Soliton Solutions for (3+1)-Dimensional Nonlinear Evolution Equations by Tanh Method

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Abstract : *In this paper, the Tanh method is employed to compute the exact solutions for nonlinear PDEs. These equations are (3+1)-dimensional Jimbo-Miwa , potential-YTSF, and generalized KP equations. The solutions obtained by this method are compared with the solutions obtained through other methods. It is shown that the Tanh method provides a powerful mathematical tool for solving nonlinear evolution equations in mathematical physics.*

Keywords Tanh method, Nonlinear system PDEs, (3+1)-dimensional Jimbo-Miwa and (3+1)-dimensional potential-YTSF equations, Exact Solutions.

1. Introduction

Nonlinear phenomena play important roles in applied mathematics, physics and also in engineering problems in which each parameter varies depending on different factors. Solving Non-Linear Evolution Equations (NLEEs) may guide authors to know the described process deeply and sometimes leads them to know some facts that are not simply understood through common observations. Moreover, obtaining exact solutions for these problems is a great purpose, He et al [1].

The nonlinear wave phenomena observed in the above mentioned scientific fields, are often modeled by the bell-shaped sech solutions and the kink-shaped tanh solutions. The availability of these exact solutions, for those nonlinear equations can greatly facilitate the verification of numerical solvers on the stability analysis of the solution. The investigation of exact solutions of Non-Linear Partial Differential Equations (NLPDEs) plays an important role in the study of these phenomena. In the past several decades, many effective methods for obtaining exact solutions of NLPDEs have been presented. In the literature, there is a wide variety of approaches to nonlinear problems for constructing traveling wave solutions, such as the variation iteration method (VIM), homotopy perturbation method (HPM) and Exp-Function method to solve these equations, see for instance, He [2-5], He et al [6], He et al [7]. Tanh method was applied in solving evolution equations. The nonlinear wave equations studied by Anwar et al [8] by applying tanh method to solve the coupled mKdV equations, Drinfeld–Sokolov equation and its generalized version. Anwar et al [9] carried out the integration of Burgers equation by the aid of tanh method. That leaded to the complex solutions for the Burgers equation, KdV–Burgers equation, coupled Burgers equation and the generalized time-delayed Burgers equation. Then the perturbed Burgers equation in (1+1) dimensions was integrated by the ansatz method.

This paper outlines the implementation of Tanh method for solving four (3+1)-dimensional evolution equations. These equations play a very important role in mathematical physics and engineering sciences. These equations are (3+1)-dimensional Jimbo-Miwa , potential-YTSF, and generalized KP equations. The hyperbolic tangent (tanh) method is a powerful technique to symbolically compute traveling wave solutions of nonlinear wave and evolution equations. In particular, the method is well suited for problems where dispersion, convection, and reaction diffusion phenomena play an important role, Wazwaz [10].

2. Outline of the Tanh Method

The tanh method will be introduced as presented by Malfliet [11] and improved and extended to tanh–coth method by Wazwaz [12]. The tanh method is based on a priori assumption that the traveling wave solutions can be expressed in terms of the tanh function to solve the coupled KdV equations.

The tanh method is developed by Malfliet [11]. The method is applied to find out exact solutions of nonlinear differential equations with (3+1)-dimensional:

$$P(u,u_x,u_t,u_y,u_z,u_{xx},u_{xy},u_{yy},u_{zz},.........)=0 \quad (1)$$

where P is polynomial of the variable u and its derivatives. If we consider $u(x,y,z,t)=u(\xi),\ \xi=kx+\lambda y+\beta z+\omega t+\theta_0$, so that $u(x,y,z,t)=U(\xi)$, we can use the following changes:

$$\frac{\partial}{\partial t}=\omega\frac{d}{d\xi},\qquad \frac{\partial}{\partial x}=k\frac{d}{d\xi},\qquad \frac{\partial}{\partial y}=\lambda\frac{d}{d\xi},\qquad \frac{\partial}{\partial z}=\beta\frac{d}{d\xi},$$

