On The Identity $d(x) = \lambda x + \zeta(x)$

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Abstract: *The main purpose of this paper is to study and investigate some results concerning generalized derivation D on semiprime ring R, where d a derivation on R and R has a cancellation property with identity. Then under certain conditions we prove that there exist $\lambda \in C$ and an additive mapping $\zeta: R \to C$ such that $d(x) = \lambda x + \zeta(x)$ for all $x \in R$.*

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1. Introduction

Many authors have studied centralizing derivations, end omorphisms, and some related additive mappings. , let R be a ring with center Z(R),a mapping F of R into itself is called centralizing if F(x)x - xF(x) ∈ Z(R) for all x ∈ R, Matej Bresar[1]proved every additive centralizing mapping F on a von Neumann algebra R is of the form F(x) = cx + ζ(x), x ∈ R, where c ∈ Z(R) and ζ is an additive mapping from R into Z(R),and also consider α-derivations and some related mappings, which are centralizing on rings and Banach algebras. In fact this research has been motivated by the work of Matej Bresar [26].The history of commuting and centralizing mappings goes back to (1955) when Divinsky [3] proved that a simple Artinian ring is commutative if it has a commuting nontrivial automorphism. Two years later, Posner [4] has proved that the existence of a non-zero centralizing derivation on prime ring forces the ring to be commutative(Posner's second theorem).Luch [5]generalized the Divinsky result, we have just mentioned above, to arbitrary prime ring. Mayne [6] prove that in case there exists a nontrivial centralizing automorphism on a prime ring, then the ring is commutative (Mayne's theorem). Chung and Luh[7] have shown that every semicommuting automorphism of a prime ring is commuting provided that R has either characteristic different from 3 or non- zero center and thus they proved the commutativity of primering having nontrivial semicommuting automorphism except in the indicated cases Generalized derivation of operators onvarious algebraic structures have been an active areaof researchsince the last fifty years due to their usefulnessinvariousfields of mathematics. Some authors have studied centralizersin the generalframeworkof semiprimerings see[8,2,10,14,19,20,21,22,23and24]).MuhammadA.C.   and Mohammed S.S.[16] proved,let R be a semiprime ring and d:R→ R a mapping satisfy d(x)y=xd(y) for all x,y ∈ R.Then d is a centralizer.Molnar [15] has proved,let R be a 2-torsion free prime ring and let d:R → R be an additive mapping.If d(xyx)=d(x)yx holds for every x,y ∈ R,then d is a left centralizer. Muhammad A.C. and A. B.Thaheem [17] proved ,let d and g be a pair of

derivations of semiprime ring R satisfying $d(x)x+xg(x) \in$ $Z(R)$,then cd and cg are central for all $c \in Z(R)$.J.Vukman [24] proved, let R be a 2-torsion free semiprime ring and let $d:R \rightarrow R$ be an additive centralizing mapping on R, in this case, d is commuting R .B.Zalar [24] has proved, let R be a 2-torsion free semiprime ring and $d:R \rightarrow R$ an additive mapping which satisfies $d(x^2) = d(x)x$ for all $x \in R$.Then d is a left centralizer.Hvala [11] initiated the algebraic study of generalized derivation and extended some results concerning derivation to generalized derivation . Majeed and Mehsin [12] proved, let R be a 2- torsion free semiprime ring, (D,d) and (G,g) be generalized derivations of R,if R admits to satisfy $[D(x),G(x)]= [d(x),g(x)]$ for all $x \in R$ and d acts as a left centralizer, then (D,d) and (G,g) are orthogonal generalized derivations of R .Recently, Mehsin Jabel [13] proved, let R be a semiprime ring and U be a non-zero ideal of R.If R admits a generalized derivation D associated with a non-zero derivation d such that $D(xy) – yx \in Z(R)$ for all $x,y \in U$, then R contains a non-zero central ideal. Mehsin Jabel[8] proved, let R be a semiprime ring with left cancellation property,(D,d) and (G,g) be a non-zero generalized derivations of R,U a non-zero ideal of R,if R admits to satisfy $[d(x),g(x)]=o$ for all $x \in U$ and a non-zero d acts as a left centralizer (resp. a non-zero g acts as a left centralizer),then R contains a non-zero central ideal, where two a additive maps $d,g:R \rightarrow R$ are called orthogonal if $d(x)Rg(y) =o=g(y)Rd(x)$ for all $x,y \in R$. And two generalized derivations (D,d) and (G,g) of R are called orthogonal if $D(x)RG(y)=o=G(y)RD(x)$ for all $x,y \in R$, and we denote by (D,d) to a generalized derivation $D:R \rightarrow R$ determined by a derivation d of R .Recently, Mehsin Jabel[27,28 and 29] proved some results concerning generalized derivations on prime and semiprime rings.

