

Mathematical Model of Permanent Magnet Synchronous Motor

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Abstract

Due to the benefits that they provide, permanent magnet synchronous motors are becoming increasingly common. The machine's mathematical modeling allows for a better understanding and provides insight into the influence of each variable on the machine's dynamics and to design an appropriate control system for the motor. This work provides a mathematical model of a PMSM (permanent magnet synchronous motor), and this model has been simulated via utilize MATLAB/Simulink tools. The results obtained verify the model's effectiveness to work under different loading conditions. This model can be used in various control strategies such as vector control or direct torque control.

Keywords: PMSM, MATLAB, modeling, and Park transformation.

1. Introduction

The development of control technology and permanent magnetic materials has led to the widespread usage of PMSMs, which have the following benefits: high efficiency, high power density, reliable, high torque/inertia ratio, and easy maintenance, in addition, compared to other AC machines, these motors' control circuits are simpler [1, 2]. PMSMs are being utilized extensively in both industrial and transportation applications [3, 4].

In order to verify a range of controlling algorithms and optimize the complete controlling system, it is crucial to construct a Simulink model of the PMSM and its controlling system [1]. Numerous researchers have conducted studies on modeling in recent years [5]. In this work, the PMSM model has been built and simulated via utilizing MATLAB Simulink tools, and the results that represent speed, torque, and (d-q) current parts of the stator were extracted.

The structure of the present paper is as follows: section 2 demonstrates the Park transformation, the two-axis model of PMSM is presented in section 3, the Simulink implementation of the PMSM model is shown in section 4, section 5 demonstrates the simulation outcomes, and paper's conclusion is shown in section 6.

2. Park transformation

Typically, the voltage and current equations in three-phase machines are used to define and assess the performance and behavior. The differential equations' coefficients that characterize the machines' dynamic behavior are time-varying except if a motionless rotor is present. Since the system is in relative motion, the flux linkages, currents, and induced voltages vary continually, making such a system difficult to mathematically model. Mathematical transformations are usually used to analyze

complex electrical machines by decoupling or separating the variables and solving equations including time-varying values by referring the whole variables to a common reference frame that might be rotating or stationary. The two most well-known transformation techniques are the Clarke transformation and the Park transformation. The complexity of the mathematical model of the machine can be greatly reduced with careful reference frame selection [6, 7]. The three-phase values ($V_a, V_b,$ and V_c) may be changed into the voltages V_d and V_q that the two-axis PMSM model requires [8].

Park's transformation is utilized to convert the three-phase voltages to (d,q) voltages. The transformation of Park is described by [9]:

$$\begin{bmatrix} V_d \\ V_q \\ V_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ -\sin(\theta) & -\sin\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad (1)$$

And its inverse is [9]:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 1 \\ \cos\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta + \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) & 1 \end{bmatrix} \begin{bmatrix} V_d \\ V_q \\ V_0 \end{bmatrix} \quad (2)$$

Where V_0 (zero-voltage component) is often ignored, and θ denotes the phase (a) electrical angle respect to the reference frame.

3. Two-axis model of PMSM

To create permanent magnet synchronous motor control, a mathematical model of the PMSM is required. The 3-phase system mathematical model is rarely used in control design. This method is known as the time-varying model. consequently, developing a control strategy becomes significantly more difficult. As a result, a 2-phase motor model with quadrature and direct axes is employed [10, 11]. The permanent magnet synchronous motor

mathematical model typically includes stator flux equations, voltage equations, mechanical equations, and electromagnetic torque equations [12, 13].

The following assumptions were used to build the PMSM model without damper winding on the rotor reference frame [14, 15].

1. The EMF generated is sinusoidal.
2. Saturation is ignored.
3. No field current dynamics
4. Hysteresis losses and eddy current are ignored.

In the d-q coordinate system, the following equations may be expressed [15].