$$\frac{\partial^2}{\partial x^2}=k^2\frac{d^2}{d\xi^2},\quad \frac{\partial^3}{\partial x^3}=k^3\frac{d^3}{d\xi^3}$$

and so on, then Eq. (1) becomes an ordinary differential equation

$$Q(U,U^{/},U^{//},U^{///},.........)=0 \quad (2)$$

with Q being another polynomial form of their arguments. Eq.(2) is called the reduced ordinary differential equations. Integrating Eq.(2) as long as all terms contain derivatives, the integration constants are considered to be zeros in view of the localized solutions. However, the nonzero constants can be used and handled as well. Now finding the traveling wave solutions to Eq.(1) is equivalent to obtaining the solution to the reduced ordinary differential equation (2). For the tanh method, we introduce the new independent variable

$$Y(x, y, z, t) = \tanh(\xi) \tag{3}$$

that leads to the change of variables:

$$\frac{d}{d\xi} = (1 - Y^2)\frac{d}{dY}$$

$$\frac{d^2}{d\xi^2} = -2Y(1 - Y^2)\frac{d}{dY} + (1 - Y^2)^2\frac{d^2}{dY^2}$$

$$\frac{d^3}{d\xi^3} = 2(1 - Y^2)(3Y^2 - 1)\frac{d}{dY} - 6Y(1 - Y^2)^2\frac{d^2}{dY^2} + (1 - Y^2)^3\frac{d^3}{dY^3} \tag{4}$$

The next crucial step is that the solution we are seeking for is expressed in the form

$$u(x, y, z, t) = U(\xi) = \sum_{i=0}^{m} a_i Y^i \tag{5}$$

where the parameter m can be found by balancing the highest-order linear term with the nonlinear terms in Eq. (2), and $k, \lambda, \beta, \omega, a_0, a_1, ..., a_m$ are to be determined. Substituting (5) into (2) will yield a set of algebraic equations for $k, \lambda, \beta, \omega, a_0, a_1, ..., a_m$ because all coefficients of Y^i have to vanish. From these relations, $k, \lambda, \beta, \omega, a_0, a_1, ..., a_m$ can be obtained. Having determined these parameters, knowing that m is positive integers in most cases, and using (5) we obtain analytic solutions $u(x,t)$ in a closed form. The tanh method seems to be powerful tool in dealing with nonlinear physical models.

3. Numerical Applications:

The tanh method is generalized on four examples including (3+1)-dimensional equations.

3.1 The potential- TSF equation

To illustrate the basic idea of the tanh method, we first consider the potential- TSF equation; see Bai *et al* [13], and Wazwaz [10] that was recently derived by Yu et al.[14] as

$$u_{xxxz} + 4u_x u_{xz} + 2u_{xx}u_z - 4u_{xt} + 3u_{yy} = 0 \qquad (6)$$

This equation was studied by Borhanifar *et al* [15] by applying the exp-function method. By using tanh method and using the traveling wave transformations in Eq(5) and Eq.(3) with

$$\xi = kx + \lambda y + \beta z + \omega t + \theta_0 \qquad (7)$$

The nonlinear system of partial differential equation (6) is carried to an ordinary differential equation

$$k^3\beta U^{////} + 6k^2\beta U^{/}U^{//} + (3\lambda^2 - 4k\omega)U^{//} = 0 \qquad (8)$$

Eq.(8) can be written as:

$$k^3\beta U^{////} + 3k^2\beta(U^{/2})^{/} + (3\lambda^2 - 4k\omega)U^{//} = 0 \qquad (9)$$

Integrating Eq.(9) once with zero constant

$$k^3\beta U^{///} + 3k^2\beta U^{/2} + (3\lambda^2 - 4k\omega)U^{/} = 0 \qquad (10)$$

we postulate the tanh series, Eq(10) reduces to

$$k^3\beta[2(1-Y^2)(3Y^2-1)\frac{dU}{dY} - 6Y(1-Y^2)^2\frac{d^2U}{dY^2} + (1-Y^2)^3\frac{d^3U}{dY^3}] + 3k^2\beta[(1-Y^2)\frac{dU}{dY}]^2 + (3\lambda^2 - 4k\omega)[(1-Y^2)\frac{dU}{dY}] = 0 \qquad (11)$$