In this paper we study and investigate some results concerning generalized derivation D on semiprime ring R, we give some results about that.

2.Preliminaries

Throughout this paper R will represent an associative ring with identity and has a cancellation property with the center Z(R).We recall that R is semiprime if xRx = (o) implies x=o and it is prime if xRy=(o) implies x=o or y=o.A prime ring is semiprime but the converse is not true in general . A ring R is 2-torsion free in case 2x = o implies that x = o for any x $\in$ R .An additive mapping d:R $\rightarrow$ R is called a derivation if d(xy)=d(x)y+xd(y) holds for all x,y $\in$ R.A mapping d is called centralizing if [d(x),x] $\in$ Z(R) for all x $\in$ R, in particular, if [d(x),x] = o for all x $\in$ R, then it is called commuting, and is called central if d(x) $\in$ Z(R) for all x $\in$ R.Every central mapping is obviously commuting but not conversely in general. In[10] Q.Deng and H.E.Bell extended the notion of commutativity to one of n-commutativity, where n is an arbitrary positive integer, by defining a mapping d to be n-commuting on U if [x^n,d(x)]=0 for all x $\in$ U, where U be a non empty subset of R .Following Bresar [9] an additive mapping D:R $\rightarrow$ R is called a generalized derivation on R if there exists a derivation d:R$\rightarrow$R such that D(xy)=D(x)y+xd(y) holds for all x,y $\in$ R. However , generalized derivation covers the concept of derivation. Also with d=o,a generalized derivation covers the concept of left multiplier (left centralizer) that is, an additive mapping D satisfying D(xy) = D(x)y for all x,y $\in$ R. A biadditive mapping B: R$_x$R$\rightarrow$ R is called a biderivation if for every u $\in$ R the mappings x $\rightarrow$B(x, u) and x $\rightarrow$B (u, x) are derivations of R. For any semiprime ring R one can construct the ring of quotients Q of R[25], As R can be embedded isomorphically in Q, we consider R as a subring of Q . If the element q $\in$ Q commutes with every element in R then q belongs to Z(Q), the center of Q. C contains the centroid of R, and it is called the extended centroid of R. In general, C is a von Neumann regular ring, and C is a field if and only if R is prime [25, Theorem 5].As usual, we write [x,y] for xy −yx and make use of the commutator identities [xy,z]=x[y,z]+[x,z]y and [x,yz]=y[x,z]+ [x,y]z, and the symbol xoy stands for the anti-commutator xy+yx.
The following Lemmas are necessary for this paper.

Lemma 2.1 [18,Corollary 9]
Any anticommutative semiprime ring R is commutative. Where A ring R is said to be anticommutative if
xy=-yx (that is, xy + yx = 0) for all x,y∈ R.

Lemma 2.2 [26,Theorem 4.1]
Let R be a semiprime ring, and let B:R$_x$R→ R be a biderivation. Then there exist an idempotent λ ∈ C and an element μ∈ C such that the algebra (1 - λ)R is commutative and λB(x,y) = μλ[x,y] for all x, y ∈ R.

3.The main results
Theorem 3.1
Let R be a semiprime ring.If R admits a non-zero generalized derivation D associated with a non- zero derivation d such that D([x,y])=[x,y] for all x,y∈ R. Then there exist λ∈C and an additive mapping ζ: R →C such that d(x) = λx + ζ(x) for all x ∈ R.

