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[0,1] Truncated Exponentiated Exponential Inverse Weibull Distribution with Applications of Carbon Fiber and COVID-19 Data

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Abstract

In the recent years the modeling of lifetime data is an essential research topic. Several research studies on this subject have appeared, aiming to introduce new statistical methodologies for dealing with lifetime phenomena. In this paper, we extension Inverse Weibull distribution by using the family [0,1] Truncated Exponentiated Exponential-G family. We get [0,1] Truncated Exponentiated Exponential Inverse Weibull distribution ([0,1]TEEIW). We provide explicit expressions for its properties like: hazard function, quantile function, moments, moment generating function, reliability function, and Order statistics. Three data sets are used to provide the flexibility of new distribution, which represents 63 observations and is about the strength of carbon fibers. And two a COVID-19 mortality rates data set from Italy and Canada. Evidence of this distribution to outperform other classes of lifetime models has been noticed.

Introduction

Many researchers introduce new families of distributions to create new flexible distributions. Eugen et al., was the first researchers used new family named Beta-G see [1-3]. Merovci (2013) introduces an expansion of this distribution Transmuted Exponentiated Exponential-G [4], Also Oguntunde et al. study of an extension of the exponential distribution using logistic-x family of distributions [5], the Weibull-G by Bourguignon et al. [6], Oguntunde and Adejumo introduce the Transmuted Inverse Exponential-G [7], Khaleel et al. introduced the Marshall-Olkin Topp Leone-G family [8], Ahmed et al. found and studied Topp Leone Marshall Olkin-G family [9], Also Type II Half Logistic Exponentiated Exponential-G by Abdulkabir and Ipinyomi [10], and Mansoor et al. introduced and study of the logistic Exponentiated Exponential-G [11], the family of Gompertz-G by Alizadeh et al. [12-13], Abid et al. [14], propose new method to find new family named the [0,1] truncated-G family. The new truncated distribution family by defining the new (cdf) and (pdf) as follows:

$$M\big(G(x,\xi)\big) = \frac{W\big(G(x,\xi)\big) - W(0)}{W(1) - W(0)}, \quad 0 < x < 1 \tag{1}$$

or Eq. (1) can be reduce to where $W(0) = 0$

$$M\big(G(x,\xi)\big) = \frac{W\big(G(x,\xi)\big)}{W(1)}, \tag{2}$$

The pdf of the [0,1] Truncated- G family can be written as:

$$m\big(G(x,\xi)\big) = \frac{w\big(G(x,\xi)\big)\, g(x,\xi)}{W(1)}, \quad 0 < x < 1 \tag{3}$$

where $G(x,\xi)$, $g(x,\xi)$ are any base line distributions.

In 2022 Khalaf and Khaleel [15] used the [0, 1] Truncated method to define and study a new family name [0,1] Truncated Exponentiated Exponential-G Family. The cdf and pdf of [0,1] Truncated Exponentiated Exponential-G Family can be written as follows respectively:

$$F(x,\alpha,\beta) = \frac{\left(1 - e^{-\alpha G(x;\xi)}\right)^{\beta}}{(1 - e^{-\alpha})^{\beta}} \qquad 0 < x < 1 \tag{4}$$

$$f(x,\alpha,\beta) = \frac{\alpha\beta g(x;\xi) e^{-\alpha G(x;\xi)}\left(1 - e^{-\alpha G(x;\xi)}\right)^{\beta-1}}{(1 - e^{-\alpha})^{\beta}} \quad ,\alpha,\beta > 0 \tag{5}$$

with parameters α (scale parameter) and β (shape parameter). In our paper a new distribution is defined by using [0,1] Truncated Exponentiated Exponential inverse Weibull ([0,1] TEEIW) distribution.

Our motivation is to study and introduce the new [0,1] TEEIW distribution for the following reasons:

- To find a very flexible distribution with different shapes of pdf and hazard rate function.
- The [0,1] TEEIW distribution is recommended for modeling the relief times.

Many researchers follow different families to define several new distributions with different parameters like [16-23].

The following sections make up the paper: in section 2 a definition of the pdf and cdf of the new distribution [0, 1] TEEIW is given with (h) hazard and (S) survival functions. The statistical features such as are covered in Section 3 a series representation of pdf function, moments, (MGF) moment generating function, quantile function, and order statistics. Section 4 addresses the estimations of parameters using ML method. Application study of our distribution for a life data is indicate in section 5. We wrap up with conclusions in section 6.

[0,1]Truncated Exponentiated Exponential inverse Weibull

Inverse Weibull (IW) distribution has received appreciable attention in recent times. Since then, a number of authors have been applying and extending the distribution [24]. The cdf and pdf of the IW distribution respectively can be written as follows:

$$G(x,\lambda,\vartheta) = e^{-\lambda x^{-\vartheta}} \tag{6}$$

$$g(x,\lambda,\vartheta) = \lambda\vartheta x^{-(\vartheta+1)} e^{-\lambda x^{-\vartheta}} \quad ,x > 0,\ \lambda,\vartheta > 0 \tag{7}$$

By inserting Eq. (6) in Eq. (4) we have a cdf of new distribution as follows:

$$F(x,\alpha,\beta,\lambda,\vartheta)=\frac{\left(1-e^{-\alpha e^{-\lambda x^{-\vartheta}}}\right)^{\beta}}{(1-e^{-\alpha})^{\beta}} \qquad x>0,\quad \alpha,\beta,\lambda,\vartheta>0 \tag{8}$$

