



## Fuzzy Weakly Preclosed Functions between Fuzzy Topological Spaces



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### Abstract

Many researchers introduced and characterize fuzzy sets, Prof. Zadeh (1965) introduced the concept of fuzzy sets, much attention has been paid in order to generalize the basic concept of classical topology in fuzzy setting and thus a modern theory of fuzzy topology is developed. In (1985) D.A. Rose defined weakly open functions in topological spaces, J.H. Park, Y.B. Park and J.S. Park (1997) introduced the notion of weakly open functions in between fuzzy topological spaces.

In this paper we discuss the concept of fuzzy weakly preclosed functions. By  $(X, \tau)$  we mean a fuzzy topological space (fts) due to Chang.

A fuzzy point  $x \in X$  with  $p$  ( $0 < p \leq 1$ ) is denoted by  $x_p$ . Two fuzzy sets  $\lambda$  and  $\beta$  are called quasi-coincident denoted by  $\lambda q \beta$  if  $\exists x \in X \ni \lambda(x) + \beta(x) > 1$  and if it is not quasi-coincident it denoted by  $\bar{q}$ .  $\lambda \leq \beta$  iff  $\lambda \bar{q} (1 - \beta)$ . A fuzzy set  $\lambda$  is said to  $q$ -neighborhood ( $q$ -nbd) of  $x_p$  if there is a fuzzy open set  $\mu$  such that  $x_p q \mu$ . [8]

If  $\lambda$  be a fuzzy set  $\in X$ , then  $pCl(\lambda) = \bigcap \{ \beta : \beta \geq \lambda, \beta \text{ is a fuzzy preclosed} \}$   
 $pInt(\lambda) = \bigcup \{ \beta : \lambda \geq \beta, \beta \text{ is a fuzzy preopen} \}$  is called a fuzzy preclosure of  $\lambda$ .

Let  $f : (X, \tau_1) \rightarrow (Y, \tau_2)$  be a function from a fts  $(X, \tau_1)$  into  $(Y, \tau_2)$ , the function  $f$  is called fuzzy preclosed if  $f(\lambda)$  is a fuzzy preclosed of  $Y$  for each fuzzy closed set  $\lambda$  in  $X$ .

A function  $f : (X, \tau_1) \rightarrow (Y, \tau_2)$  is said to be fuzzy weakly preclosed if  $pCl(f(Int(\beta))) \leq f(\beta)$  for each fuzzy closed subset  $\beta$  of  $X$ . A function  $f : (X, \tau_1) \rightarrow (Y, \tau_2)$  is called fuzzy contra-closed if  $f(\lambda)$  is a fuzzy open set in  $Y$  for each fuzzy closed set  $\lambda$  in  $X$ .

In this paper some modification on the fuzzy preclosed functions between topologies is presented and the conclusions are listed at the end.

### 1-Some Definitions

A fuzzy set  $\lambda$  in a fts  $X$  is called :

1. Fuzzy preopen [3] if  $\lambda \leq Int(cl(\lambda))$ .
2. Fuzzy preclosed [3] if  $cl(Int(\lambda)) \leq \lambda$ .
3. Fuzzy regular open [1] if  $\lambda = Int(cl(\lambda))$ .
4. Fuzzy regular closed [1] if  $\lambda = cl(Int(\lambda))$ .
5. Fuzzy  $\alpha$ -open [3] if  $\lambda \leq Int(cl(Int(\lambda)))$ .
6. Fuzzy  $\alpha$ -closed [3] if  $cl(Int(cl(\lambda))) \leq \lambda$ .

Where  $Int(\lambda)$ ,  $cl(\lambda)$  and  $(1 - \lambda)$  represent the interior, closure and the complement of  $\lambda$  respectively.



**Definitions 1-1 [2]**

Let  $f : (X, \tau_1) \rightarrow (Y, \tau_2)$  be a function from a fts  $(X, \tau_1)$  into a fts  $(Y, \tau_2)$ . The function  $f$  is called :

1. fuzzy preclosed [ 3 ] if  $f(\lambda)$  is a fuzzy preclosed of  $Y$  for each fuzzy closed set  $\lambda$  in  $X$ .
2. fuzzy weakly open [ 8 ] if  $f(\lambda) \leq Int(f(cl(\lambda)))$  for each fuzzy open set  $\lambda$  in  $X$ .
3. fuzzy contra-closed if  $f(\lambda)$  is a fuzzy open set of  $Y$  for each fuzzy closed set  $\lambda$  in  $X$ .