Proof: For x,y∈ R ,we have D([x,y])=[x,y] for all x,y∈ R, which gives:

D(x)y+xd(y)-D(y)x-yd(x)-[x,y]=0.   (1)

Replacing y by yz in (1),we obtain

D(x)yz+xd(y)z+xyd(z)-D(y)zx-yd(z)x-yzd(x)-y[x,z]-[x,y]z=0
                for all x,y∈ R.   (2)

Substituting (1) in (2) gives:

D(y)[x,z]+yd(x)z +xd(y)z+xyd(z)-yd(z)x-yzd(x)-y[x,z]=0
                for all x,y∈ R.   (3)

Replacing z by x in (3), we obtain:

xd(y)x+xyd(x) -yxd(x)=0 for all x,y∈ R.   (4)

Replacing y by x in (4), we get:

xd(x)x=0   for all x ∈ R.   (5)

By using the cancellation property on x ,from left,we obtain

d(x)x=0   for all x ∈ R.   (6)

Again by using the cancellation property on x ,from right,we get

xd(x)=0   for all x ∈ R.   (7)

Subtracting (6) and (7) ,we obtain[d(x),x]=o for all x ∈ R.

Linearizing [d(x), x] = 0 we obtain [d(x), y] = [x, d(y)]. Hence, we see that the mapping (x, y) → [d(x), y] is a biderivation. By Lemma 2.2 ,there exist an idempotent λ ∈C and an element μ ∈C such that the algebra (1 - λ)R is commutative (hence, (1 - λ)R ⊆ C), and λ[d(x), y] = λμ[x,y] holds for all x, y ∈ R. Thus, λd(x) - μλ x commutes with every element in R, so that λd(x) - μλ x ∈ C. Now, let λd(x) =μλ and define a mapping

ζ by ζ(x) = (λd(x) - λx) + (1 - λ)d(x). Note that ζ maps in C and that d(x) = λx + ζ(x) holds for every x ∈ R, by this we complete our proof.

A slight modification in the proof of the above theorem yields the following:

Theorem 3.2

   Let R be a semiprime ring. If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that D([x,y])+[x,y]=0 for all x,y∈ R. Then there exist λ∈C and an additive mapping ζ: R →C such that d(x) = λx + ζ(x) for all x ∈ R.

Theorem 3.3

   Let R be a semiprime ring.If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that D(xoy)=(xoy) for all x,y∈ R. Then there exist λ∈C and an additive mapping ζ: R →C such that d(x) = λx + ζ(x) for all x ∈ R.

Proof: For any x,y∈R,we have

D(xoy)=(xoy) for all x,y∈ R.

This can be written as

D(x)y+xd(y)+D(y)x+yd(x)-(xoy) =0   for all x,y∈ R.   (8)

Replacing y by yx in above equation ,we obtain:

$$D(x)yx+xd(y)x+xyd(x)+D(y)x^2 +yd(x)x-(xoy)x =0$$
$$\text{for all } x,y∈ R. \qquad (9)$$

According to (8) the relation above reduced to:

(xoy)d(x)=0   for all x,y∈ R.

By using the cancellation property on d(x),we get:

(xoy)=0 for all x,y∈ R.   (10)

By Lemma 2.1,we get :

[x,y]=0 for all x,y∈ R.   (11)

Replacing y by d(x),we get:

[d(x),x]=o for all x ∈ R. Linearizing [d(x), x] = 0 we obtain [d(x), y] = [x, d(y)]. Hence, we see that the mapping (x, y) → [d(x), y] is a biderivation. By Lemma 2.2 ,there exist an idempotent λ ∈ C and an element μ ∈ C such that the algebra (1 - λ)R is commutative (hence, (1 - λ)R ⊆ C), and λ[d(x), y] = λμ[x,y] holds for all x, y ∈ R. Thus, λd(x) - μλ x commutes with every element in R, so that λd(x) - μλ x ∈ C. Now, let λd(x) =μλ and define a mapping ζ by ζ(x) = (λd(x) - λx) + (1 - λ)d(x). Note that ζ maps in C and that d(x) = λx + ζ(x) holds for every x ∈R, by this we complete our proof. We complete our proof.

