On Discrete Frechet Distribution : Estimation and Application

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ABSTRACT

A discrete two-parameter Fréchet distribution corresponding to the continuous Fréchet distribution is introduced and derived by using the approach of discretization of continuous distribution functions . The Survival and Hazard functions of the proposed distribution were also found and examined . Two methods were discussed to estimate the parameters , Maximum Likelihood and the Bayesian method , and then compare the performance using the simulation method . In addition, the compatibility of the proposed distribution was investigated through real data , and then the estimates of the survival and Hazard functions were reached . The superiority of the Bayesian method in data modelling was shown .

Keywords : Fréchet distribution , survival function , hazard function , Bayesian estimation , MLE .

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1.Introduction

In most survival studies , continuous data are often used , although discrete data sets are more common in real situations due to the presence of discrete measurements of time . Accordingly , the counter-part was proposed to model the discrete life data and then define it based on the shape and characteristics similar to the continuous distribution .

Nowadays , with the increasing development in data collection and storage due to technological advancement , counting data has become widely available in many areas , such as transportation safety , number of highway breakdowns , number of calls to the call center , and others .

Discrete distributions with continuous corresponding parts can be obtained either by considering the characteristic of a continuous distribution and then constructing a similar distribution in discrete time , such as a geometric distribution , or by looking at the integer part of the continuous time as discrete time . Nakagawa and Osaki (1975) [5] are proposed the discrete distributions based on their continuous counter-parts and suggested discrete Weibull distribution . Vila et al. (2018) [7] discussed the important theoretical results on the discrete Weibull distribution of Nakagawa and Osaki. Krishna and Pundir (2009) [4] are studied estimation of the parameters of the discrete Maxwell distribution . Chakraborty and Chakravarty (2012) [2] are studied the moments , Reliability Characteristics and the estimation of the parameters of discrete gamma distribution . Kamari et al . (2016) [3] studied Bayesian estimation of discrete Burr distribution with two parameters . Alamatsaz et al. (2016) [1] presented discrete generalized Rayleigh (DGR) distribution moments , and derive the Order Statistics of this distribution.

In our research , the mechanism of converting the continuous distribution into the discrete one was employed for the Fréchet distribution.

Distribution 2. Discrete Fréchet

The Fréchet mathematician (Maurice René Fréchet, 1920) presented the continuous Fréchet distribution Fr (α, λ) , which is a special case of the Generalized Extreme Value distribution. It is also called the inverse Weibull distribution. The researchers (Kotz and Nadarajah) also discussed the possibility of applying this distribution in various fields such as natural disasters, horse racing , queues , wind speed , and others. It was proposed to use this distribution to construct its counter-part , which is the discrete Fréchet distribution.

Assuming that the random variable X follows a continuous Fréchet distribution , then its probability density function is defined as follows [6]:

$$\boldsymbol{f(x,\alpha,\lambda) = \lambda\,\alpha\; x^{-(\alpha+1)} e^{-\lambda x^{-\alpha}} \qquad x > 0\,, \alpha, \lambda > 0} \qquad (2.1)$$

With the cumulative distribution function for Fr **(α , λ)** is given by:

$$\boldsymbol{F(x) = \int_0^x \lambda\,\alpha\; u^{-(\alpha+1)} e^{-\lambda u^{-\alpha}}\; du}$$
$$\boldsymbol{= e^{-\lambda x^{-\alpha}}} \qquad (2.2)$$

And using the conversion to the discretization approach defined as :

$$\boldsymbol{p(Y=y) = S(y) - S(y+1)} \qquad (2.3)$$

Where, $\boldsymbol{S\,(y) = 1 - F(y)}$

Then, the probability mass function (p.m.f.) of the discrete Fréchet distribution DFr **(α,q)** with two parameters is given as follows:

$$\boldsymbol{p(Y=y) = \left(1 - e^{-\lambda y^{-\alpha}}\right) - \left(1 - e^{-\lambda (y+1)^{-\alpha}}\right)}$$

$$= e^{-\lambda(y+1)^{-\alpha}} - e^{-\lambda y^{-\alpha}}$$

$$= q^{(y+1)^{-\alpha}} - q^{(y)^{-\alpha}} \quad , \quad y = 0, 1, 2, \ldots \qquad (2.5)$$

Where $\boldsymbol{\alpha}$ is the shape parameter , $\mathbf{q}$ is the scale parameter ,