Substituting Eq. (6) and Eq. (7) into Eq. (5) yields the appropriate pdf:

$$f(x,\alpha,\beta,\lambda,\vartheta)=\frac{\alpha\beta\lambda\vartheta x^{-(\vartheta+1)}e^{-\lambda x^{-\vartheta}}e^{-\alpha e^{-\lambda x^{-\vartheta}}}\left(1-e^{-\alpha e^{-\lambda x^{-\vartheta}}}\right)^{\beta-1}}{(1-e^{-\alpha})^{\beta}} \tag{9}$$

The survival function and hazard function of [0,1]TEEIW distribution, respectively,

$$S(x,\alpha,\beta,\lambda,\vartheta)=1-\frac{\left(1-e^{-\alpha e^{-\lambda x^{-\vartheta}}}\right)^{\beta}}{(1-e^{-\alpha})^{\beta}} \tag{10}$$

$$H(x,\alpha,\beta,\lambda,\vartheta)=\left(\frac{\alpha\beta\lambda\vartheta x^{-(\vartheta+1)}e^{-\lambda x^{-\vartheta}}e^{-\alpha e^{-\lambda x^{-\vartheta}}}\left(1-e^{-\alpha e^{-\lambda x^{-\vartheta}}}\right)^{\beta-1}}{(1-e^{-\alpha})^{\beta}-\left(1-e^{-\alpha e^{-\lambda x^{-\vartheta}}}\right)^{\beta}}\right) \tag{11}$$

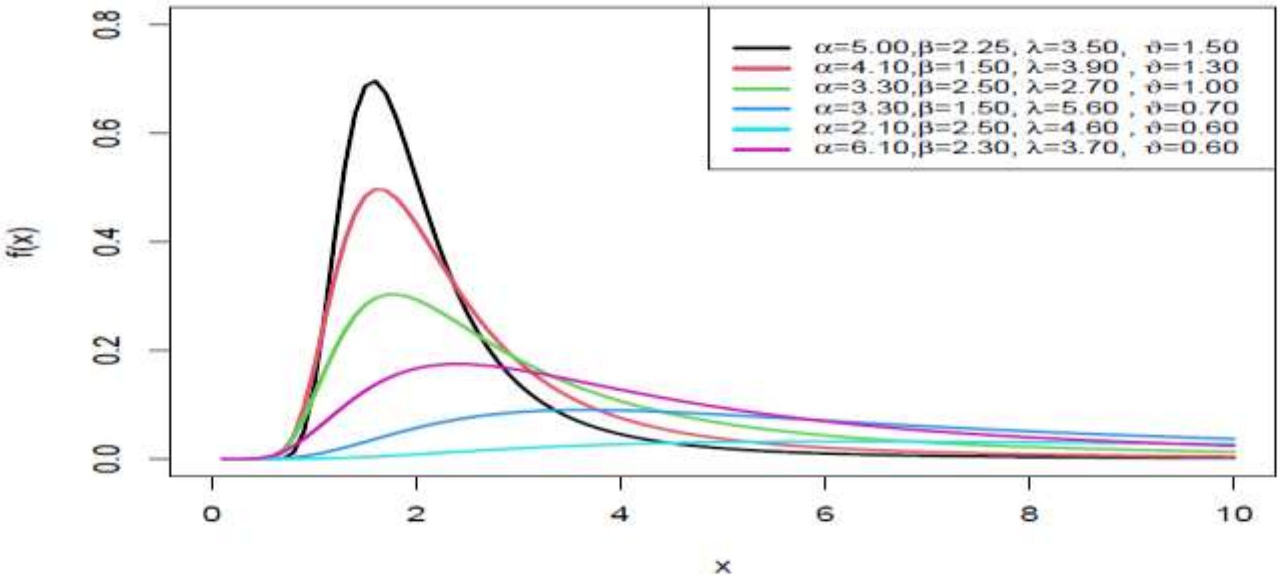

Figure 1: Different shapes of the pdf, with different value of parameters.