3.1 Voltage equations

In the rotor reference frame, the voltage formulas of the PMSM are shown [15]:

$$V_q = R_s i_q + \omega_r \lambda_d + \frac{d}{dt} \lambda_q \quad (3)$$

$$V_d = R_s i_d - \omega_r \lambda_q + \frac{d}{dt} \lambda_d \quad (4)$$

3.2 Flux linkage equations

The flux linkage equations are given by [15]:

$$\lambda_q = L_q i_q \quad (5)$$

$$\lambda_d = L_d i_d + \lambda_f \quad (6)$$

Substituting equations (5) and (6) into (3) and (4), gives [15]:

$$V_q = R_s i_q + \omega_r (L_d i_d + \lambda_f) + \frac{d}{dt} L_q i_q \quad (7)$$

$$V_d = R_s i_d - \omega_r L_q i_q + \frac{d}{dt} (L_d i_d + \lambda_f) \quad (8)$$

3.3 Torque equation

The electromagnetic torque for PMSM in the (d-q) axes may be calculated using the equation [15]:

$$T_e = \frac{3P}{2} (\lambda_d i_q - \lambda_q i_d) \quad (9)$$

The equation that represents the mechanical torque is given as [15]:

$$\omega_m = \int \frac{(T_e - T_L - B\omega_m)}{J} dt \quad (10)$$

The equation that represents the mechanical speed is [15]:

$$\omega_m = \frac{2}{p} \omega_r \quad (11)$$

where,

R_s	Stator resistance
L_d, L_q	The inductances on d-q axis
i_d, i_q	The stator currents on d-q axis
V_d, V_q	The stator voltages on d-q axis
λ_d, λ_q	The flux linkages of the stators on d-q axis.
λ_f	Permanent magnet flux

T_e, T_l	Electromagnetic and load torques
ω_r	Electrical speed
ω_m	Mechanical speed
J	Moment of inertia
P	Number of poles
B	Fraction factor

4. Simulink implementation of the PMSM model

The Simulink implementation of the PMSM model is demonstrated in figure 1. This figure contains four parts, the first one represents the (abc) to (d,q) transformation block, the second part demonstrates the (i_d, i_q) current calculation block, the torque calculation block is shown in the part three, part four contain the speed calculation block.

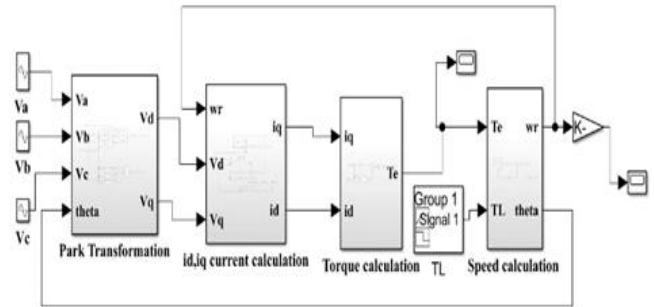


Fig. 1 Simulink implementation of the PMSM model.

The internal construction of each part is demonstrated in figures (2-5), respectively.