$q = e^{-\lambda}$ so that $\alpha > 0, \; 0 < q < 1.$

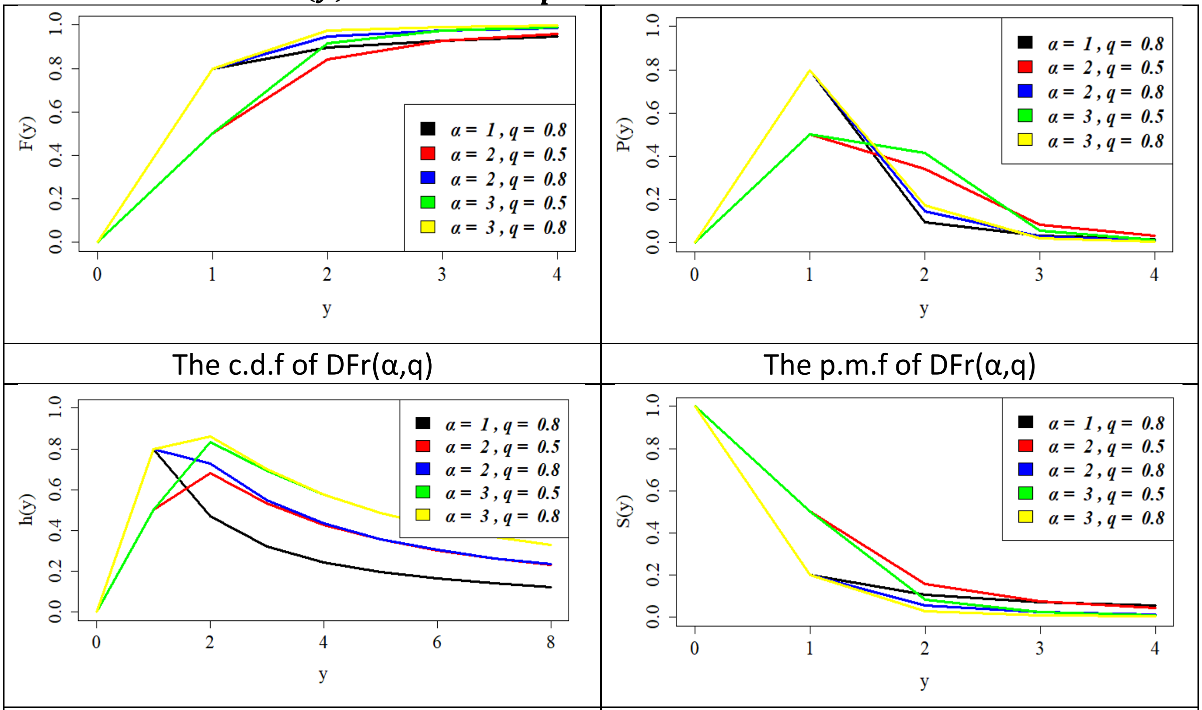
The cumulative distribution function of DFr**(α ,q)** is given by:

$$F(y) = \sum_{i=0}^{y} p(i) = q^{(y)^{-\alpha}} \quad , \qquad y = 0, 1, 2, \ldots.. \qquad (2.6)$$

And the survival and hazard functions of DFr **(α ,q)** are defined as follows :

$$S(y) = 1 - F(y) \qquad = 1 - q^{(y)^{-\alpha}} \qquad (2.7)$$

$$h(y) = \frac{p(y)}{S(y)} = \frac{q^{(y+1)^{-\alpha}} - q^{(y)^{-\alpha}}}{1 - q^{(y)^{-\alpha}}} \qquad (2.8)$$



The c.d.f of DFr(α,q)

The p.m.f of DFr(α,q)

The $h(y)$ of DFr(α,q)

The $S(y)$ of DFr(α,q)

Figure (2.1): Graph of the functions of the discrete DFr(α,q)

1.2 Maximum Likelihood Estimation:

Assuming that $y_1 , \ldots ., \; y_n$ is a (i.i.d.) random sample of size (n) , which follows the discrete Fréchet distribution with a probability mass function $p(y\,/\theta)$ and **that** $y = (y_1, \ldots ., y_n)$ and $\theta = (\alpha, q)$, then the likelihood function based on equation (2.3) is as follows:

$$L(\alpha, q; y_1, \ldots ., y_n) = \prod_{i=1}^{n} \left[q^{(y_i+1)^{-\alpha}} - q^{(y_i)^{-\alpha}}\right] \qquad (2.9)$$

And by taking the natural logarithm of both sides of the above equation

$$logL(\alpha, q; y_1, \ldots ., y_n) = \sum_{i=1}^{n} Log\left[q^{(y_i+1)^{-\alpha}} - q^{(y_i)^{-\alpha}}\right] \; (2.10)$$