A slight modification in the proof of the Theorem (3.3) yields the following:

Theorem 3.4

    Let R be a semiprime ring.If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that D(xoy)+(xoy)=0 for all x,y∈ R. Then there exist λ∈C and an additive mapping ζ: R →C such that d(x) = λx + ζ(x) for all x ∈ R.

Theorem 3.5

    Let R be a semiprime ring.If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that d(x)oD(y)=0 for all x,y∈ R. Then there exist λ∈C and an additive mapping ζ: R →C such that d(x) = λx + ζ(x) for all x ∈ R .

Proof: We have d(x)oD(y)=0 for all x,y∈ R.   (12)

Replacing y by yr,we obtain

(d(x)oy)d(r)-y[d(x),d(r)] +(d(x)oD(y))r-D(y)[d(x),r]=0

for all $x,y \in U, r \in R$.   (13)
According to (12),then (13) reduced to:

$(d(x)oy)d(r)-y[d(x),d(r)] -D(y)[d(x),r]=0$ for all $x,y \in U, r \in R$.
Replacing r by d(x), we get:

$(d(x)oy)d^2(x)-y[d(x),d^2(x)]=0$ for all $x,y \in R$.   (14)
Replacing y by zy in (14), with using (14),we obtain
$[d(x),z]yd^2(x)=0$ for all $x,y,z \in R$.   (15)
By using the cancellation property on (15) ,from right, we obtain:
$[d(x),z]y=0$ for all $x,y,z \in R$.   (16)
Since R is semiprime from above relation , we get:
$[d(x),z]=0$ for al l$x,y \in R$.   (17)
Replacing z by x, we obtain, $[d(x),x]=o$ for all $x \in R$. Linearizing $[d(x), x] = 0$ we obtain $[d(x), y] = [x, d(y)]$. Hence, we see that the mapping $(x, y) \rightarrow [d(x), y]$ is a biderivation. By Lemma 2.2,there exist an idempotent $\lambda \in C$ and an element $\mu \in C$ such that the algebra $(1 - \lambda)R$ is commutative (hence, $(1 - \lambda)R \subseteq C$), and $\lambda[d(x), y] = \lambda\mu[x,y]$ holds for all $x, y \in R$. Thus, $\lambda d(x) - \mu\lambda x$ commutes with every element in R, so that $\lambda d(x) - \mu\lambda x \in C$. Now, let $\lambda d(x) = \mu\lambda$ and define a mapping ζ by $\zeta(x) = (\lambda d(x) - \lambda x) + (1 - \lambda)d(x)$. Note that ζ maps in C and that $d(x) = \lambda x + \zeta(x)$ holds for every $x \in R$, by this we complete our proof.

Theorem 3.6

Let R be a semiprime ring. If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that $[d(x),D(y)]=0$ for all $x,y \in R$. Then there exist $\lambda \in C$ and an additive mapping $\zeta: R \rightarrow C$ such that $d(x) = \lambda x + \zeta(x)$ for all $x \in R$.

Proof: We have $[d(x),D(y)]=0$ for all $x,y \in R$.   (18)

Replacing y by yz in(18) and using the result with (18),we obtain
$D(y)[d(x),z]+y[d(x),d(z)]+[d(x),y]d(z)=0$ for all $x,y \in R$.   (19)
Replacing z by zd(x) in (19) and using the result with (19), we get
$yz[d(x),d^2(x)] +y[d(x),z]d^2(x)+[d(x),y]zd^2(x) =0$

for all x,y ∈ R.   (20)

Again replacing y by ry in (20) and using the result with (20), we obtain:

$[d(x),z]yd^2(x)=0$ for all x,y,z ∈ R.

By using similar arguments as in the proof of Theorem 3.5, we obtain the required result.

Theorem 3.7

Let R be a semiprime ring. If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that d(x)oD(y)=xoy   for all x,y ∈ R. Then there exist λ ∈ C and an additive mapping ζ: R →C such that d(x) = λx + ζ(x) for all x ∈ R.