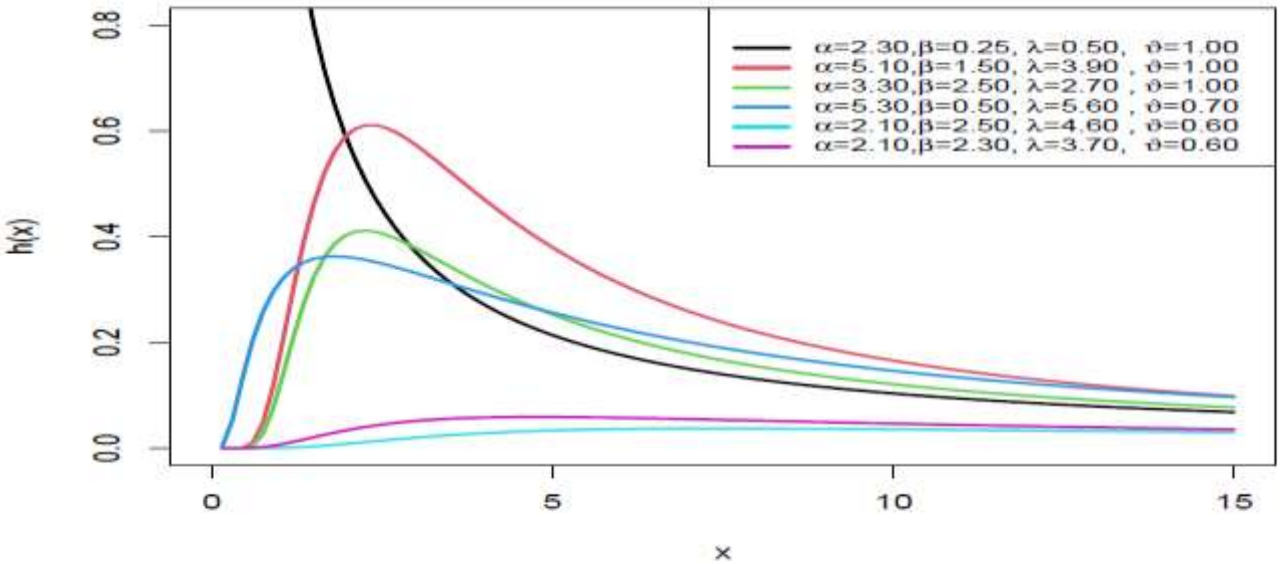

Figure 2: Different shapes of the h, function with different value of parameters.

Figures 1, and 2, show plots of the pdf, and hazard rate function for the [0, 1] TEEIW distribution. Figure 1 shows decreasing curves and unimodal curves right skewed and revise J shapes. Hence, the hazard rate shape of the [0, 1] TEEIW distribution could decreasing or bathtub, increasing decreasing increasing depending on the parameter values.

Mathematical Properties of [0, 1] TEEIW

In this section, many important properties of new distribution will find. It can help us to understand the used of distribution in real life phenomena. These properties are such as follows:

Useful Expansion

Let us use the generalized binomial [20] $(1-z)^{w-1}=\sum_{j=0}^{\infty}\binom{w-1}{j}z^j$ for expansion for the PDF of new distribution on Eq. (9):

$$\left(1-e^{-\alpha e^{-\lambda x^{-\vartheta}}}\right)^{\beta-1}=\sum_{j=0}^{\infty}(-1)^j\binom{\beta-1}{j}\left(e^{-\alpha e^{-\lambda x^{-\vartheta}}}\right)^j \tag{12}$$

Appling Eq. (12) in Eq. (9) we have

$$f(x,\alpha,\beta,\lambda,\vartheta)=\sum_{j=0}^{\infty}(-1)^j\binom{\beta-1}{j}\frac{\alpha\beta\lambda\vartheta x^{-(\vartheta+1)}e^{-\lambda x^{-\vartheta}}e^{-(1+j)\alpha e^{-\lambda x^{-\vartheta}}}}{(1-e^{-\alpha})^{\beta}} \tag{13}$$

Then by using exponential Taylor series to produce an expansion for the pdf $e^{-\omega x}=\sum_{i=0}^{\infty}\frac{(-1)^i\omega^i x^i}{i!}$ [15], [21], and the on Eq. (13) we get :

$$e^{-(1+j)\alpha e^{-\lambda x^{-\vartheta}}}=\sum_{m=0}^{\infty}\frac{(-1)^m}{m!}\alpha^m(1+j)^m\left(e^{-\lambda x^{-\vartheta}}\right)^m \tag{14}$$

Then the pdf can be write like the form:

$$f(x,\alpha,\beta,\lambda,\vartheta)=\sum_{j,m=0}^{\infty}\frac{(-1)^{j+m}\alpha^{m+1}\beta\lambda(1+j)^m}{(1-e^{-\alpha})^{\beta}m!}\binom{\beta-1}{j}\vartheta x^{-(\vartheta+1)}e^{-(m+1)\lambda x^{-\vartheta}} \tag{15}$$

Or can reduse Eq. (14) to the form

$$f(x,\alpha,\beta,\lambda,\vartheta)=\sum_{j,m=0}^{\infty}¥_{j,m}\,\vartheta x^{-(\vartheta+1)}e^{-(m+1)\lambda x^{-\vartheta}} \tag{16}$$

where

$$¥_{j,m}=\frac{(-1)^{j+m}\alpha^{m+1}\beta\lambda(1+j)^m}{(1-e^{-\alpha})^{\beta}m!}\binom{\beta-1}{j}$$

Equation (16) is very important to study and find many properties that need integration and it is like (exponential-G) family distribution with different parameters.

Moments

The r^{th} moments of the [0,1]TEEIW distributions are expressed as

$$E(X^r)=\int_0^{\infty}x^r f(x,\alpha,\beta,\lambda,\vartheta)dx$$

Using the PDF of the [0,1]TEEIW in Eq. (16), we get

$$E(X^r) = \sum_{j=m=0}^{\infty} ¥_{j,m} \int_0^{\infty} x^r \, x^{-(\vartheta+1)} e^{-ℷ(m+1)x^{-\vartheta}} dx \tag{17}$$

Let $y = ℷ(m+1)x^{-\vartheta}$: then after some mathematical steps we can reduce Eq. (17) to

$$E(X^r) = \sum_{j=m=0}^{\infty} ¥_{j,m} \frac{-1}{\vartheta\left(ℷ(m+1)\right)^{1-\frac{r}{\vartheta}}} \int_0^{\infty} y^{-\frac{r}{\vartheta}} e^{-y} dy$$

The end result as

$$\acute{\mu}_r = E(X^r) = \sum_{j=m=0}^{\infty} ¥_{j,m} \frac{-\Gamma(1-\frac{r}{\vartheta})}{\vartheta\left(ℷ(m+1)\right)^{1-\frac{r}{\vartheta}}} \tag{18}$$

Equation (18) very important to find many statistical concepts like mean, the 4th moments, variance, CV, Moment generating function (MGF) and so on.