To obtain the likelihood function, the function is derived with respect to the unknown parameters, as follows

$$\frac{\partial logL}{\partial\, \alpha} = \sum_{i=1}^{n} \frac{\left(k_1\, q^{(y_i+1)^{-\alpha}} Logq\right) - \left(k_2\, q^{(y_i)^{-\alpha}} Logq\right)}{q^{(y_i+1)^{-\alpha}} - q^{(y_i)^{-\alpha}}} = 0 \quad (2.11)$$

Where

$$k_1 = [-(y_i+1)^{-\alpha}\log(y_i+1)]$$

$$k_2 = [-(y_i)^{-\alpha}\log(y_i)]$$

$$\frac{\partial logL}{\partial q} = \sum_{i=1}^{n} \frac{(y_i+1)^{-\alpha} q^{(y_i+1)^{-\alpha}-1} - (y_i)^{-\alpha}\, q^{(y_i)^{-\alpha}-1}}{q^{(y_i+1)^{-\alpha}} - q^{(y_i)^{-\alpha}}} = 0 \qquad (2.12)$$

$$\frac{\partial^2 logL}{\partial \alpha^2} = \sum_{i=0}^{n} \frac{A_2 + A_3}{(A_1)^2} \qquad (2.13)$$

Where

$$A_1 = q^{(y_i+1)^{-\alpha}} - q^{(y_i)^{-\alpha}}$$

$$A_2 = Log(q)\{Log(y_i+1)\}^2 (y_i+1)^{-\alpha} q^{(y_i+1)^{-\alpha}} [(y_i+1)^{-\alpha} Log(q) + 1]$$

$$A_3 = -Log(q)\,[Log(y_i)]^2 (y_i)^{-\alpha} q^{(y_i)^{-\alpha}} [(y_i)^{-\alpha} + 1]$$

$$\frac{\partial^2 logL}{\partial q^2} = \sum_{i=1}^{n} \frac{A_4 + A_5}{(A_1)^2} \qquad (2.14)$$

Where

$$A_4 = (y_i+1)^{-\alpha}[(y_i+1)^{-\alpha} - 1]\, q^{(y_i+1)^{-\alpha-2}}$$

$$A_5 = (y_i)^{-\alpha}[(y_i)^{-\alpha} - 1] q^{(y_i)^{-\alpha-2}}$$

The equations (2.11) and (2.12) can be solved using one of the numerical methods.

2.2 Bayes Estimation:

Assuming that $y_1, \ldots, y_n$ is a (i.i.d.) random sample of size (n) , which follows the discrete Fréchet distribution with vector parameter $\theta = (\alpha, q)$ and likelihood function is defined by (2.9). According to Jeffrey's law and assuming q and α are independent, the prior distributions function for q and α is defined as follows:

$$\pi_1(\alpha) = \frac{d^c}{\Gamma(c)}\, \alpha^{c-1} e^{-d\alpha} \qquad , \alpha > 0 \qquad (2.15)$$

$$\pi_2(q) = \frac{[\Gamma(a+b)]}{\Gamma(a)\Gamma(b)} q^{a-1}(1-q)^{b-1} \qquad , 0 < q < 1 \qquad (2.16)$$

And the joint prior distribution of q and α is given by

$$\pi(\alpha, q) = \pi_1(\alpha)\,\pi_2(q) = \frac{d^c[\Gamma(a+b)]}{\Gamma(a)\Gamma(b)\Gamma(c)}\, \alpha^{c-1} e^{-d\alpha}\, q^{a-1}(1-q)^{b-1} \qquad (2.17)$$

And the joint posterior conditional probability function of q and α is given by

$$h(\alpha, q/y) = \frac{L(\alpha, q/y)\, \pi(\alpha, q)}{\int_0^1 \int_0^\infty L(\alpha, q/y)\, \pi(\alpha, q)\, d\alpha dq}$$

$$= \frac{\prod_{i=1}^{n}\left[q^{(y_i+1)^{-\alpha}} - q^{(y_i)^{-\alpha}}\right] \frac{d^c[\Gamma(a+b)]}{\Gamma(a)\Gamma(b)\Gamma(c)} q^{a-1}(1-q)^{b-1}\ \alpha^{c-1}e^{-d\alpha}}{\int_0^1\int_0^\infty \prod_{i=1}^{n}\left[q^{(y_i+1)^{-\alpha}} - q^{(y_i)^{-\alpha}}\right] \frac{d^c[\Gamma(a+b)]}{\Gamma(a)\Gamma(b)\Gamma(c)} q^{a-1}(1-q)^{b-1}\ \alpha^{c-1}e^{-d\alpha}\ d\alpha dq} \quad (2.18)$$

