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Estimate survival function for censored sample type two Bladder cancer

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Abstract :

This research cares to estimate the unlabeled parameters for generalized Rayleigh distribution model depend on censored samples type two ; Utilizing the maximum likelihood estimator method to get the derivation of the point estimation for all unlabeled parameters depend on iterative technique , as Newton – Raphson method , then to derive Lindley approximation estimator method . At last , checking whether this model (GRD) suits to a set of real data .

Keywords :

Survival function maximum likelihood estimator method , censored sample type two , Lindley approximation estimator method .

1:Introduction

Recently , Surles and Padgeet introduced the generalized Rayleigh distribution in (2001 and 2004) that contains two parameters (shape and scale) which may be used quite influentially in modeling strength data and also modeling general lifetime data [7,8] .

Pay attention that the two parameters generalized Rayleigh distribution is a specific member of the generalized Weibull distribution [4] .

The survival function and Reliability function are two of the most important subjects in statistical measurements which are used in lifetime of human and lifetime of equipments respectively to get results close to real life .

Kundu and Raqab in (2005) [4] studied different methods to estimate the parameters of this distribution : maximum likelihood method , modified moment method , weighted least square method , L-moment method and compare their performance through Monte Carlo simulation .

Al-Nachawati and Abu-Youssef in (2009) [2] estimated the shape parameter only by using standard Bayes method which is based on quadratic fault loss function and linear – exponential loss function (LINEX) , then found the risk function and efficiency measure to make comparison between estimators . Numerical and simulation examples were included .

Lio , Ding-Geng and Tzong-Ru in (2011) [5] estimated the parameters of generalized Rayleigh distribution by using maximum likelihood method , moment method and probability plot method and try to find point and interval estimation for progressively type one censored samples . Their work were compared depend on simulation outcomes in term of the mean squared error and bias .

Dhwyia , Faten , Nathier and Hani in (2012) [3] developed the moment generating function which was derived to get the moment , and with the use of cumulative distribution function , then obtaining the least squares

estimators for the unlabeled parameters and moment estimator method .

Parvin , Ali and Hossein in (2013) [6] regarded the estimation of $R = p(y < x)$ where x and y have two parameter of generalized Rayleigh distribution , then obtained the maximum likelihood estimations of parameters with simple iterative technique for many values of parameters and compute the MLE of $R = p(y < x)$ with Simpson integration using maple codes .

The aim of the research

This research aims to study censored sample type two Bladder cancer by using maximum likelihood estimator method and Lindley approximation estimator method ; and through comparing the two methods .

2:Maximum likelihood estimator method for censored data type two

The maximum likelihood method is the most familiar method to estimate the parameter θ which determines a probability function $f(t:\theta)$, depend on the observations $t_1, t_2, \ldots\ldots, t_n$ which were independently sample from the distribution .

The likelihood function of type two censored data is :

$$L = \frac{n!}{(n-r)!}[\prod_{i=1}^{r} f(t_i)][1-F(T_r)]^{n-r} \qquad 0 \le t_{(1)} \le t_{(2)} \le \ldots\ldots \le t_{(n)} \le t_{(r)} \ldots\ldots\ldots\ldots\ldots(1)$$

Let $\frac{n!}{(n-r)!} = a$

$$L = a[\prod_{i=1}^{r} 2\alpha\beta t_i e^{-\beta t_i^2}(1-e^{-\beta t_i^2})^{\alpha-1}][1-(1-e^{-\beta T_r^2})^{\alpha})]^{n-r}$$

$$L = a 2^r \alpha^r \beta^r \prod_{i=1}^{r} t_i e^{-\beta\sum_{i=1}^{r} t_i^2} \prod_{i=1}^{r}(1-e^{-\beta t_i^2})^{\alpha-1}[1-(1-e^{-\beta T_r^2})^{\alpha})]^{n-r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$\ln L = \ln a + r\ln 2 + r\ln\alpha + r\ln\beta + \sum_{i=1}^{r}\ln t_i - \beta\sum_{i=1}^{r} t_i^2$$
$$+(\alpha-1)\sum_{i=1}^{r}\ln(1-e^{-\beta t_i^2}) + (n-r)\ln[1-(1-e^{-\beta T_r^2})^{\alpha})]\ldots(3)$$

$$\frac{\partial \ln L}{\partial \alpha} = \frac{r}{\alpha} + \sum_{i=1}^{r}\ln(1-e^{-\beta t_i^2})$$
$$-\frac{(n-r)(1-e^{-\beta T_r^2})^{\alpha}\ln(1-e^{-\beta T_r^2})}{1-(1-e^{-\beta T_r^2})^{\alpha}}\ldots\ldots\ldots\ldots\ldots(4)$$