Now, to determine the parameter m , balance the linear term of highest-order with the highest order nonlinear terms. So, in Eq. (11) ,balance U''' with U'^2 , to obtain

3 + m = 2m+2 , then m = 1

The tanh method admits the use of the finite expansion for :

$$u(x,t) = U(Y) = a_0 + a_1 Y \ , \ a_1 \neq 0 \qquad (12)$$

Substituting U, U', U'', U''' in Eq. (11), then equating the coefficient of Y^i , i= 0, 2 leads

$$\overset{0}{Y} : -2k^3\beta + 3k^2\beta a_1 + (3\lambda^2 - 4k\omega) = 0$$

$$\overset{2}{Y} : 6k^3\beta - 3k^2\beta a_1 = 0 \qquad (13)$$

Solving the nonlinear system of equations (13) to get:

$a_1 = 2k$ provided that $\omega = \dfrac{4k^3\beta + 3\lambda^2}{4k}$

and

$$u(x,y,z,t) = a_0 + 2k \tanh\{kx + \lambda y + \beta z + \frac{4k^3\beta + 3\lambda^2}{4k} t + \theta_0\} \qquad (14)$$

Result in Eq.(14) is the same result that was obtained by Borhanifar et al [15]. For $a_0 = 1, k = \lambda = \beta = 1 \ \theta_0 = 0$, at $y = 0$

$$u(x,0,z,t) = 1 + 2\tanh\{x + z + \frac{7}{4} t\} \qquad (15)$$

the solitary wave and behavior of the solution in Eq.(15) is shown in Figures (1) and (2).

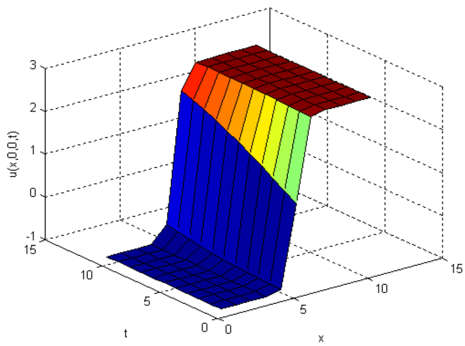

Figure (1) represents Solution at z = 0

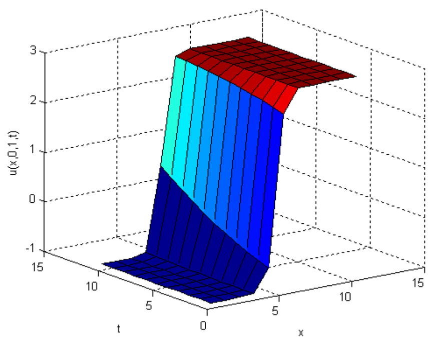

Figure (2) represents Solution at z = 1

3.2 (3+1)-dimensional Jimbo-Miwa Equation

Let us consider the Jimbo-Miwa equation in the form :

$$u_{xxxy} + 3u_x u_{xy} + 3u_{xx}u_y + 2u_{yt} - 3u_{xz} = 0 \qquad (16)$$

This equation was studied by Borhanifar [15] by applying the exp-function method. By using tanh method and using the traveling wave transformations in Eq(5) and Eq.(3) with the transformation in Eq.(7), the nonlinear system of partial differential equation (16) is carried to an ordinary differential equation

$$k^3\lambda U^{////} + 6k^2\lambda U^{/}U^{//} + (2\lambda\omega - 3k\beta)U^{//} = 0 \qquad (17)$$

Eq.(17) can be written as:

$$k^3\lambda U^{////} + 3k^2\lambda(U^{/2})^{/} + (2\lambda\omega - 3k\beta)U^{//} = 0 \qquad (18)$$