Thus, the marginal posterior distribution function for α when q and y are given

$$\pi_1(\alpha/q, y) = \int_0^1 \left[\frac{\prod_{i=1}^{n}\left[q^{(y_i+1)^{-\alpha}} - q^{(y_i)^{-\alpha}}\right]\ q^{a-1}(1-q)^{b-1}\ \alpha^{c-1}e^{-d\alpha}}{\int_0^1\int_0^\infty \prod_{i=1}^{n}\left[q^{(y_i+1)^{-\alpha}} - q^{(y_i)^{-\alpha}}\right]\ q^{a-1}(1-q)^{b-1}\ \alpha^{c-1}e^{-d\alpha}\, d\alpha dq}\right] dq \ , \ \alpha > 0 \quad (2.19)$$

And the marginal posterior distribution function for q when α and y are given by

$$\pi_2(q/\alpha, y) = \int_0^\infty \left[\frac{\prod_{i=1}^{n}\left[q^{(y_i+1)^{-\alpha}} - q^{(y_i)^{-\alpha}}\right]\ q^{a-1}(1-q)^{b-1}\ \alpha^{c-1}e^{-d\alpha}}{\int_0^1\int_0^\infty \prod_{i=1}^{n}\left[q^{(y_i+1)^{-\alpha}} - q^{(y_i)^{-\alpha}}\right]\ q^{a-1}(1-q)^{b-1}\ \alpha^{c-1}e^{-d\alpha}\, d\alpha dq}\right] d\alpha \quad (2.20)$$

Using the squared error loss function when (**q**) and (**y**) are given, the expectation and variance for (**α**) are as follows:

$$E(\alpha/q, y) = \int_0^\infty \alpha \left\{\int_0^1 \left[\frac{\prod_{i=1}^{n}\left[q^{(y_i+1)^{-\alpha}} - q^{(y_i)^{-\alpha}}\right]\ q^{a-1}(1-q)^{b-1}\ \alpha^{c-1}e^{-d\alpha}}{\int_0^1\int_0^\infty \prod_{i=1}^{n}\left[q^{(y_i+1)^{-\alpha}} - q^{(y_i)^{-\alpha}}\right]\ q^{a-1}(1-q)^{b-1}\ \alpha^{c-1}e^{-d\alpha}\, d\alpha dq}\right] dq\right\} d\alpha \quad (2.21)$$

$$Var(\alpha/q, y) = E(\alpha^2/q, y) - [E(\alpha/q, y)]^2$$
$$= \int_0^\infty \alpha^2 \left\{\int_0^1 \left[\frac{\prod_{i=1}^{n}\left[q^{(y_i+1)^{-\alpha}} - q^{(y_i)^{-\alpha}}\right]\ q^{a-1}(1-q)^{b-1}\ \alpha^{c-1}e^{-d\alpha}}{\int_0^1\int_0^\infty \prod_{i=1}^{n}\left[q^{(y_i+1)^{-\alpha}} - q^{(y_i)^{-\alpha}}\right]\ q^{a-1}(1-q)^{b-1}\ \alpha^{c-1}e^{-d\alpha}\, d\alpha dq}\right] dq\right\} d\alpha - [E(\alpha/q, y)]^2 \quad (2.22)$$

As for the expectation and variance of **(α)** when (q) and **(y)** are given, they are as follows:

$$E(q/\alpha, y) = \int_0^1 q \left\{\int_0^\infty \left[\frac{\prod_{i=1}^{n}\left[q^{(y_i+1)^{-\alpha}} - q^{(y_i)^{-\alpha}}\right]\ q^{a-1}(1-q)^{b-1}\ \alpha^{c-1}e^{-d\alpha}}{\int_0^1\int_0^\infty \prod_{i=1}^{n}\left[q^{(y_i+1)^{-\alpha}} - q^{(y_i)^{-\alpha}}\right]\ q^{a-1}(1-q)^{b-1}\ \alpha^{c-1}e^{-d\alpha}\, d\alpha dq}\right] d\alpha\right\} dq \quad (2.23)$$

$$Var(q/\alpha, y) = E(q^2/\alpha, y) - [E(q/\alpha, y)]^2$$
$$= \int_0^1 q^2 \left\{\int_0^\infty \frac{\prod_{i=1}^{n}\left[q^{(y_i+1)^{-\alpha}} - q^{(y_i)^{-\alpha}}\right]\ q^{a-1}(1-q)^{b-1}\ \alpha^{c-1}e^{-d\alpha}}{\int_0^1\int_0^\infty \prod_{i=1}^{n}\left[q^{(y_i+1)^{-\alpha}} - q^{(y_i)^{-\alpha}}\right]\ q^{a-1}(1-q)^{b-1}\ \alpha^{c-1}e^{-d\alpha}\, d\alpha dq} d\alpha\right\} dq - [E(q/\alpha, y)]^2 \quad (2.24)$$

Using numerical methods , a solution to the above equations can be obtained.