Moments Generating Function

By using Eq.(18) and Taylor expanstion $e^{tx=} \sum_{l=0}^{\infty} \frac{(t)^l}{l!} x^l$ we can get the MGF of new distribution as follows:

$$M_x(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} f(x, \alpha, \beta, ℷ, \vartheta) dx$$

$$M_x(t) = \sum_{l=0}^{\infty} \frac{t^l}{l!} \int_0^{\infty} x^l f(x) dx = \sum_{l=0}^{\infty} \frac{t^l}{l!} E(X^l) = \sum_{l=0}^{\infty} \frac{t^l}{l!} [E(X^r)]$$

By using Eq. (18), it is same moment's function we have

$$M_x(t) = \sum_{l=0}^{\infty} \frac{t^l}{l!} \left[\sum_{j=m=0}^{\infty} ¥_{j,m} \frac{\Gamma(1-\frac{r}{\vartheta})}{-\vartheta\left(ℷ(m+1)\right)^{1-\frac{r}{\vartheta}}} \right] \tag{19}$$

Order Statistics

Let $X_1, X_2, X_3, \ldots, X_n$ denote a random sample of size n drawn from the [0,1]TEEIW distribution, and $X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$ denote the order statistics. If $X_{s:n}$ denotes statistics of the ith order, then the probability density function of $X_{s:n}$is given by[25].

$$f_{s:n}(x) = \frac{n!}{(s-1)!\,(n-s)!} [F(x, \alpha, \beta, ℷ, \vartheta)]^{s-1} [1 - F(x, \alpha, \beta, ℷ, \vartheta)]^{n-s} f(x, \alpha, \beta, ℷ, \vartheta) \tag{20}$$

By using (8) and (9) we have

$$f_{s:n}(x) = \frac{n!}{(s-1)!\,(n-s)!}\left[\frac{\left(1-e^{-\alpha e^{-\lambda x^{-\vartheta}}}\right)^{\beta}}{(1-e^{-\alpha})^{\beta}}\right]^{s-1}\left[1-\frac{\left(1-e^{-\alpha e^{-\lambda x^{-\vartheta}}}\right)^{\beta}}{(1-e^{-\alpha})^{\beta}}\right]^{n-s} * \left[\frac{\alpha\beta\lambda\vartheta x^{-(\vartheta+1)}e^{-\lambda x^{-\vartheta}}e^{-\alpha e^{-\lambda x^{-\vartheta}}}\left(1-e^{-\alpha e^{-\lambda x^{-\vartheta}}}\right)^{\beta-1}}{(1-e^{-\alpha})^{\beta}}\right] \tag{21}$$

Equation (21) very useful to find the 1st order when $s = 1$ and the large order when $s = n$.

Quantile Function

By inverting Eq. (8), we get the [0,1]TEEIW quantile function, say, $x = Q(u)$, as shown below [26].

$$\frac{\left(1-e^{-\alpha e^{-\lambda x^{-\vartheta}}}\right)^{\beta}}{(1-e^{-\alpha})^{\beta}} = u \tag{22}$$

Which yield

$$Q(u) = \left[\frac{-1}{\lambda}\ln\left(\frac{-1}{\alpha}ln\left(1-\left(u\times(1-e^{-\alpha})^{\beta}\right)^{\frac{1}{\beta}}\right)\right)\right]^{\frac{-1}{\vartheta}} \tag{23}$$

Equation (23) is very important to find the Kurtosis and Skewness based on quartiles of the [0,1]TEEIW distribution. With this understanding, we can generate random sample for the [0,1]TIWE distribution using:

$$x = \left[\frac{-1}{\lambda}\ln\left(\frac{-1}{\alpha}ln\left(1-\left(U\times(1-e^{-\alpha})^{\beta}\right)^{\frac{1}{\beta}}\right)\right)\right]^{\frac{-1}{\vartheta}} \tag{24}$$

Equation (24) very important to study simulation of the [0, 1] TEEIW distribution where U is Uniform (0,1).