Integrating Eq.(18) once with zero constant

$$k^3\lambda U^{///} + 3k^2\lambda U^{/2} + (2\lambda\omega - 3k\beta)U^{/} = 0 \qquad (19)$$

we postulate the tanh series, Eq(19) reduces to

$$k^3\lambda[2(1-Y^2)(3Y^2-1)\frac{dU}{dY} - 6Y(1-Y^2)^2\frac{d^2U}{dY^2} +$$
$$(1-Y^2)^3\frac{d^3U}{dY^3}] + 3k^2\lambda[(1-Y^2)\frac{dU}{dY}]^2 + \qquad (20)$$
$$(2\lambda\omega - 3k\beta)[(1-Y^2)\frac{dU}{dY}] = 0$$

Now, to determine the parameter m , balance the linear term of highest-order with the highest order nonlinear terms. So, in Eq. (20) , balance $U^{///}$ with $U^{/2}$, to obtain

3 + m = 2m+2 , then m = 1

The tanh method admits the use of the finite expansion for :

$$u(x,t) = U(Y) = a_0 + a_1 Y \ , a_1 \neq 0 \qquad (21)$$

Substituting $U, U^{/}, U^{//}, U^{///}$ in Eq. (20), then equating the coefficient of Y^i , i= 0, 2 leads

$$\overset{0}{Y} : -2k^3\lambda + 3k^2\lambda a_1 + (2\lambda\omega - 3k\beta) = 0$$
$$\overset{2}{Y} : 6k^3\lambda - 3k^2\lambda a_1 = 0 \qquad (22)$$

Solving the nonlinear system of equations (22) to get:

$$a_1 = 2k \quad , \ \omega = \frac{3k\beta - 4k^3\lambda}{2\lambda}$$

and

$$u(x,y,z,t) = a_0 + 2k\tanh\{kx + \lambda y + \beta z + \frac{3k\beta - 4k^3\lambda}{2\lambda}t + \theta_0\} \quad (23)$$

Result in Eq.(23) is the same result that obtained by Borhanifar [13]. For $a_0 = 1, k = \lambda = \beta = 1$ $\theta_0 = 0$ $y = 0$

$$u(x,0,z,t) = 1 + 2\tanh\{x + z - \frac{1}{2}t\} \quad (24)$$

the solitary wave of the solution in Eq.(24) is shown in Figures (3) and(4).

3.3 (3+1)-dimensional generalized KP-I equation

Let us consider the generalized KP-I equation in the form :

$$u_{xxxy} + 3(u_x u_y)_x + u_{tx} - u_{ty} - u_{zz} = 0 \quad (25)$$

This equation was studied by Wen-Xiu *et al* [16] by applying the Wronskian and Grammian formulations.

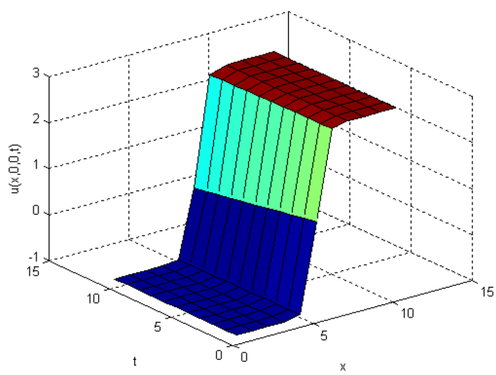

Figure (3) represents Solution at z = 0

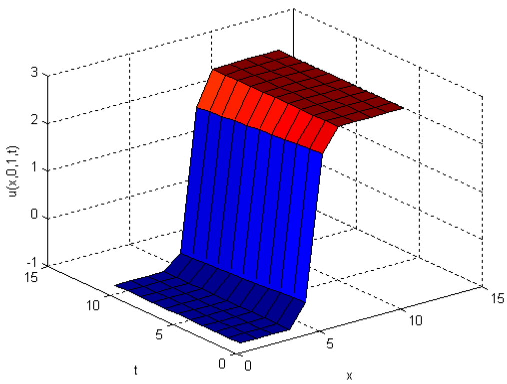

Figure (4) represents Solution at z = 1

By using tanh method and using the traveling wave transformations in Eq(5) and Eq.(3) with the transformation in Eq.(7), the nonlinear system of partial differential equation (16) is carried to an ordinary differential equation

$$k^3\lambda U'''' + 6k^2\lambda U' U'' + (\lambda\omega + k\omega - \beta^2)U'' = 0 \quad (26)$$