3. simulation study

Different supposed values were chosen for parameters such that $(\alpha, q) = (0.7, 0.6)$, $(1, 0.8)$, $(0.5, 0.1)$, $(2, 0.8)$ with sample sizes **(50,100,150)**. The times of the Survival and Hazard functions **(t=1, 2,…,5).** Each experiment was repeated 1000 times. The estimation of the parameters of the Fréchet distribution was carried out using the methods of maximum Likelihood and the Bayesian through the Markov Chain Monte Carlo **(MCMC)** algorithm.

For comparison between the two estimation methods, the absolute value of bias **(|Bias|)** and the mean squared error **(MSE)** were calculated as follows

$$|BIAS(\hat{\phi})| = \left| \frac{1}{r} \sum_{j=1}^{r} [\hat{\phi}_j - \phi] \right| \quad (3.1)$$

$$MSE(\hat{\phi}) = Var(\hat{\phi}) + [BIAS(\hat{\phi})]^2 \quad (3.2)$$

Where (r) is the number of repetitions for each experiment, **($\hat{\phi}_j$)** an estimate of the parameter **φ** at j times and $Var(\hat{\phi})$ the variance of the estimated values and then an estimate of the survival and hazard functions. The simulation results were as follows:

1. Through the values of table (1.3) and in the case of the supposed values **(α = 0.7, q = 0.6), ($\alpha = 0.5, q = 0.1$) and ($\alpha = 2, q = 0.8$)** it was noted that the Bayes method is preferred based on the values of |Bias| and MSE . but , in the case of supposed values for the set **(α = 1, q = 0.8),** it is noted that the Bayes method is preferred to the estimator $\hat{\alpha}$ and the two estimation methods alternate in preference to the estimator $\hat{q}$ depending on the criterion values |Bias| and MSE values.
2. And the values of (MSE) in general decrease with the increase in the sample size as well.
3. Through the values of Tables (2.3) and (3.3) it is noted that the estimates of the survival and hazard functions are close to their true values when using the values of all supposed parameters. Also , it is noted that the results of the two estimation methods are close , and that the values of the survival and hazard functions decrease with the increase of time.

Table (1.3) Estimates of the two parameters of the discrete Fréchet distribution

Sample Size	Parameters		α=0.7, q =0.6		α=1, q =0.8		α=0.5, q =0.1		α=2, q =0.8	
			MLE	Bayes	MLE	Bayes	MLE	Bayes	MLE	Bayes
n=50	α	$\hat{\alpha}$	0.7771	0.7455	1.0895	0.9839	0.4879	0.4897	2.8683	2.0262
		$\|Bias\|$	0.0771	0.0455	0.0895	0.0161	0.0121	0.0103	0.8683	0.0262
		MSE	0.0420	0.0330	0.1444	0.0955	0.0057	0.0055	1.0712	0.5608
	q	$\hat{q}$	0.6294	0.6282	0.7884	0.7837	0.0984	0.1015	0.7949	0.7997
		$\|Bias\|$	0.0294	0.0282	0.0116	0.0163	0.0016	0.0015	0.0051	0.0003
		MSE	0.0032	0.0030	0.0032	0.0030	0.0012	0.0011	0.0028	0.0026
n=100	α	$\hat{\alpha}$	0.7442	0.7304	1.0266	0.9754	0.5061	0.5015	2.123	1.9759
		$\|Bias\|$	0.0442	0.0304	0.0266	0.0246	0.0061	0.0015	0.1230	0.0241
		MSE	0.0180	0.0160	0.0611	0.0501	0.0022	0.0019	0.3475	0.2618
	q	$\hat{q}$	0.6055	0.6052	0.7991	0.7966	0.0940	0.0978	0.7951	0.7956
		$\|Bias\|$	0.0055	0.0052	0.0009	0.0034	0.0060	0.0022	0.0049	0.0044
		MSE	0.0021	0.0020	0.0018	0.0017	0.0005	0.0004	0.0013	0.0011
n=150	α	$\hat{\alpha}$	0.7140	0.7057	1.0414	1.0083	0.5078	0.5049	2.0747	1.9788
		$\|Bias\|$	0.0140	0.0057	0.0414	0.0083	0.0078	0.0049	0.0747	0.0212
		MSE	0.0082	0.0077	0.0415	0.0359	0.0014	0.0013	0.1255	0.1054
	q	$\hat{q}$	0.6006	0.6004	0.7950	0.7933	0.1041	0.1036	0.8016	0.7989
		$\|Bias\|$	0.0006	0.0004	0.0050	0.0067	0.0041	0.0036	0.0016	0.0011
		MSE	0.0018	0.0017	0.00103	0.00100	0.0003	0.0002	0.0012	0.0011