Parameter Estimation of [0, 1] TEEIW distribution

In this section, we continue by demonstrating the maximum likelihood (ML) estimation method for estimating parameters of the [0,1]TEEIW class of distributions. Several methods for estimating model parameters have been proposed in the literature, but the ML estimation method is the most widely used. Let $X_1, X_2, \dots, X_n$ represent a random sample of observed values $x_1, x_2, \dots, x_n$ from the [0,1]TEEIW distribution. Consider $\Omega = (\alpha, \beta, \lambda, \vartheta)^T$. The log-likelihood (LL) function is given as [27-29]

$$l = nlog\alpha + nlog\beta + nlog\lambda + nlog\vartheta - n\beta\log(1-e^{-\alpha}) - (\vartheta+1)\sum_{i=1}^{n}logx_i + \sum_{i=1}^{n}\left(-\lambda x_i^{-\vartheta}\right) + \sum_{i=1}^{n}\left(-\alpha e^{-\lambda x_i^{-\vartheta}}\right) + (\beta-1)\sum_{i=1}^{n}\log\left(1-e^{-\alpha e^{-\lambda x_i^{-\vartheta}}}\right) \tag{25}$$

The ML equations of the [0,1]TEEIW are obtained as follows:

$$\frac{\partial l}{\partial\alpha} = \frac{n}{\alpha} - \frac{n\beta e^{-\alpha}}{1-e^{-\alpha}} + \sum_{i=1}^{n}\left(-e^{-\lambda x_i^{-\vartheta}}\right) + (\beta-1)\left(\sum_{i=1}^{n}\frac{e^{-\lambda x_i^{-\vartheta}}e^{-\alpha e^{-\lambda x_i^{-\vartheta}}}}{1-e^{-\alpha e^{-\lambda x_i^{-\vartheta}}}}\right) \tag{26}$$

$$\frac{\partial l}{\partial \beta} = \frac{n}{\beta} - n\log(1-e^{-\alpha}) + \sum_{i=1}^{n} \log\left(1 - e^{-\alpha e^{-\lambda x_i^{-\vartheta}}}\right) \tag{27}$$

$$\frac{\partial l}{\partial \lambda} = \frac{n}{\lambda} + \sum_{i=1}^{n} \left(-x_i^{-\vartheta}\right) + \sum_{i=1}^{n} \left(\alpha x_i^{-\vartheta} e^{-\lambda x_i^{-\vartheta}}\right) + (\beta - 1)\left(\sum_{i=1}^{n} \frac{-\alpha x_i^{-\vartheta} e^{-\lambda x_i^{-\vartheta}} e^{-\alpha e^{-\lambda x_i^{-\vartheta}}}}{1 - e^{-\alpha e^{-\lambda x_i^{-\vartheta}}}}\right) \tag{28}$$

$$\begin{aligned}\frac{\partial l}{\partial \vartheta} &= \frac{n}{\vartheta} - \sum_{i=1}^{n} logx_i + \sum_{i=1}^{n} \lambda x_i^{-\vartheta}\, logx_i + \sum_{i=1}^{n} \left(-\alpha\lambda x_i^{-\vartheta} logx_i e^{-\lambda x_i^{-\vartheta}}\right) \\ &\quad + (\beta - 1)\left(\sum_{i=1}^{n} \frac{\alpha\lambda x_i^{-\vartheta} logx_i e^{-\lambda x_i^{-\vartheta}} e^{-\alpha e^{-\lambda x_i^{-\vartheta}}}}{1 - e^{-\alpha e^{-\lambda x_i^{-\vartheta}}}}\right)\end{aligned} \tag{29}$$

By solving the nonlinear equations $\frac{\partial l}{\partial \alpha} = \frac{\partial l}{\partial \beta} = \frac{\partial l}{\partial \lambda} = \frac{\partial l}{\partial \vartheta} = 0$, using a numerical method such as the Newton-Raphson procedure. Because it is difficult to solve manually, we used here the package in R.

Application

This section discusses the proposed model's applicability and goodness of fit. We looked at three datasets, the first data is carbon fiber and the second and third data on COVID-19 in Italy and Canada that described mortality rates. The [0,1]TEEIW distribution was compared with the distributions shown in the following table:

Table (1): Statistical criteria equations that used in application

Statistical criteria and w^*, A^*, K-S	Abbreviation	Law
Akaike Information Criteria	AIC	$AIC = -2l + 2K$
Consistent Akaike Information Criteria	CAIC	$CAIC = AIC + \frac{2K(K+1)}{n-K-1}$
Bayesian Information Criteria	BIC	$BIC = -2l + klog(n)$
Hanan and Quinn Information Criteria	HQIC	$HQIC = 2kln[\ln(n)] - 2l$
Kolmogorov–Smirnov criterion	KS	$\text{KS} = max\left\{\frac{i}{n} - \hat{F}(x_i), \hat{F}(x_i) - \frac{i-1}{n}\right\}$
Cramér-von Mises criterion	W	$W^\star = \frac{1}{12n} + \sum_{i=1}^{n} \left[\frac{2i-1}{2n} - \hat{F}(x_i)\right]^2$
Anderson–Darling criterion	A	$A^\star = -n - \frac{1}{n}\sum_{i=1}^{n} (2i-1)\left[\log \hat{F}(x_i) + \log\left(1 - \hat{F}(x_{n+1-i})\right)\right]$

Tables (1) show the values of Akaike Information Criteria (AIC), Consistent Akaike Information Criteria (CAIC), Bayesian Information Criteria (BIC), Hanan and Quinn Information Criteria (HQIC), Kolmogorov-Smirnov (KS), P-Value, Cramér-von Mises (W), and Anderson-Darling (A) statistics for all models fitted using three real data sets.