Eq.(26) can be written as:

$$k^3\lambda U'''' + 3k^2\lambda (U'^2)' + (\lambda\omega + k\omega - \beta^2)U'' = 0 \quad (27)$$

Integrating Eq.(27) once with zero constant

$$k^3\lambda U''' + 3k^2\lambda U'^2 + (\lambda\omega + k\omega - \beta^2)U' = 0 \quad (28)$$

we postulate the tanh series, Eq(28) reduces to

$$\begin{aligned} &k^3\lambda[2(1-Y^2)(3Y^2-1)\frac{dU}{dY} - 6Y(1-Y^2)^2\frac{d^2U}{dY^2} \\ &+(1-Y^2)^3\frac{d^3U}{dY^3}] + 3k^2\lambda[(1-Y^2)\frac{dU}{dY}]^2 + \\ &(\lambda\omega + k\omega - \beta^2)[(1-Y^2)\frac{dU}{dY}] = 0 \end{aligned} \quad (29)$$

Now, to determine the parameter m , balance the linear term of highest-order with the highest order nonlinear terms. So, in Eq. (29) , balance $U^{///}$ with $U^{/2}$, to obtain
3 + m = 2m+2 , then m = 1
The tanh method admits the use of the finite expansion for :

$$u(x,t) = U(Y) = a_0 + a_1 Y \ , \ a_1 \neq 0 \qquad (30)$$

Substituting $U, U^{/}, U^{//}, U^{///}$ in Eq. (28), then equating the coefficient of Y^i , i= 0, 2 leads

$$\overset{0}{Y} : -2k^3\lambda + 3k^2\lambda a_1 + (\lambda\omega + k\omega - \beta^2) = 0$$

$$\overset{2}{Y} : 6k^3\lambda - 3k^2\lambda a_1 = 0 \qquad (31)$$

Solving the nonlinear system of equations (31) to get:

$$a_1 = 2k \ , \quad \omega = \frac{\beta^2 - 4k^3\lambda}{\lambda + k}$$

and

$$u(x,y,z,t) = a_0 + 2k \tanh\{kx + \lambda y + \beta z + \frac{\beta^2 - 4k^3\lambda}{\lambda + k} t + \theta_0\} \qquad (32)$$

for

$$a_0 = 1, k = \lambda = \beta = 1 \quad \theta_0 = 0 \qquad y = 0$$

$$u(x,0,z,t) = 1 + 2\tanh\{x + z - \frac{3}{2}t\} \qquad (33)$$

the solitary wave and behavior of the solution in Eq.(33) is shown in Figures (5) and (6).

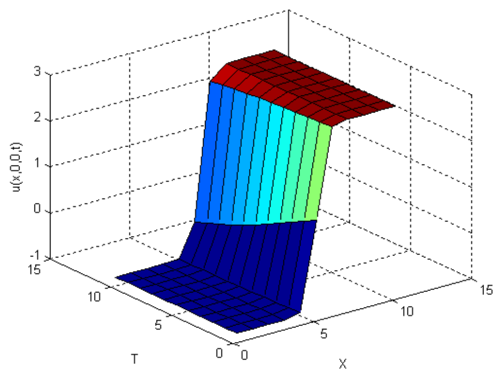

Figure (5) represents Solution at z = 0

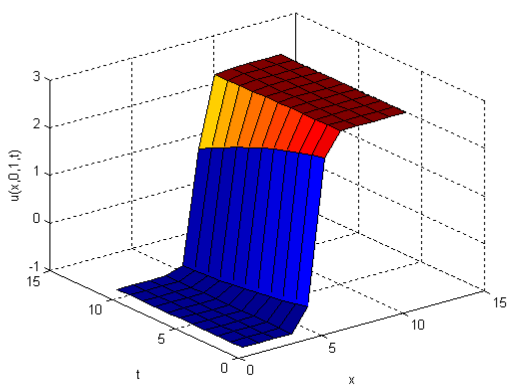

Figure (6) represents Solution at z = 1

3.4 (3+1)-dimensional generalized KP-II equation

Other form of the (3 + 1)-dimensional KP-II equation is given by Ablowitz *etal* [17], and Senatorski *et al* [18]:

$$(u_t + 6uu_x + u_{xxx})_x - 3(u_{yy} + u_{zz}) = 0 \qquad (34)$$

This explains wave propagation in the field of plasma physics, fluid dynamics, etc. Soliton simulation studies for Eq.(34) have been done by Senatorski et al. [18].