Table (2.3) Estimates of survival and hazard function for the discrete Fréchet distribution

Survival function when α = 0.7, q = 0.6							
t	Real	n=50		n=100		n=150	
		MLE	Bayes	MLE	Bayes	MLE	Bayes
1	0.2698	0.2389	0.2440	0.2596	0.2619	0.2677	0.2691
2	0.2108	0.1827	0.1887	0.2001	0.2029	0.2085	0.2102
3	0.1760	0.1508	0.1568	0.1656	0.1686	0.1737	0.1756
4	0.1526	0.1498	0.1357	0.1528	0.1557	0.1505	0.1524
5	0.1356	0.1399	0.1353	0.1451	0.1477	0.1290	0.1309
Hazard function when α = 0.7, q = 0.6							
1	0.3255	0.3576	0.3456	0.3421	0.3368	0.3301	0.3268
2	0.2187	0.2428	0.2336	0.2317	0.2276	0.2225	0.2200
3	0.1652	0.1841	0.1768	0.1755	0.1723	0.1683	0.1664
4	0.1329	0.1484	0.1423	0.1414	0.1388	0.1355	0.1339
5	0.1113	0.1143	0.1107	0.1052	0.1053	0.1203	0.1187
Survival function when α = 1, q = 0.8							
1	0.10557	0.10863	0.11779	0.10578	0.11042	0.10637	0.10948
2	0.07168	0.07387	0.08261	0.07231	0.07680	0.07194	0.07494
3	0.05426	0.05643	0.06439	0.05520	0.05932	0.05447	0.05722
4	0.04365	0.04591	0.05318	0.04478	0.04857	0.04392	0.04642
5	0.03651	0.03886	0.04555	0.03776	0.04125	0.03685	0.03914
Hazard function when α = 1, q = 0.8							
1	0.47214	0.53500	0.49238	0.50624	0.48875	0.49443	0.48287
2	0.32102	0.38276	0.34453	0.35229	0.33804	0.34001	0.33079
3	0.24307	0.29701	0.26442	0.26949	0.25791	0.25870	0.25130
4	0.19556	0.24242	0.21440	0.21805	0.20838	0.20869	0.20255
5	0.16358	0.20470	0.18025	0.18305	0.17478	0.17486	0.16962

Table (3.3) Estimates of survival and hazard function for the discrete Fréchet distribution

Survival function when α = 0.5, q = 0.1							
t	Real	n=50		n=100		n=150	
		MLE	Bayes	MLE	Bayes	MLE	Bayes
1	0.8037	0.8120	0.8036	0.8118	0.8073	0.8011	0.7982
2	0.7354	0.7472	0.7389	0.7439	0.7396	0.7316	0.7288
3	0.6838	0.6977	0.6899	0.6922	0.6882	0.6791	0.6766
4	0.6429	0.6582	0.6510	0.6512	0.6475	0.6377	0.6354
5	0.6094	0.5987	0.5926	0.5985	0.5955	0.5908	0.5887
Hazard function when α = 0.5, q = 0.1							
1	0.1070	0.0996	0.1018	0.1043	0.1054	0.1089	0.1096
2	0.0850	0.0799	0.0805	0.0839	0.0842	0.0869	0.0870
3	0.0702	0.0663	0.0664	0.0697	0.0696	0.0718	0.0717
4	0.0598	0.0567	0.0565	0.0596	0.0594	0.0612	0.0611
5	0.0521	0.0562	0.0557	0.0525	0.0524	0.0514	0.0513
Survival function when α = 2, q = 0.8							
1	0.05426	0.04982	0.06100	0.05349	0.05899	0.05286	0.05655
2	0.02449	0.02353	0.03100	0.02496	0.02877	0.02406	0.02661
3	0.01385	0.01406	0.01945	0.01472	0.01746	0.01383	0.01564
4	0.00889	0.00953	0.01366	0.00984	0.01193	0.00903	0.01039
5	0.00618	0.00697	0.01028	0.00712	0.00877	0.00640	0.00745
Hazard function when α = 2, q = 0.8							
1	0.72871	0.80371	0.75446	0.73671	0.70938	0.72825	0.71294
2	0.54866	0.66234	0.59728	0.56101	0.53234	0.55259	0.53712
3	0.43445	0.56064	0.48810	0.44718	0.42117	0.43944	0.42564
4	0.35839	0.48731	0.41119	0.37046	0.34735	0.36349	0.35134
5	0.30461	0.43214	0.35471	0.31579	0.29519	0.30952	0.29876