The first data set

The first data set contains 63 observations and is about the strength of carbon fibers tested under tension at gauge lengths of 10 mm [30-32]. (1.901, 2.132, 2.203, 2.228, 2.257, 2.350, 2.361, 2.396, 2.397, 2.445, 2.454, 2.474, 2.518, 2.522, 2.525, 2.532, 2.575, 2.614, 2.616, 2.618, 2.624, 2.659, 2.675, 2.738, 2.740, 2.856, 2.917, 2.928, 2.937, 2.937, 2.977,2.996, 3.030, 3.125, 3.139, 3.145, 3.220, 3.223, 3.235, 3.243, 3.264, 3.272, 3.294, 3.332, 3.346, 3.377, 3.408, 3.435, 3.493, 3.501, 3.537, 3.554, 3.562, 3.628, 3.852, 3.871, 3.886, 3.971, 4.024, 4.027, 4.225, 4.395, 5.020).

The second data set (COVID-19)

The second data set represents a COVID-19 mortality rates data set from Italy for 59 days, from February 27 to April 27, 2021[33]. (4.571, 7.201, 3.606, 8.479, 11.410, 8.961, 10.919, 10.908, 6.503, 18.474, 11.010, 17.337, 16.561, 13.226, 15.137, 8.697, 15.787, 13.333, 11.822, 14.242, 11.273, 14.330, 16.046, 11.950, 10.282, 11.775, 10.138, 9.037 ,12.396, 10.644, 8.646, 8.905, 8.906, 7.407, 7.445, 7.214, 6.194, 4.640, 5.452 ,5.073, 4.416, 4.859, 4.408, 4.639, 3.148, 4.040, 4.253, 4.011, 3.564, 3.827, 3.134, 2.780, 2.881 ,3.341, 2.686 , 2.814, 2.508, 2.450, 1.518).

The third data set (COVID-19)

The third data set represents a COVID-19 drought mortality rates data set from Canada for 36 days, from 10 April to 15 May 2021[34]. (3.1091, 3.3825, 3.1444, 3.2135, 2.4946, 3.5146, 4.9274, 3.3769, 6.8686, 3.0914, 4.9378, 3.1091 ,3.2823, 3.8594, 4.0480, 4.1685, 3.6426, 3.2110, 2.8636, 3.2218, 2.9078, 3.6346, 2.7957, 4.2781, 4.2202, 1.5157, 2.6029, 3.3592, 2.8349, 3.1348, 2.5261, 1.5806, 2.7704, 2.1901, 2.4141, 1.9048)

Table (2): MLEs of [0,1]TEEIW for carbon fibers data.

Distribution	Parameters estimation			
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\theta}$
[0,1]TEEIW	**26.6583**	**2.5170**	**11.2542**	**1.2317**
EGIW	8.7088	11.8167	5.4843	1.3532
WEIW	4.6037	8.4671	12.5866	9.3475
GOIW	0.0314	1.9869	2.4147	2.4409
IW	16.1724	2.9225		

Table (3): -LL, AIC, CAIC, BIC, KS, P-Value, W, A. for carbon fibers data.

Distribution	-LL	AIC	CAIC	BIC	HQIC	KS	P-value	W	A
[0,1]TEEIW	**56.65**	**120.98**	**121.67**	**129.55**	**124.35**	**0.081**	**0.798**	**0.065**	**0.349**
EGIW	56.94	121.88	122.57	130.45	125.25	0.101	0.539	0.081	0.424
WEIW	56.95	121.90	122.59	130.47	125.27	0.082	0.787	0.066	0.363
GOIW	62.92	133.89	134.58	142.46	137.26	0.117	0.348	0.124	0.865
IW	76.73	157.60	157.80	161.88	159.28	0.193	0.017	0.075	0.394

Table (2) include the MLE of the parameters for the models. Table (3) demonstrates that our recommended distribution fits the data substantially better than the other distributions. Under consideration these Statistics show that the **[0,1]TEEIW** distribution outperforms all fitted models. It has the lowest W, A, KS, AIC, HQIC, BIC, and CAIC values using real dataset,

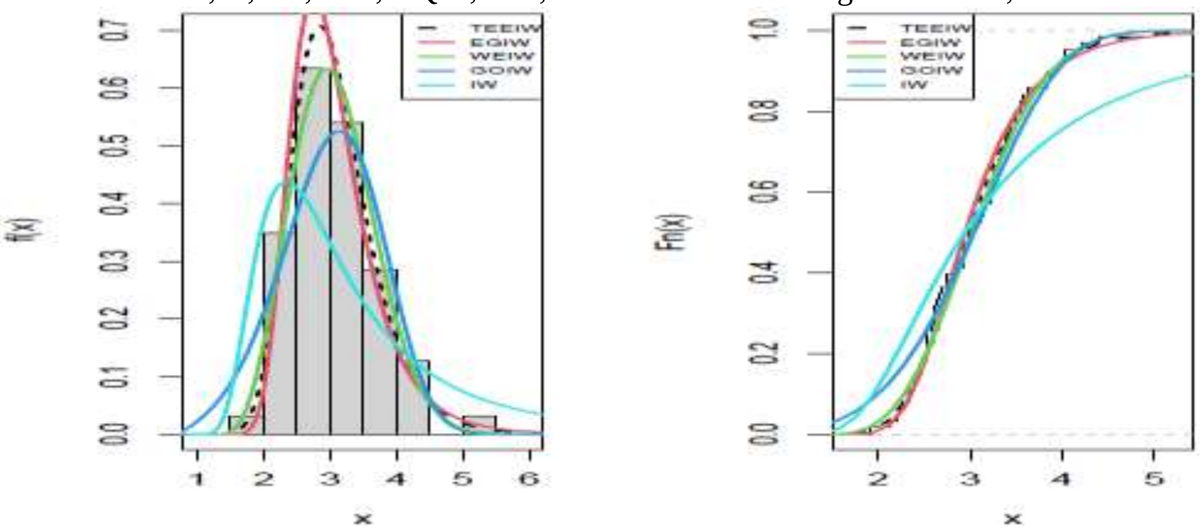

Figure 3: The estimated PDF, CDF, QQ and Boxplot of [0,1]TEEIW for Data1

Figures (3) show the fitted [0,1]TEEIW, PDF, and CDF, of the first data sets.