**Definition 1-2 [6]**

A fuzzy point  $x_p$  in a fts  $X$  is said to be a fuzzy  $\theta$ -cluster point of a fuzzy set  $\lambda$  if and only if for every fuzzy open  $q$ -nbd  $\mu$ -of  $x_p$ ,  $Cl(\mu)$  is  $q$ -coincident with  $\lambda$ . The set of all fuzzy  $\theta$ -cluster points of  $\lambda$  is called the fuzzy  $\theta$ -cluster of  $\lambda$  and is denoted by  $Cl_\theta(\lambda)$ . A fuzzy set  $\lambda$  is fuzzy  $\theta$ -closed if and only if  $\lambda = Cl_\theta(\lambda)$ .

**Definition 1-3 [7]**

A function  $f : (X, \tau_1) \rightarrow (Y, \tau_2)$  is said to be fuzzy weakly preclosed if  $pCl(f(Int(\beta))) \leq f(\beta)$  for each fuzzy closed subset  $\beta$  of  $X$ .

Clearly, every fuzzy preclosed function is fuzzy weakly preclosed function since  $pCl(f(Int(\beta))) \leq pCl(f(\beta)) = f(\beta)$  for every fuzzy closed subset  $\beta$  of  $X$ , but the converse is not generally true, as the next example shows.

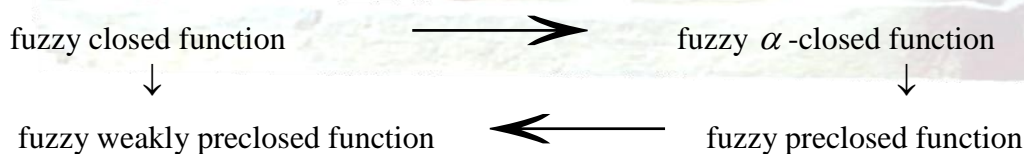
**Example 1-4 [5]**

Let  $X = \{a, b\}$  and  $Y = \{x, y\}$ . Fuzzy sets  $A$  and  $B$  are defined as :

$$A(x) = 0.4 \quad A(y) = 0.3 \quad A(a) = 0.5 \quad A(b) = 0.6$$

Let  $\tau_1 = \{0, B, 1_x\}$  and  $\tau_2 = \{0, A, 1_y\}$ . Then the function  $f : (X, \tau_1) \rightarrow (Y, \tau_2)$

defined by  $f(a) = x$ ,  $f(b) = y$  is fuzzy weakly preclosed but not fuzzy preclosed. Thus, every fuzzy closed function is fuzzy  $\alpha$ -closed and every fuzzy  $\alpha$ -closed is fuzzy preclosed, but the reverse implications not be true in general [ 5 ]. We have the following diagram and the converses of these implication do not hold, in general as is showed.



**2-Main Reselts**

**Theorem 2-1**

Let  $f : (X, \tau_1) \rightarrow (Y, \tau_2)$  be a function, the following conditions equivalent :

- i)  $f$  is fuzzy weakly preclosed.
- ii)  $pCl(f(\lambda)) \leq f(Cl(\lambda))$  for every fuzzy open set  $\lambda$  in  $X$ .

**Proof**

(i)  $\rightarrow$  (ii)

Let  $\lambda$  be any fuzzy open subset of  $X$ . Then ,  $pCl(f(\lambda)) = pCl(f(Int(\lambda))) \leq pCl(f(Int(Cl(\lambda)))) \leq f(Cl(\lambda))$ .

(ii)  $\rightarrow$  (i)



Let  $\beta$  be any fuzzy closed subset of  $X$ . Then ,  
 $pCl(f(Int(\beta))) \leq f(Cl(Int(\beta))) \leq f(Cl(\beta)) = f(\beta)$ .

### Theorem 2-2

For a function  $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ , the following conditions are equivalent:

- (i)  $f$  is fuzzy weakly preclosed .
- (ii)  $pCl(f(\beta)) \leq f(Cl(\beta))$  for every fuzzy open subset  $\beta$  of  $X$  .
- (iii)  $pCl(f(Int(\beta))) \leq f(\beta)$  for each fuzzy closed subset  $\beta$  of  $X$  .
- (iv)  $pCl(f(Int(\beta))) \leq f(\beta)$  for each fuzzy preclosed subset  $\beta$  of  $X$  .
- (v)  $pCl(f(Int(\beta))) \leq f(\beta)$  for each fuzzy  $\alpha$ -closed subset  $\beta$  of  $X$  .

### Theorem 2-3

For a function  $f : (X, \tau_1) \rightarrow (Y, \tau_2)$  the following conditions are equivalent :

- (i)  $f$  is fuzzy weakly preclosed .
- (ii)  $pCl(f(Int(Cl(\lambda)))) \leq f(Cl(\lambda))$  for each fuzzy set  $\lambda$  in  $X$  .
- (iii)  $pCl(f(Int(Cl_\theta(\lambda)))) \leq f(Cl_\theta(\lambda))$  for each fuzzy set  $\lambda$  in  $X$  .