To study the travelling wave solutions to Eq.(34), substitute $u(x,y,z,t) = U(\zeta)$, and $\xi = kx + \lambda y + \beta z + \omega t + \theta_0$ into Eq.(34) and integrating twice, we have:

$$k^4U^{//} + [k\omega - 3(\lambda^2 + \beta^2)]U + 3k^2U^2 = 0 \qquad (35)$$

postulate tanh series , and the transformation given in Eq.(3), so that Eq.(35) reduces to:

$$k^4[-2Y(1-Y^2)\frac{dU}{dY} + (1-Y^2)^2\frac{d^2U}{dY^2}] +$$
$$[k\omega - 3(\lambda^2 + \beta^2)]U + 3k^2U^2 = 0$$

(36)

Now, to determine the parameter *m*, we balance the linear term of highest-order with the highest order nonlinear terms. So, in Eq.(36) we balance U^2 with $U^{//}$, to obtain:

m+2 = 2*m* , then *m* = 2.

The tanh-coth method admits the use of the finite expansion for :

$$U = a_0 + a_1Y + a_2Y^2 + b_1Y^{-1} + b_2Y^{-2}$$

and

$$U^{/} = a_1 + 2a_2Y - b_1Y^{-2} - 2b_2Y^{-3}$$

and

$$U^{//} = 2a_2 + 2b_1Y^{-3} + 6b_2Y^{-4}$$

(37)

Substituting $U^{/}, U^{//}$ from Eq.(37) in Eq.(36), then equating the coefficient of Y^i , i= 0, 1, 2, 3, 4, -1, -2, -3, -4 leads to the following nonlinear system of algebraic equations:

$$\overset{0}{Y} : 2k^4[a_2 + b_2] + [k\omega - 3(\lambda^2 + \beta^2)]a_0 + 3k^2(a_0^2 + 2a_1b_1 + 2a_2b_2) = 0$$

$$\overset{1}{Y} : -2k^4a_1 + [k\omega - 3(\lambda^2 + \beta^2)]a_1 + 6k^2(a_0a_1 + a_2b_1) = 0$$

$$\overset{2}{Y} : -8k^4a_2 + [k\omega - 3(\lambda^2 + \beta^2)]a_2 + 3k^2(a_1^2 + 2a_0a_2) = 0$$

$$\overset{3}{Y} : 2k^4a_1 + 6k^2a_1a_2 = 0$$

$$\overset{4}{Y} : 6k^4a_2 + 3k^2a_2^2 = 0$$

(38)

$$\overset{-1}{Y}: -2k^4 b_1 + [k\omega - 3(\lambda^2+\beta^2)]b_1 + 6k^2(a_0 b_1 + a_1 b_2) = 0$$

$$\overset{-2}{Y}: -8k^4 b_2 + [k\omega - 3(\lambda^2+\beta^2)]b_2 + 3k^2(2a_0 b_2 + b_1^2) = 0$$

$$\overset{-3}{Y}: 2k^4 b_1 + 6k^2 b_1 b_2 = 0$$

$$\overset{-4}{Y}: 6k^4 b_2 + 3k^2 b_2^2 = 0$$

Solving the nonlinear systems of equations (38) we can get $a_1 = 0$ and $b_1 = 0$ with the following cases:

Case 1

$$a_0 = 4k^2,\ a_2 = -2k^2,\ b_2 = -2k^2,$$

$$\omega = -16k^3 + \frac{3(\alpha^2+\beta^2)}{k}$$

$$u_1 = 2k^2[2 - \tan h^2(kx+\lambda y + \beta z + [\frac{3(\alpha^2+\beta^2)}{k} - 16k^3]t + \theta_0) - \coth^2(kx+\lambda y + \beta z + [\frac{3(\alpha^2+\beta^2)}{k} - 16k^3]t + \theta_0)] \quad (39)$$

or

$$u_1 = 2k^2 \operatorname{csc}h^2(kx+\lambda y + \beta z + [\frac{3(\alpha^2+\beta^2)}{k} - 16k^3]t + \theta_0)$$

