Modified Equations for Water Flow through Packed Bed for different types of packing systems

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Abstract

Semi-empirical equations for water flow through packed bed have been estimated, for different types of packing systems (mono size spherical particle system, mono size non spherical particle system, binary sized spherical particle system, ternary sized spherical particle system, quaternary sized spherical particle system, quinary sized spherical particle) depending on Buckingham π theorem. The working range of the modified equation is within the fixed region of the fluid flow diagram, i.e., the modified equation can be used for fluid flow up to the fluidization point.

The results of all calculations for the modified equations have been compared with many documented experimental literatures. This comparison gave a very good agreement, and has been represented in curves. The results from Ergun equation using similar conditions have been represented in the curves for the sake of comparison.

Introduction

Fluid flow through packed bed is a frequent occurrence in the chemical industry and therefore expressions are needed to predict pressure drop across beds due to the resistance caused by the presence of the particles [1].

A typical packed bed setup is a cylindrically-shaped column filled with packing materials. The column can vary in diameter, height, and material. The packing material can vary in shape, roughness, and particle size [2, 3]. The most important factor in concerning the bed from a mechanical perspective is the pressure drop required for the liquid or the gas to flow through the column at a specified flow rate[4].

The fluid flow through packed bed has attracted considerable attention from many investigators. Darcy [5] derived a semi-empirical equation describing fluid transport in packed bed for single-phase flow. Carman and Kozeny [6] derived an expression for pressure drop under viscous flow. Burke and Plummer [7] derived an expression for change in pressure at turbulent flow resulting from kinetic energy .Ergun [8] proposed a semi-empirical equation covers any flow type and condition (laminar, transitional and turbulent) by adding the Carman-Kozeny equation for laminer flow to the Burke-Plummer equation derived for the fully turbulent. Ergun equation applies to a broad spectrum of fluids and packing materials, but it does not predict pressure drop behavior after the point of fluidization because of bed expansion and changes in packing void fraction [9]. Ergun's equation does not take in consideration wall effects, which represents pipe like flow around the edges of the column [10, 11]. Leva [12] predicted the pressure drop of flow rate based on the study of single incompressible fluids through an incompressible bed of granular salts. Dullien and MacDonald addressed the problem of multi-sized particles present in a porous media. Dullien [13] modified Kozeny equation assuming pores with periodic step changes in their diameter. MacDonald [14] generalized the Blake-Kozeny equation for multi sized spherical particles. Bey and Eigenberger [15] have represented the pressure drop in the packing by modifying the Ergun equation for a cylindrical coordinated

system. Shenoy et. al [16] developed a theoretical model for the prediction of velocity and pressure drop for the flow of a viscous power law fluid through a bed packed with uniform spherical particles. Hellström and Lundström[17] suggested a model for flow through packed bed taking into consideration the inertia-effects. They compared their results with Ergun equation, and it fits well to Ergun equation.

Theoretical Part: Water Flow Semi-Empirical Equations

Semi-empirical formulas for modeling water flow through packed bed were modiefied for the parameters affecting the pressure drop using <u>Buckingham π theorem</u> [18]. This formula consists of multiplied dimensionless terms raised to certain powers [19]; these powers were evaluated from experimental data taken from literatures with statistical fitting.

The method of modeling used to derive an expression for the pressure drop was based on curve fitting of the available literatures experimental data, and by implementing dimensional analysis. This analysis can be summarized as follows:

The pressure drop was assumed to be dependent on fluid velocity (u),packing diameter (d_p),bed length (L),fluid density (ρ),fluid viscosity (μ),porosity (ε),and sphericity (ϕ), and can be written in the following expression:

$$\Delta P = f\left(u, d_p, L, \rho, \mu, \varepsilon, \phi\right)$$

... (1)

The Buckingham's π theorem [18] was used to write the semi-empirical formula of the fluid flow equation. In this theorem the dimensions of a physical quantity are associated with mass, length and time ,represented by symbols M,L and T respectively, each raised to rational powers[20]. The Buckingham's π theorem [18] forms the basis of the central tool of the dimensional analysis. This theorem describes how every physically meaningful equation involving n variables can be equivalently rewritten as an equation of *n-m* dimensionless parameters, whereas, the number of fundamental dimensions used. Furthermore, and the most important is that it proves a method for computing these

dimensionless parameters from the given variables [19]. According to this theorem n=8 and m=3, then this theorem gave us five dimensionless groups.