2.4.Application

The following data represents the duration of stay of (100) patients with stroke in a hospital , measured in days from the date of their admission to the date of their discharge for the year 2022, as in the following table:

Table (1.4) shows the length of stay of patients with stroke in the hospital in days

4	2	4	21	7	7	155	11	19	1
0	101	2	0	4	14	6	5	24	17
4	15	10	8	19	2	186	1	2	3
10	61	7	6	49	3	0	0	518	23
2	11	15	159	2	6	18	8	0	91
217	23	20	1	0	4	1	12	1	9
1	5	54	107	78	182	28	820	1	5
39	2	17	4	25	215	362	16	4	14
38	35	14	34	10	0	6	11	18	5
21	43	18	4	143	4	2	9	89	360

To find out the distribution of the observations, the (c.d.f.) function was calculated and then the Kolmogorov-Smirnov test was used, where the value of the statistic was equal to (0.09335), and this means that the observations follow the discrete Fréchet distribution, Through the R program the results were obtained as in the histogram drawing and the distribution curve shown in the following figure .

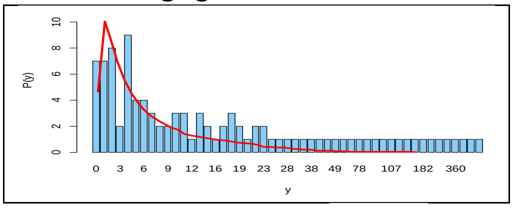


Figure1.4 Histogram of discrete Fréchet distribution

Table (2.4) The results of estimating the parameters of the discrete Fréchet distribution

Parameters	MLE	Bayes
$\hat{\alpha}$	0.65478	0.64632
$\hat{q}$	0.052012	0.05615
MSE	0.0008934	0.0008903
AIC	1157.486	1157.441
BIC	1162.696	1162.652

The survival and hazard functions of the distribution were also estimated using the (MLE) and (Bayes) methods for different periods starting from (t = 0) until (t = 20). The following table shows the estimated values of the survival function when applying the discrete Fréchet distribution.

Table (3.4) Results of survival function estimation for the discrete Fréchet distribution

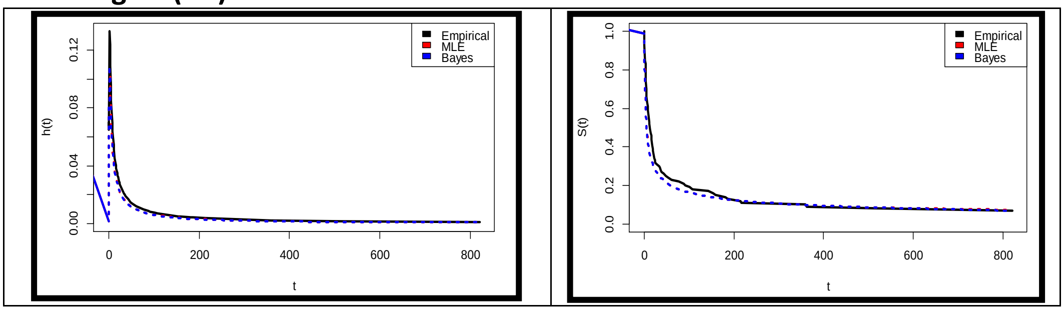
t	Empirical	Estimated		t	Empirical	Estimated	
		MLE	Bayes			MLE	Bayes
0	1	1	1	34	0.3	0.24928	0.24895
1	0.93	0.88842	0.87418	35	0.29	0.24645	0.24612
2	0.85	0.73753	0.72927	38	0.28	0.23860	0.23828
3	0.83	0.64032	0.63577	39	0.27	0.23617	0.23585
4	0.74	0.57527	0.57245	43	0.26	0.22726	0.22695
5	0.7	0.52829	0.52635	49	0.25	0.21586	0.21556
6	0.66	0.49237	0.49093	54	0.24	0.20776	0.20746
7	0.63	0.46373	0.46260	61	0.23	0.19802	0.19773
8	0.61	0.44019	0.43925	78	0.22	0.17974	0.17947
9	0.59	0.42037	0.41957	89	0.21	0.17064	0.17037