### Proof

It is clear that (i)  $\rightarrow$  (ii) , (i)  $\rightarrow$  (iii) and (ii)  $\rightarrow$  (i) . To show that (iii)  $\rightarrow$  (i) : It suffices see that  $Cl_\theta(\lambda) = Cl(\lambda)$  for every fuzzy open sets  $\lambda$  in  $X$  .

### Definition 2-4 [4]

A fuzzy set  $\lambda$  in  $(X, \tau)$  is called pre- $q$ -nbd of  $x_\alpha$  if there exists a fuzzy preopen subset  $\mu$  in  $X$  such that  $x_\alpha q \mu \leq \lambda$  .

### Theorem 2-5

If  $f : (X, \tau_1) \rightarrow (Y, \tau_2)$  is fuzzy weakly preclosed and if for each fuzzy closed subset  $\beta$  of  $X$  and each fiber  $f^{-1}(y_p) \leq 1_x - \beta$  there exists a fuzzy open  $q$ -nbd  $\mu$  of  $X$  such that  $f^{-1}(y_p) \leq \mu \leq Cl(\mu) \leq 1_x - \beta$  . Then  $f$  is fuzzy preclosed .

### Proof

Let  $\beta$  is any fuzzy closed subset of  $X$  and let  $y_p \in 1_y - f(\beta)$  . Then  $f^{-1}(y_p) \bar{q} \beta$  and hence  $f^{-1}(y_p) \leq 1_x - \beta$  . By hypothesis , there exists a fuzzy open  $q$ -nbd  $\mu$  of  $X$  such that  $f^{-1}(y_p) \leq \mu \leq Cl(\mu) \leq 1_x - \beta$  . Since  $f$  is fuzzy weakly preclosed , there exists a fuzzy preopen  $q$ -nbd  $\nu$  in  $Y$  with  $y_p \in \nu$  and  $f^{-1}(\nu) \leq Cl(\mu)$  . Therefore , we obtain  $f^{-1}(\nu) \bar{q} \beta$  and hence  $\nu \bar{q} f(\beta)$  , this shows that  $y_p \notin pCl(f(\beta))$  . Therefore,  $f(\beta)$  is fuzzy preclosed in  $Y$  and  $f$  is fuzzy preclosed.

### Theorem 2-6

If  $f : (X, \tau_1) \rightarrow (Y, \tau_2)$  is fuzzy preclosed and fuzzy contra-closed , then  $f$  is fuzzy weakly preclosed .

### Proof

Let  $\beta$  be a fuzzy closed subset of  $X$  . Since  $f$  is fuzzy preclosed  $Cl(Int(f(\beta))) \leq f(\beta)$  and since  $f$  is fuzzy contra-closed  $f(\beta)$  is fuzzy open. Therefore,  $pCl(f(Int(\beta))) \leq pCl(f(\beta)) \leq Cl(Int(f(\beta))) \leq f(\beta)$  .



**Theorem 2-6**

If  $f : (X, \tau_1) \rightarrow (Y, \tau_2)$  is fuzzy weakly preclosed , then for every fuzzy subset  $\beta$  in  $Y$  and every fuzzy open set  $\lambda$  in  $X$  with  $f^{-1}(\beta) \leq \lambda$ , there exists a fuzzy preclosed set  $\delta$  in  $Y$  such that  $\beta \leq \delta$  and  $f^{-1}(\delta) \leq Cl(\lambda)$ .

**Proof**

Let  $\beta$  be a fuzzy subset of  $Y$  and let  $\lambda$  be a fuzzy open subset of  $X$  with  $f^{-1}(\beta) \leq \lambda$ . Put  $\delta = pCl(f(Int(Cl(\lambda))))$ , then  $\delta$  is a fuzzy preclosed set of  $Y$  such that  $\beta \leq \delta$  since  $\beta \leq f(\lambda) \leq f(Int(Cl(\lambda))) \leq pCl(f(Int(Cl(\lambda)))) = \delta$ .

And since  $f$  is fuzzy weakly preclosed,  $f^{-1}(\delta) \leq Cl(\lambda)$ .

**Corollary 2-7**

If  $f : (X, \tau_1) \rightarrow (Y, \tau_2)$  is fuzzy weakly preclosed , then for every fuzzy point  $y_p$  in  $Y$  and every fuzzy open set  $\lambda$  in  $X$  with  $f^{-1}(y_p) \leq \lambda$ , there exists a fuzzy preclosed set  $\delta$  in  $Y$  containing  $y_p$  such that  $f^{-1}(\delta) \leq Cl(\lambda)$ .

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