Table (4): MLEs for Italy data.

Distribution	Parameter estimation			
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\theta}$
[0,1]TEEIW	**47.7721**	**0.1620**	**51.1572**	**0.9411**
EGIW	0.6733	3.8531	4.8345	2.3463
WEIW	2.8402	1.8417	2.2958	1.2080
GOIW	0.0530	1.2917	2.1589	1.4575
IW	7.1240	1.3009		
E	0.1226			

Table (5): -LL, AIC, CAIC, BIC, KS, P-Value, W, A. for Italy data.

Distribution	-LL	AIC	CAIC	BIC	HQIC	KS	P-value	W	A
[0,1]TEEIW	**165.7**	**339.45**	**340.19**	**347.76**	**342.69**	**0.106**	**0.483**	**0.112**	**0.627**
EGIW	175.2	358.58	359.32	366.89	361.82	0.164	0.072	0.282	1.666
WEIW	168.2	344.43	345.17	352.74	347.67	0.120	0.329	0.167	0.938
GOIW	167.0	342.18	342.92	350.49	345.43	0.115	0.384	0.127	0.753
IW	177.7	359.55	359.76	363.71	361.17	0.148	0.135	0.243	1.402
E	182.8	367.65	367.72	369.73	368.46	0.242	0.002	0.147	0.848

Table (4) include the MLE of the parameters for the models. Table (5) demonstrates that our recommended distribution fits the data substantially better than the other distributions. Under consideration these Statistics show that the **[0,1]TEEIW** distribution outperforms all fitted models. It has the lowest W, A, KS, AIC, HQIC, BIC, and CAIC values using COVID 19 dataset.

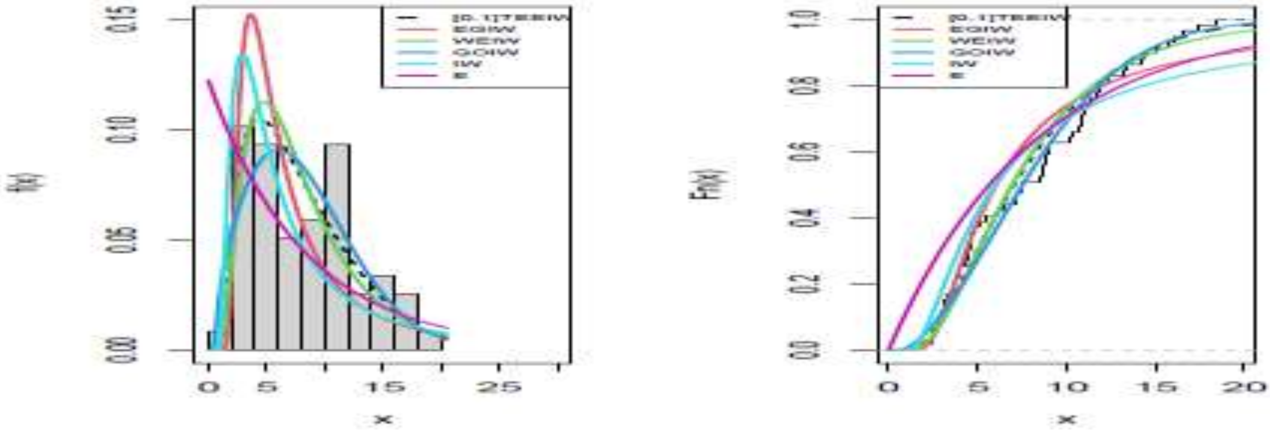

Figure 4: Estimated PDF and CDF for Data 2

Figures (4) show the fitted [0,1]TEEIW, PDF, and CDF, of the first data sets, respectively.

Table (6): MLEs for Canada COVID 19 data.

Distribution	Parameter estimation			
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\lambda}$	$\hat{\theta}$
[0,1]TEEIW	**22.0640**	**0.8947**	**14.2086**	**1.2029**
EGIW	4.5081	3.9705	5.0300	1.3521
WEIW	3.1630	5.3552	5.6274	5.7108
GOIW	0.3697	1.8826	4.6765	1.7705
IW	11.978	2.5780		

Table (7) -LL, AIC, CAIC, BIC, KS, P-Value, W, A. for Canad data.