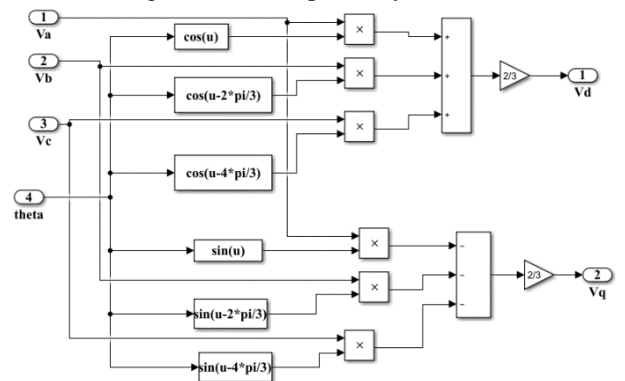


Fig.2 Park transformation

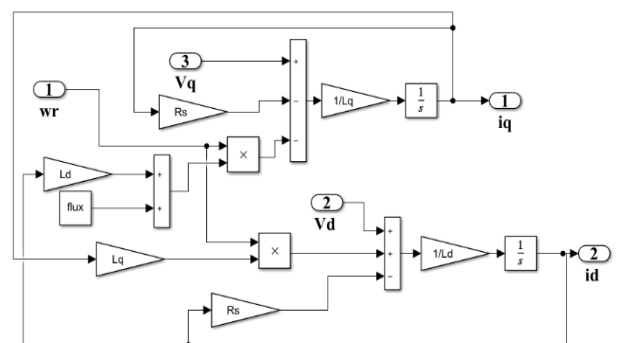


Fig. 3 i_d and i_q currents calculation

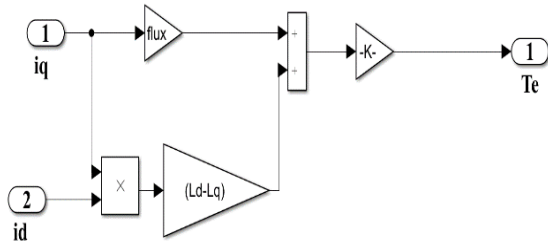


Fig. 4 Torque calculation

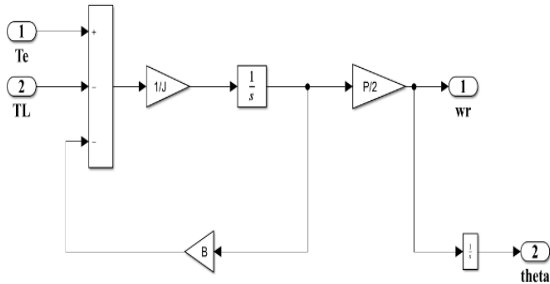


Fig. 5 Speed calculation

5. Simulation outcomes

To validate the 3-phase PMSM model's efficacy (the motor parameters used are demonstrated in table 1), The MATLAB software package was utilized to simulate the model. The no load properties are plotted in the figures (6-9), which demonstrate the speed, torque, and (q,d) axis currents of the stator.

Table 1 Parameters of the PMSM

No.	Parameters	Value
1	R_S	4.765Ω
2	L_q	0.014 H
3	L_d	0.014 H
4	P	2
5	B	4.047*10-5 N.m/rad/s
6	J	0.0001051 Kg.m2
7	λ_f	0.1848 Wb