Proof: For any x,y ∈ R, we have

d(x)oD(y)=xoy for all x,y ∈ R. Replacing y by yr, we get:

(d(x)oy)d(r)-y[d(x),d(r)]+(d(x)oD(y))r-D(y)[d(x),r]=(xoy)r-y[x,r]
for all x,y ,r ∈ R. Using our relation ,we obtain
(d(x)oy)d(r)-y[d(x),d(r)]-D(y)[d(x),r] +y[x,r] =0
for all x,y ,r ∈ R.   (21)

In (21) replacing r by d(x), we obtain:
$(d(x)oy)d^2(x)-y[d(x),d^2(x)]+y[x,d(x)] =0$ for all x,y ∈ R.   (22)

Replacing y by zy in (22), we obtain:

$(z(d(x)oy)+[d(x),z]y)d^2(x)-zy[d(x),d^2(x)]+zy[x,d(x)]=0$
for all x,y ∈ R.   (23)

According to(22),above relation reduced to:

$[d(x),z]yd^2(x)=0$ for all x,y,z ∈ R.   (24)

By using similar arguments as in the proof of Theorem 3.5, we obtain the required result.

A slight modification in the proof of the Theorem(3.7), yields the following

Theorem 3.8

Let R be a semiprime ring.If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that d(x)oD(y)+xoy=0 for all x,y∈ R. Then there exist λ∈C and an additive mapping ζ: R →C such that d(x) = λx + ζ(x) for all x ∈ R.

Theorem 3.9

Let R be a semiprime ring.If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that d(x)D(y)-xy∈ Z(R) for all x,y∈ R. Then there exist λ∈C and an additive mapping ζ: R →C such that d(x) = λx + ζ(x) for all x ∈ R.

Proof: For any x,y∈R,we have

d(x) D(y)-xy∈ Z(R) ,replacing y by yr,we obtain

(d(x)D(y)-xy)r+d(x)yd(r) ∈ Z(R) for all x,y ,r∈R.   (25)

This implies that

[d(x)yd(r),r]=0 for all x,y ,r∈R.   (26)

Hence it follows that:

d(x)[yd(r),r]+[d(x),r]yd(r) =0   for all x,y ,r∈R.   (27)

In (27) replacing y by d(x)y, we obtain:

[d(x),r]d(x)yd(r)=0 for all x,y,r∈ R.   (28)

By using the cancellation property on d(x)yd(r),we obtain

[d(x),r]=0 for all x,r∈ R.   (29)

Replacing r by x in above relation, we obtain   :

[d(x),x] =0 for all x∈ R.   (30)

Then according to (30), we obtain:

d is commuting on R.

By the same method that we used in above theorem, we can prove the theorem

Theorem 3.10

Let R be a semiprime ring.If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that:

d(x)D(y)+xy$\in$ Z(R) for all x,y$\in$ R. Then there exist λ$\in$C and an additive mapping ζ: R →C such that:

d(x) = λx + ζ(x) for all x $\in$ R.

Theorem 3.11

Let R be a semiprime ring.If R admits a non-zero generalized derivation D associated with a non- zero derivation d such that [d(x),D(y)]=[x,y]   for all x,y$\in$ R. Then there exist λ$\in$C and an additive mapping ζ: R →C such that d(x) = λx + ζ(x) for all x $\in$ R.

Proof: For any x,y$\in$R, we have:

[d(x),D(y)]=[x,y]   for all x,y$\in$ R.   (31)

Replacing y by yz in (31), with using the result with (31), we obtain

D(y)[d(x),z]+y[d(x),d(z)]+[d(x),y]d(z)=y[x,z]

$\qquad\qquad\qquad\qquad\qquad$ for all x,y$\in$ R.   (32)

Again replacing z by zd(x) in (32) with using the result with (32), we obtain:

y[d(x),z]d^2(x)+yz[d(x),d^2(x)]+[d(x),y]zd^2(x)=yz[x,d(x)]

$\qquad\qquad\qquad\qquad$ for all x,y$\in$ R.   (33)

Replacing y by ry in(33),we obtain:

ryz[d(x),d^2(x)]+ry[d(x),z]d^2(x)+r[d(x),y]zd^2(x)+[d(x),r]yzd^2

(x)=ryz[x,d(x)] for all x,y,r$\in$ R.   (34)

According to (33),the relation(34) reduced to:

[d(x),r]yzd^2(x)=0 for all x,y,r$\in$ R.   (35)

Thus by same method in Theorem 3.5, we complete our proof.
Proceeding on the same lines with necessary variations, we can prove the following.