Distribution	-LL	AIC	CAIC	BIC	HQIC	KS	P-value	W	A
[0,1]TEEIW	**47.96**	**103.92**	**105.21**	**110.25**	**106.13**	**0.113**	**0.743**	**0.094**	**0.552**
EGIW	49.76	107.54	108.83	113.87	109.75	0.141	0.463	0.154	0.936
WEIW	48.85	105.71	107.00	112.04	107.92	0.128	0.596	0.130	0.775
GOIW	51.00	110.04	11.33	116.37	112.25	0.134	0.529	0.160	0.916
IW	54.73	113.51	113.88	116.68	114.62	0.198	0.117	0.208	1.254

Table (6) include the MLE of the parameters for the models. Table (7) demonstrates that our recommended distribution fits the data substantially better than the other distributions. Under consideration these Statistics show that the **[0,1]TEEIW** distribution outperforms all fitted models. It has the lowest W, A, KS, AIC, HQIC, BIC, and CAIC values using COVID 19 dataset.

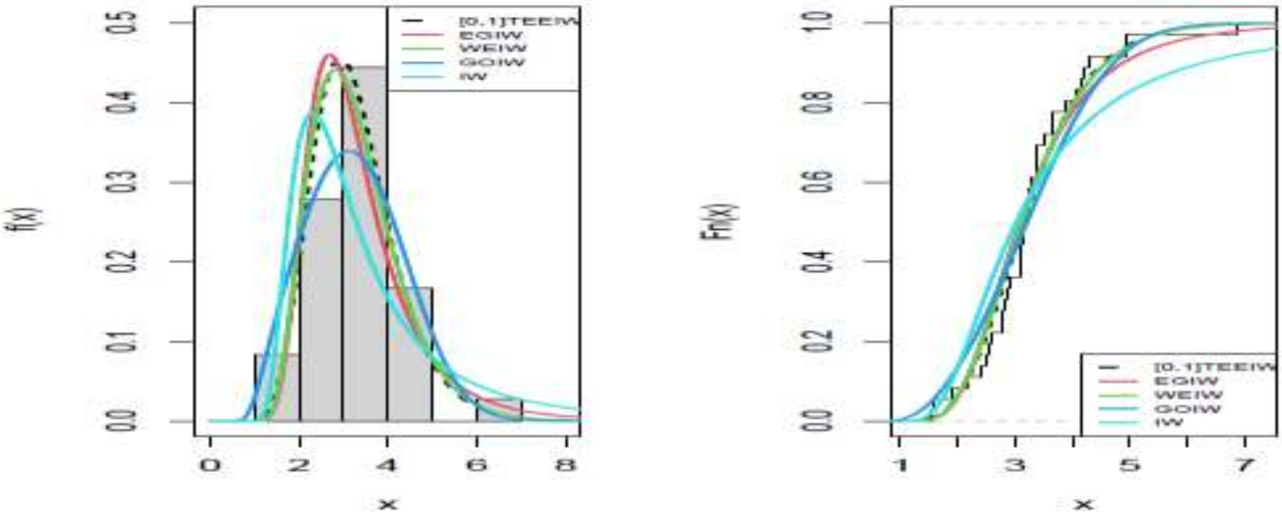

Figure 4: Estimated PDF and CDF for Data 3

Figures (4) shows the histogram plot for COVID-19 data. It indicates that the [0, 1] TEEIW distribution has the greatest peak and best matches the dataset's histogram. As a consequence, our distribution is a superior fit for the above real-world data.

Conclusion

In this study, the [0, 1] Truncated Exponentiated Exponential inverse Weibull ([0, 1] TEEIW) distribution was effectively created and its different statistical features were investigated. The form of the model might be unimodal, declining, or rising. The maximum likelihood technique is offered as a method for estimating unknown model parameters. Failure rates may be used to explain and model real-world events such as bathtubs, inverted bathtubs, increasing and decreasing. When applied to real-world data sets, the [0, 1] TEEIW distribution is found to be an improvement and a better option than other distributions.

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[0,1] توزيع ويبل الأسي المعكوس المبتور مع تطبيقات ألياف الكربون وبيانات كوفيد-19

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المستخلص

ان نمذجة البيانات الحقيقية او الطبيعية يعد من المواضيع المهمة في السنوات الأخيرة، حيث تعد نمذجة بيانات البقاء موضوعًا بحثيًا أساسيًا. ظهرت العديد من الدراسات البحثية حول هذا الموضوع والهدف من ذلك هو تقديم طريقة إحصائية جديدة للتعامل مع ظواهر بيانات البقاء. في هذا البحث، قمنا بتوسيع توزيع معكوس ويبل باستخدام احدى العوائل المستمرة الا و هي العائلة Truncated Exponentiated Exponential-G [0،1]. حيث سنحصل على توزيع جديد اسمة [0،1] معكوس ويبل الأسي المعمم المبتور ([0،1] TEEIW). نقدم بعض الخواص لهذا التوزيع مثل دالة الخطر ، الدالة الكمية، العزوم، الدالة المولدة للعزوم، دالة البقاء، والإحصاءات المرتبة. و لغرض بيان مرونة التوزيع نستخدم ثلاث مجموعات بيانات، والذي يمثل 63 مشاهدة الذي يتعلق بقوة ألياف الكربون. واثنان من بيانات لمعدلات وفيات COVID-19 في إيطاليا وكندا. وقد لوحظ انه من خلال النتائج ان التوزيع الجديد يتفوق في النمذجة على انواع أخرى من التوزيعات.

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[0،1] مقتطع، لحظات، توزيع ويبل العكسي، إحصائيات الترتيب، التقدير.

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