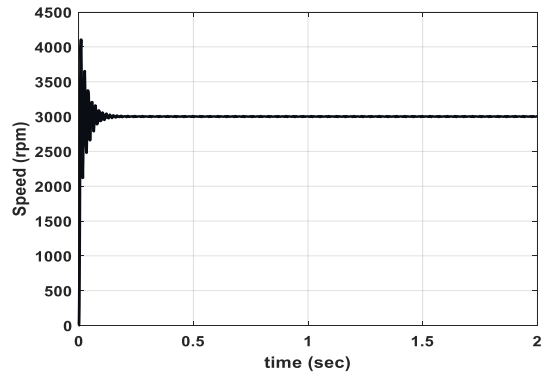


Fig. 6 The speed response at no load

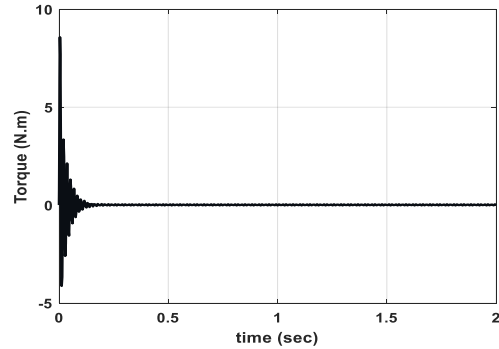


Fig. 7 Torque response at no load

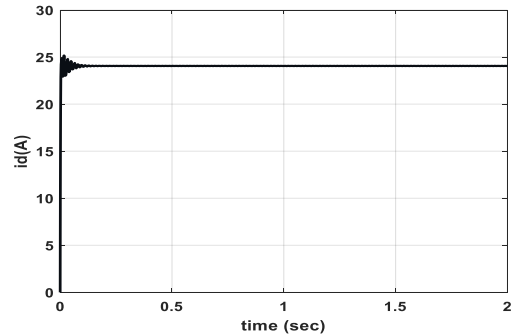


Fig. 8 The stator current on the direct-axis at no load

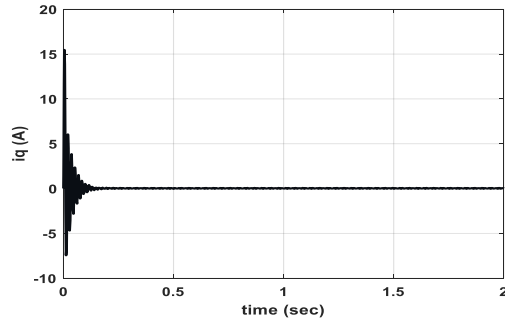


Fig. 9 The stator current on the quadrature-axis at no load

In figures (10-13) a rated load (1.7 N.m) has been applied to the machine after 0.5 seconds to see the responses of the electromagnetic torque, speed, and stator d-q axis current components under variation of load. It demonstrates that when a sudden load is added to the machine the speed of the motor and i_d current of the stator are decreased, but the motor torque and i_q current of the stator are increased.

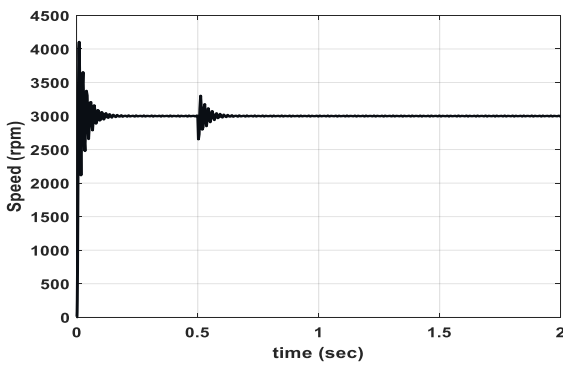


Fig. 10 The speed response at load

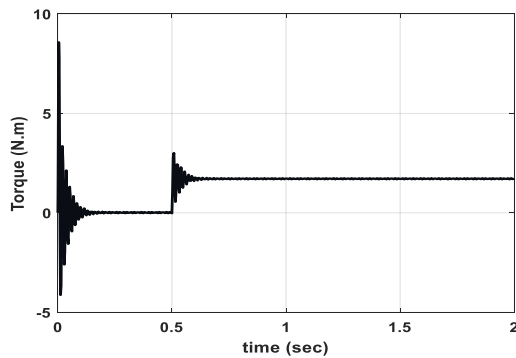


Fig. 11 Torque response at load

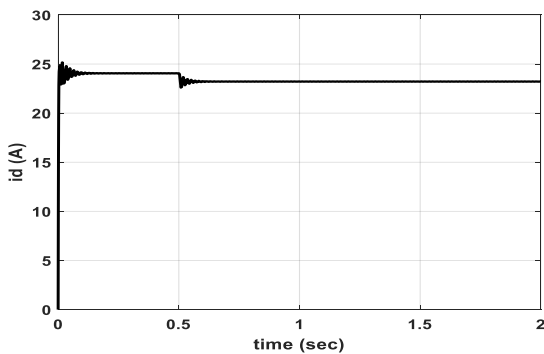


Fig. 12 The stator current on the direct-axis at load

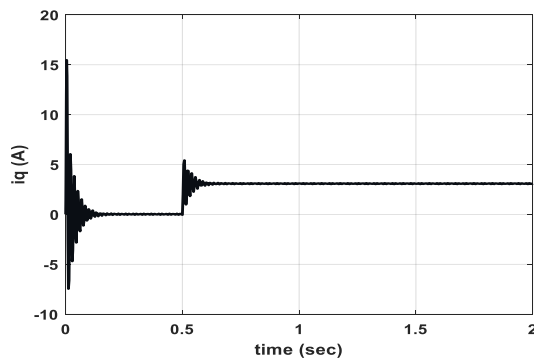


Fig. 13 The stator current on the quadrature-axis at load

6. Conclusion

In this study, a two-axis mathematical model of the PMSM was built and simulated utilizing the MATLAB

software package. The simulation results have been extracted such as speed, torque, and (d,q) current components of the stator. The mathematical motor model has been proven effective under various loading conditions (no-load and load conditions). Also, this model can be used with different control strategies that are used with the PMSM.

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