10	0.56	0.40338	0.40267	91	0.2	0.16915	0.16889
11	0.53	0.38858	0.38794	101	0.19	0.16235	0.16209
12	0.52	0.37553	0.37495	107	0.18	0.15870	0.15844
14	0.49	0.35346	0.35296	143	0.17	0.14157	0.14132
15	0.47	0.34400	0.34352	155	0.16	0.13714	0.13690
16	0.46	0.33538	0.33493	159	0.15	0.13577	0.13553
17	0.44	0.32748	0.32704	182	0.14	0.12874	0.12850
18	0.41	0.32020	0.31977	186	0.13	0.12764	0.12741
19	0.39	0.31346	0.31305	215	0.12	0.12056	0.12033
20	0.38	0.30720	0.30680	217	0.11	0.12012	0.11989
21	0.36	0.30135	0.30096	360	0.1	0.09840	0.09820
23	0.34	0.29075	0.29038	362	0.09	0.09819	0.09799
24	0.33	0.28592	0.28555	518	0.08	0.08526	0.08508
25	0.32	0.28137	0.28100	820	0.07	0.07115	0.07098
28	0.31	0.26909	0.26874	-	-	-	-

Table (4.4) Results of hazard function estimation for the discrete Fréchet distribution

z	Empirical	Estimated		t	Empirical	Estimated	
		MLE	Bayes			MLE	Bayes
0	0.07	0	0	34	0.02081	0.01665	0.01643
1	0.06502	0.05202	0.05615	35	0.02027	0.01622	0.01600
2	0.13308	0.10647	0.10880	38	0.01882	0.01505	0.01485
3	0.12398	0.09919	0.09976	39	0.01837	0.01470	0.01450
4	0.10887	0.08709	0.08704	43	0.01680	0.01344	0.01326
5	0.09583	0.07666	0.07636	49	0.01490	0.01192	0.01176
6	0.08527	0.06821	0.06779	54	0.01362	0.01089	0.01074
7	0.07670	0.06136	0.06089	61	0.01215	0.00972	0.00959
8	0.06967	0.05573	0.05525	78	0.00965	0.00772	0.00761
9	0.06381	0.05105	0.05057	89	0.00851	0.00681	0.00672
10	0.05886	0.04709	0.04662	91	0.00833	0.00667	0.00658
11	0.05463	0.04370	0.04324	101	0.00755	0.00604	0.00595
12	0.05097	0.04078	0.04033	107	0.00714	0.00571	0.00564
14	0.04496	0.03597	0.03556	143	0.00541	0.00433	0.00427
15	0.04247	0.03398	0.03358	155	0.00500	0.00400	0.00395
16	0.04024	0.03219	0.03181	159	0.00488	0.00391	0.00385
17	0.03824	0.03059	0.03022	182	0.00428	0.00343	0.00338
18	0.03642	0.02914	0.02878	186	0.00420	0.00336	0.00331
19	0.03478	0.02782	0.02748	215	0.00365	0.00292	0.00288
20	0.03328	0.02662	0.02629	217	0.00361	0.00289	0.00285
21	0.03190	0.02552	0.02520	360	0.00220	0.00176	0.00174
23	0.02947	0.02358	0.02328	362	0.00219	0.00175	0.00173
24	0.02839	0.02272	0.02242	518	0.00154	0.00123	0.00122
25	0.02739	0.02191	0.02163	820	0.00098	0.00078	0.00077
28	0.02478	0.01982	0.01956	-	-	-	-

Figure (2.4): Survival and risk functions of the discrete Fréchet distribution



5. Conclusions

In the experimental side of the discrete Fréchet distribution, and through table (1.3), the preference was given to the Bayes method for the two parameters α and , and the values of (MSE) in general decrease with the increase in the sample size as well.

On the applied side, and when fitting the discrete Fréchet distribution on observations of the survival of patients with stroke, the preference of the Bayes method was deduced because it had the lowest criteria (MSE, AIC, BIC) compared to the (ML) method, as shown in table (2.4).

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حول توزيع فريجيت المتقطع : تقدير وتطبيق

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المستخلص

تم تقديم واشتقاق توزيع فريجيت المتقطع بمعلمتين باستخدام النهج العام المبني على الجزء المقابل للتوزيع المستمر . ومن ثم تم ايجاد وفحص دالتي البقاء والمخاطرة للتوزيع المقترح . كما نوقشت طريقتين لتقدير المعلمات هما ، الامكان الاعظم وطريقة بيز مع اجراء مقارنة الاداء باستخدام اسلوب المحاكاة . فضلا عن ذلك ، تم التحقق من توافق التوزيع المقترح من خلال البيانات الحقيقية والتوصل الى دالتي البقاء والمخاطرة . وقد استنتج ان طريقة بيز هي الافضل في نمذجة البيانات المستخدمة .

الكلمات المفتاحية : توزيع فريجيت ، دالة البقاء ، دالة المخاطرة ، طريقة بيز ، طريقة الامكان الاعظم .

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