Table 1: The Dimensions of Parameters Used In Expression (1)

Variable	Dimension
Pressure drop	$M\,L^{-1}T^{-2}$
Fluid velocity	$L\,T^{-1}$
Particle diameter	L
Bed length	L
Fluid density	$M\,L^{-3}$
Fluid viscosity	$M\,L^{-1}T^{-1}$
Porosity	-
Sphericity	-

Selecting the variables particle diameter, fluid velocity, and fluid density.

The particle diameter (d_p) has the dimension L therefore $L = d_p$

The fluid velocity (u) has dimensions $L\,T^{-1}$ therefore $T = d_p\,u^{-1}$

The fluid density (ρ) has dimensions $M\,L^{-3}$ therefore $M = \rho\,d_p^{3}$

The first group (π_1) = ΔP ($M^{-1}\,L\,T^{2}$)

$$\pi_1 = \frac{\Delta P}{\rho u^2}$$ … (2)

The second group (π_2) = L (L^{-1})

$$\pi_2 = \frac{L}{d_p}$$ … (3)

The third group (π_3) = μ ($M^{-1}\,L\,T$)

$$\pi_3 = \frac{\mu}{\rho u d_p}$$ … (4)

The fourth group (π_4) = ε

$$\pi_4 = \varepsilon$$ … (5)

The fifth group (π_5) = ϕ

$$\pi_5 = \phi \qquad \qquad \ldots (6)$$

Therefore the equation for the pressure drop dependence on fluid velocity (u), packing diameter (d_p), bed length (L), fluid density (ρ), fluid viscosity (μ), porosity (ε), and sphericity (ϕ) will be as follows:

$$\frac{\Delta P}{\rho u^2} = b_1 \left(\frac{L}{d_p} \right)^{b_2} \left(\frac{\mu}{\rho\, d_p\, u} \right)^{b_3} \varepsilon^{b_4}\ \phi^{b_5} \qquad \qquad \ldots (7)$$

While Reynolds number is defined as:

$$\mathrm{Re} = \frac{\rho\, d_p u}{\mu} \qquad \qquad \ldots (8)$$

Then equation (7) can be written as follows:

$$\frac{\Delta P}{\rho u^2} = b_1 \left(\frac{L}{d_p} \right)^{b_2} \left(\frac{1}{\mathrm{Re}} \right)^{b_3} \varepsilon^{b_4}\ \phi^{b_5} \qquad \qquad \ldots (9)$$

Where b_1, b_2, b_3, b_4 and b_5 are constants which can be evaluated from experiments data taken from literature by statistical fitting. The above equation can be used for different types of packing system.

Since ($\Delta P/\rho u^2$) describes fluid flow through packed bed, therefore; equation 9 can be considered as a semi-empirical equation of fluid flow through packed bed. Each term of this equation is a dimensionless group, because ($\Delta P/\rho u^2$) is dimensionless number.

Results and Discussions

The results of the modified semi-empirical equations are presented, discussed and compared with experimental results taken from literatures, as well as with results taken by using Ergun equation for water flow through packed bed.

Ergun believed that the pressure drop over the length of the packing was dependent upon rate of fluid flow, viscosity and density of the fluid, closeness and

orientation of packing, size, shape, and surface of the packing material [21]. **Ergun** equation can be expressed as:

$$\frac{\Delta P}{L} = 150 \; \frac{\mu u}{\phi^2 d_p^2} \frac{(1-\varepsilon)^2}{\varepsilon^3} + 1.75 \; \frac{\rho u^2}{\phi d_p} \frac{(1-\varepsilon)}{\varepsilon^3} \qquad \dots (10)$$

Where ΔP, ε, ρ, d_p, Φ, u, L, and μ are the pressure drop, void fraction of the bed, density of the fluid, particle diameter, sphericity of the particle, fluid velocity, height of the bed, and the fluid viscosity respectively. For beds consisting of a mixture of different particle diameters, the effective particle diameter (dp_{eff}) can be used instead of d_p (in equation 10) as:

$$dp_{eff} = \frac{1}{\sum\limits_{i=1}^{n} \dfrac{x_i}{d_{pi}}} \qquad \dots (11)$$

Where x_i is the weight fraction for particle of size d_{pi} [22, 23].

The sphericity of a particle is the ratio of the surface area of this sphere having the same volume as the particle to the actual surface area of the particle, as shown below:

$$\Phi = \frac{a_{sphere}}{a_{particale}} = \frac{6/d_p}{S_{particle}/V_{particle}} \qquad \dots (12)$$

For a sphere, the surface area $S_p = \pi d_P^2$ and the volume is $V_p = \pi d_p^3/6$ [21].

Several equations were modified by implementing different shapes of packing materials in equation 9; these equations are illustrated in table 2. This table shows all the information's found of the present work.