Case 2

$$a_0 = \frac{-4}{3}k^2,\ a_2 = -2k^2,\ b_2 = -2k^2,$$

$$\omega = 16k^3 + \frac{3(\alpha^2+\beta^2)}{k}$$

$$u_2 = -2k^2[\frac{2}{3} + \tanh^2(kx+\lambda y + \beta z + [\frac{3(\alpha^2+\beta^2)}{k} + 16k^3]t + \theta_0) + \coth^2(kx+\lambda y + \beta z + [\frac{3(\alpha^2+\beta^2)}{k} + 16k^3]t + \theta_0)] \quad (40)$$

Case 3

$$a_0 = \frac{4+\sqrt{14}}{3}k^2, a_2 = -2k^2, b_2 = \frac{-k^2}{3}$$

$$\omega = -2\sqrt{14}k^3 + \frac{3(\alpha^2+\beta^2)}{k}$$

$$u_3 = k^2[\frac{4+\sqrt{14}}{3} - 2\tanh^2(kx+\lambda y+\beta z+[\frac{3(\alpha^2+\beta^2)}{k} - 2\sqrt{14}k^3]t+\theta_0)$$
$$-\frac{1}{3}\coth^2(kx+\lambda y+\beta z+[\frac{3(\alpha^2+\beta^2)}{k} - 2\sqrt{14}k^3]t+\theta_0)] \quad (41)$$

Case 4

$$a_0 = \frac{4-\sqrt{14}}{3}k^2, a_2 = -2k^2, b_2 = \frac{-k^2}{3}$$

$$\omega = 2\sqrt{14}k^3 + \frac{3(\alpha^2+\beta^2)}{k}$$

$$\mathrm{u}_4 = k^2[\frac{4-\sqrt{14}}{3} - 2\tanh^2\left(kx+\lambda y+\beta z+[\frac{3(\alpha^2+\beta^2)}{k} + 2\sqrt{14}k^3]t+\theta_0\right)$$
$$-\frac{1}{3}\coth^2\left(kx+\lambda y+\beta z+[\frac{3(\alpha^2+\beta^2)}{k} + 2\sqrt{14}k^3]t+\theta_0\right)] \quad (42)$$

Case 5

$$a_0 = \frac{4-\sqrt{14}}{3}k^2, a_2 = \frac{-k^2}{3}, b_2 = -2k^2$$

$$\omega = 2\sqrt{14}k^3 + \frac{3(\alpha^2+\beta^2)}{k}$$

$$u_5 = k^2[\frac{4-\sqrt{14}}{3} - \frac{1}{3}\tanh^2\left(kx+\lambda y + \beta z + [\frac{3(\alpha^2+\beta^2)}{k} + 2\sqrt{14}k^3]t+\theta_0 \right)$$
$$-2\coth^2\left(kx+\lambda y + \beta z + [\frac{3(\alpha^2+\beta^2)}{k} + 2\sqrt{14}k^3]t+\theta_0 \right)] \quad (43)$$

Case 6

$$a_0 = \frac{4+\sqrt{14}}{3}k^2, \quad a_2 = \frac{-k^2}{3}, \quad b_2 = -2k^2$$

$$\omega = -2\sqrt{14}k^3 + \frac{3(\alpha^2+\beta^2)}{k}$$

$$u_6 = k^2[\frac{4+\sqrt{14}}{3} - \frac{1}{3}\tanh^2\left(kx+\lambda y + \beta z + [\frac{3(\alpha^2+\beta^2)}{k} - 2\sqrt{14}k^3]t+\theta_0 \right)$$
$$-2\coth^2\left(kx+\lambda y + \beta z + [\frac{3(\alpha^2+\beta^2)}{k} - 2\sqrt{14}k^3]t+\theta_0 \right)] \quad (44)$$

