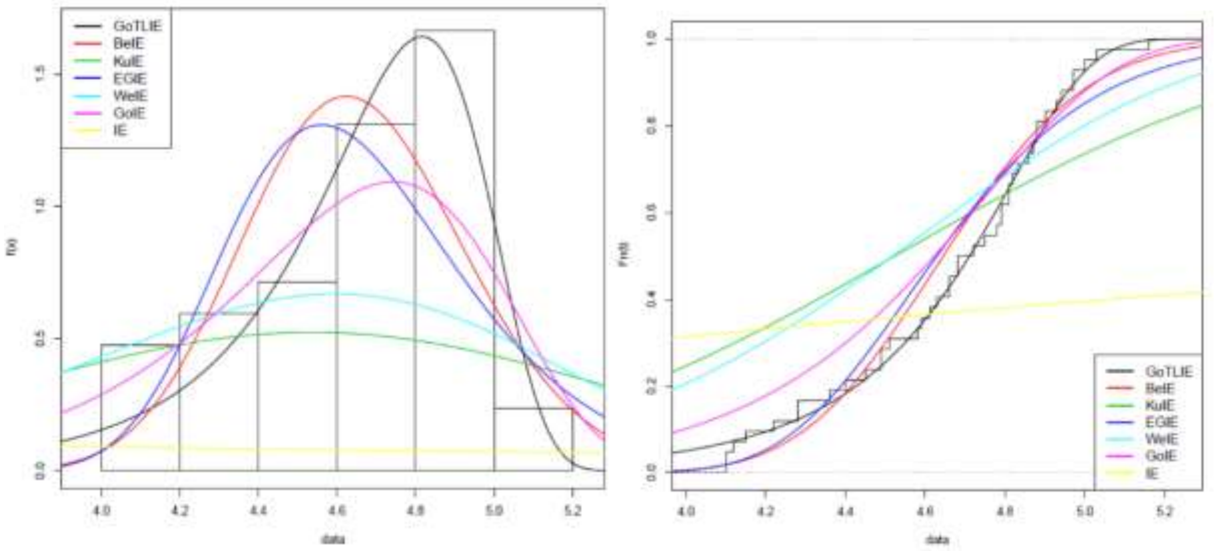

Figure 4: Estimated PDF and CDF for Data 2

Figures (4) show the fitted GoTLIE, PDF, and CDF, of the first data sets, respectively. Figures (4) shows the histogram plot for wireless mobile data set. It indicates that the GoTLIE distribution has the greatest peak and best matches the dataset's histogram. As a consequence, our distribution is a superior fit for the above real-world data.

Conclusion

In this study, the Gompertz Topp Leone Inverse Exponential (GoTLIE) distribution was effectively created and its different statistical features were investigated. The form of the model might be unimodal, declining, or rising. The maximum likelihood technique is offered as a method for estimating unknown model parameters. Failure rates may be used to explain and model real-world events such as bathtubs, inverted bathtubs, and the increase and decrease of bathtubs. When applied to real-world data sets, the GoTLIE distribution is found to be an improvement and a better option than other distributions.

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بعض خصائص وتطبيقات التوزيع الأسي العكسي Gompertz Topp-Leone

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المستخلص

في هذا البحث سنقدم توزيع جديد من ثلاث معلمات و يسمى بتوزيع كومبيرتز توب ليون معكوس الاسي يمكن اختصاره The Gompertz Topp Leone Inverse Exponential (GoTLIE). يمكن دمج معكوس التوزيع الاسي بمعلمتين لتوليد العديد من التوزيعات الجديدة. العديد من الخصائص الاحصائية مثل العزوم و الدالة المولدة للعزوم و الدالة التجزيئية و الدالة المعولية و دالة المخاطرة و بعض الدوال الاخرى تم اشتقاقها. تم تطبيق طريقة الامكان الاعظم لتقدير معلمات التوزيع الجديد. و لبيان مرونة التوزيع الجديد تم استخدام مجموعتين من البيانات الحقيقية لتفسير نمذجة التوزيع الجديد.

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