Theorem 3.12

Let R be a semiprime ring. If R admits a non-zero generalized derivation D associated with a non-zero derivation d such that [d(x),D(y)]+[x,y] =0 for all x,y$\in$ R. Then there exist λ$\in$C and an additive mapping ζ: R →C such that d(x) = λx + ζ(x) for all x $\in$ R.

Remark 2.13

In our theorems we cannot exclude the condition cancellation property , the following example explain that .

Example 2.14

Let R be a ring of matrices 2×2 with cancellation property, then

$R=\{\begin{pmatrix} a & 0 \\ o & b \end{pmatrix} / a^2 = a, a,b \in Z \}$. Where Z is the set of integers ,let m

be fixed element of Z and the additive map D define as the following

$$D\left(\begin{pmatrix} x & y \\ 0 & z \end{pmatrix}\right) = \begin{pmatrix} 0 & mx + mz \\ 0 & 0 \end{pmatrix}, and\ the\ derivation\ d\ defin\ as$$

$$d\left(\begin{pmatrix} x & y \\ 0 & z \end{pmatrix}\right) = \begin{pmatrix} 0 & mx - mz \\ 0 & 0 \end{pmatrix}.$$

$then\ it\ is\ easy\ that\ D\ is\ generalized\ derivation\ on\ R.\ then$

$$d(x)x = xd(x), where\ x = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} then$$

$$\begin{pmatrix} 0 & ma - mc \\ 0 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}\begin{pmatrix} 0 & ma - mc \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & cma - mc^2 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & ama - amc \\ 0 & 0 \end{pmatrix} \qquad (*)$$

$$\begin{pmatrix} 0 & m(ca - c^2 - a^2 + ac) \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, then$$

$$m\begin{pmatrix} 0 & 2ac - c^2 - a^2 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$m\begin{pmatrix} 0 & 2ac - c^2 - a^2 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 2ac - c^2 - a^2 \\ 0 & 0 \end{pmatrix}, then$$

by using the cancellation property on $\begin{pmatrix} 0 & 2ac - c^2 - a^2 \\ 0 & 0 \end{pmatrix}$ we

obtain $m = 0$, therefore , by substituting this result in (*) ,we get d is commuting . The additive mapping

$\zeta: R \rightarrow C$, by $\zeta(x)= \zeta\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$

(where it easy to prove that $\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$ is commuting with $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$),

i.e. $\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \in C$.

Also, we have $\lambda \in C$, then $\lambda = \begin{pmatrix} v & 0 \\ 0 & 0 \end{pmatrix}$, where $v \in Z$ the set of integers.

We have the $d(x)=d(\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix})=\begin{pmatrix} 0 & 0 \\ a-b & 0 \end{pmatrix}$. At first must be show that

d is central (i.e $d(x) \in Z(R)$),so for $r \in R$,we have

$$d(x)r\text{-}rd(x)=\begin{pmatrix} 0 & 0 \\ a-b & 0 \end{pmatrix}\begin{pmatrix} s & 0 \\ 0 & t \end{pmatrix}-\begin{pmatrix} s & 0 \\ 0 & t \end{pmatrix}\begin{pmatrix} 0 & 0 \\ a-b & 0 \end{pmatrix}=\begin{pmatrix} 0 & 0 \\ (a-b)s & 0 \end{pmatrix}-$$
$$\begin{pmatrix} 0 & 0 \\ (a-b)t & 0 \end{pmatrix}=\begin{pmatrix} 0 & 0 \\ (a-b)s-(a-b)t & 0 \end{pmatrix}=(a\text{-}b)\begin{pmatrix} 0 & 0 \\ s-t & 0 \end{pmatrix}=(a\text{-}b)\begin{pmatrix} 0 & 0 \\ s & 0 \end{pmatrix}$$
$$=(a\text{-}b)\begin{pmatrix} 0 & 0 \\ t & 0 \end{pmatrix}.$$