Case 7

$$a_0 = \frac{4+\sqrt{14}}{3}k^2, \quad a_2 = \frac{-k^2}{3}, \quad b_2 = \frac{-k^2}{3}$$

$$\omega = -2\sqrt{14}k^3 + \frac{3(\alpha^2+\beta^2)}{k}$$

$$u_7 = k^2[\frac{4+\sqrt{14}}{3} - \frac{1}{3}\tanh^2\left(kx+\lambda y + \beta z + [\frac{3(\alpha^2+\beta^2)}{k} - 2\sqrt{14}k^3]t+\theta_0 \right)$$
$$-\frac{1}{3}\coth^2\left(kx+\lambda y + \beta z + [\frac{3(\alpha^2+\beta^2)}{k} - 2\sqrt{14}k^3]t+\theta_0 \right)] \quad (45)$$

Case 8

$$a_0 = \frac{4-\sqrt{14}}{3}k^2, \quad a_2 = \frac{-k^2}{3}, \quad b_2 = \frac{-k^2}{3}$$

$$\omega = 2\sqrt{14}k^3 + \frac{3(\alpha^2+\beta^2)}{k}$$

$$u_8 = k^2[\frac{4-\sqrt{14}}{3} - \frac{1}{3}\tanh^2\left(kx+\lambda y+\beta z+[\frac{3(\alpha^2+\beta^2)}{k}+2\sqrt{14}k^3]t+\theta_0 \right)$$
$$-\frac{1}{3}\coth^2\left(kx+\lambda y+\beta z+[\frac{3(\alpha^2+\beta^2)}{k}+2\sqrt{14}k^3]t+\theta_0 \right)]$$
(46)

Results of solving (3+1)-dimensional KP-II in this paper are compatible with that results obtained by Wazwaz [10] for the (2+1)-dimensional KP equation. For $k=\lambda=\beta=1, \theta_0=0$, the solitary solution of case 1, in Eq.(39) becomes:

$$u_1(x,y,z,t) = 2\operatorname{csch}^2\left(x+y+z-10t \right) \qquad (47)$$

and is shown in figure (7).

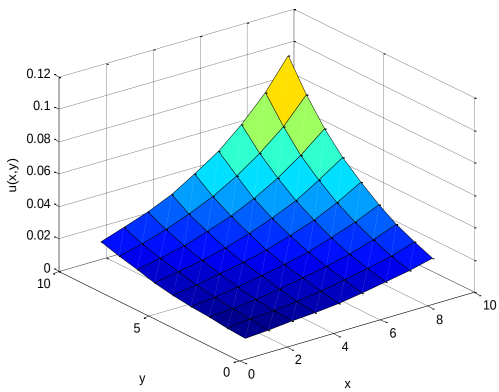

Figure (7) Solitary solution of case 1
at $0.1 \le y \le 1$, $0.1 \le x \le 1$, $t=0.5, z=1$

4. Results and Conclusion:

The Tanh Method was employed for analytic treatment of (3+1)- dimensional nonlinear partial differential equations. The tanh method requires transformation formulas. Traveling wave solutions of the four examples show that these solutions had the same trend and shape. Solutions of four examples by the Tanth

Method demonstrate the effectiveness and convenience in solving Nonlinear PDEs compared with other methods.

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حلول الموجة المنعزلة عن (3 +1) الأبعاد لمعادلات التطور اللاخطية باستخدام طريقة الظل الزائدي

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المستخلص:

تم في هذا البحث تطبيق طريقة الظل الزائدي للحصول على حلول الموجات المنعزلة في معادلات التطور غيرخطية من النوع (3+1) الابعاد ومعروفة الاستخدام في المجالات العلمية الهندسية والفيزياوية ، وهي معادلات:

Jimbo-Miwa , potential-YTSF, and generalized KP

تم مقارنة الحلول المستخرجة من طريقة الظل الزائدي مع تلك المستخرجة من طرائق اخرى واثبتت طريقة الظل الزائدي كفائتها في الحل التام وبالتالي فانها اسلوب جيد لحل معادلات التطور غير الخطية.