Then by using the cancellation property on (a-b) from left, we get
$\begin{pmatrix} 0 & 0 \\ s & 0 \end{pmatrix}=\begin{pmatrix} 0 & 0 \\ t & 0 \end{pmatrix}$, which give s=t, then

$$d(x)r\text{-}rd(x)=\begin{pmatrix} 0 & 0 \\ a-b & 0 \end{pmatrix}\begin{pmatrix} s & 0 \\ 0 & t \end{pmatrix}-\begin{pmatrix} s & 0 \\ 0 & t \end{pmatrix}\begin{pmatrix} 0 & 0 \\ a-b & 0 \end{pmatrix}=\begin{pmatrix} 0 & 0 \\ (a-b)s & 0 \end{pmatrix}-$$
$$\begin{pmatrix} 0 & 0 \\ (a-b)t & 0 \end{pmatrix}=\begin{pmatrix} 0 & 0 \\ (a-b)s-(a-b)t & 0 \end{pmatrix}=\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.\text{Then}$$

$d(x)=d(\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix})=\begin{pmatrix} 0 & 0 \\ a-b & 0 \end{pmatrix}=\begin{pmatrix} 0 & 0 \\ a & 0 \end{pmatrix}-\begin{pmatrix} 0 & 0 \\ b & 0 \end{pmatrix}$,which give

$\begin{pmatrix} 0 & 0 \\ a & 0 \end{pmatrix}=\begin{pmatrix} 0 & 0 \\ b & 0 \end{pmatrix}$.Now multiplying by b form left, we obtain

$b\begin{pmatrix} 0 & 0 \\ a & 0 \end{pmatrix}=b\begin{pmatrix} 0 & 0 \\ b & 0 \end{pmatrix}.$

$b\begin{pmatrix} 0 & 0 \\ a & 0 \end{pmatrix} = b^2 \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$.By using the cancellation Property from left and since $b = b^2$

$\begin{pmatrix} 0 & 0 \\ a & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ this give $a = 1$,by substituting in above ,we get $b = 1$.Thus

$d(x) = d(\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}) = \begin{pmatrix} 0 & 0 \\ a-b & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, and by same method ,we get

$\lambda x + \zeta(x) = \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} + \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.Thus

$d(x) = \lambda x + \zeta(x)$ for all $x \in R$.

Theorem 3.15.

Let R be a semiprime ring, and let d:R→R be a central additive mapping. Then there exist $\lambda \in C$ and an additive mapping ζ: R →C such that $d(x) = \lambda x + \zeta(x)$ for all $x \in R$.

Proof: We have that d:R→R be a central additive mapping, then we get [d(x), x]= 0 for all $x \in R$. Linearizing [d(x),x] = 0 we obtain [d(x),y] = [x,d(y)]. Hence, we see that the mapping (x,y) → [d(x),y] is a biderivation. By Theorem 4.1 ,there exist an idempotent $\lambda \in C$ and an element $\mu \in C$ such that the algebra (1 - λ)R is commutative (hence, (1- λ)R $\subseteq$ C), and λ[d(x),y] = $\lambda\mu$[x,y] holds for all x, y $\in$ R. Thus, λd(x) - $\mu\lambda$ x commutes with every element in R, so thatλd(x) - $\mu\lambda$ x$\in$ C. Now, let λd(x) =$\mu\lambda$ and define a mapping ζ by ζ(x) = (λd(x) - λx) + (1- λ)d(x). Note that ζ maps in C and that d(x) = λx + ζ(x) holds for every x $\in$R.

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حول المعادلة المتطابقة(x)ζ + λx = d(x)

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المستخلص:

الغرض الرئيسي من هذا البحث هو دراسة والتحقيق في بعض النتائج المتعلقة بالاشتقاقات العامة D على حلقة شبه الأولية R حيث إن d اشتقاق على R حيث أنتمتلك عنصر محايد وخاصية الحذف وبوضع شروط معينة برهنا R بحيث$C \rightarrow R$: ζ والتطبيق الجمعي$C \in \lambda$ توجد(x)ζ $+ \lambda x = d(x)$, لكل $x \in R$