$$\frac{\partial \ln L}{\partial \alpha} = 0$$

$$\frac{r}{\hat{\alpha}} + \sum_{i=1}^{r}\ln(1-e^{-\hat{\beta} t_i^2})$$
$$-\frac{(n-r)(1-e^{-\hat{\beta} T_r^2})^{\hat{\alpha}}\ln(1-e^{-\hat{\beta} T_r^2})}{1-(1-e^{-\hat{\beta} T_r^2})^{\hat{\alpha}}} = 0\ldots\ldots\ldots(5)$$

$$\hat{\alpha} = \frac{r[1-(1-e^{-\hat{\beta} T_r^2})^{\hat{\alpha}}]}{(1-e^{-\hat{\beta} T_r^2})^{\hat{\alpha}}[(n-r)\ln(1-e^{-\hat{\beta} T_r^2}) + \sum_{i=1}^{r}\ln(1-e^{-\hat{\beta} t_i^2})] - \sum_{i=1}^{r}\ln(1-e^{-\hat{\beta} t_i^2})}\ldots\ldots\ldots\ldots\ldots(6)$$

$$\frac{\partial \ln L}{\partial \beta} = \frac{r}{\beta} - \sum_{i=1}^{r} t_i^2 + (\alpha-1)\sum_{i=1}^{r}\frac{t_i^2 e^{-\beta t_i^2}}{1-e^{-\beta t_i^2}}$$
$$-\frac{(n-r)\alpha T_r^2 e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}}{1-(1-e^{-\beta T_r^2})^{\alpha}}\ldots\ldots\ldots\ldots(7)$$

$$\frac{\partial \ln L}{\partial \beta} = 0$$

$$\frac{r}{\hat{\beta}} - \sum_{i=1}^{r} t_i^2 + (\hat{\alpha}-1)\sum_{i=1}^{r}\frac{t_i^2 e^{-\hat{\beta} t_i^2}}{1-e^{-\hat{\beta} t_i^2}} -$$
$$\frac{(n-r)\hat{\alpha} T_r^2 e^{-\hat{\beta} T_r^2}(1-e^{-\hat{\beta} T_r^2})^{\hat{\alpha}-1}}{1-(1-e^{-\hat{\beta} T_r^2})^{\hat{\alpha}}} = 0\ldots\ldots(8)$$

$$\hat{\beta} = \frac{r}{\sum_{i=1}^{r} t_i^2 - (\hat{\alpha}-1)\sum_{i=1}^{r}\frac{t_i^2 e^{-\hat{\beta} t_i^2}}{1-e^{-\hat{\beta} t_i^2}} + \frac{(n-r)\hat{\alpha} T_r^2 e^{-\hat{\beta} T_r^2}(1-e^{-\hat{\beta} T_r^2})^{\hat{\alpha}-1}}{1-(1-e^{-\hat{\beta} T_r^2})^{\hat{\alpha}}}}\ldots(9)$$

Since we cannot find the estimators for the parameters (α, β) , because hardness solve this nonlinear equations simultaneously , so we resort to iterative technique in numerical analysis .

Consider Newton-Raphson method [1] is one of the best iterative methods in numerical analysis because it's very fast and the fault of this iterative technique is quadratic approximation . An iterative procedure is a technique of successive approximations , and each approximation is called iteration .

If the successive approximations approach the solution very closely then the iterations converge .

The Newton – Raphson method requires an initial value of each unknown parameters (α, β) .

The steps of this method are as follows :

$$\begin{bmatrix}\alpha_{k+1}\\ \beta_{k+1}\end{bmatrix} = \begin{bmatrix}\alpha_k\\ \beta_k\end{bmatrix} - J_k^{-1}\begin{bmatrix}f_1(\alpha)\\ f_2(\beta)\end{bmatrix}\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

The two – functions $f_1(\alpha)$ and $f_2(\beta)$ are the first derivative of log-likelihood function with reference to unlabeled parameters α and β successively .

$$f_1(\alpha) = \frac{r}{\alpha} + \sum_{i=1}^{r}\ln(1-e^{-\beta t_i^2})$$
$$-\frac{(n-r)(1-e^{-\beta T_r^2})^{\alpha}\ln(1-e^{-\beta T_r^2})}{1-(1-e^{-\beta T_r^2})^{\alpha}}\ldots\ldots\ldots\ldots\ldots(11)$$

$$f_2(\beta)=\frac{r}{\beta}-\sum_{i=1}^{r}t_i^2+(\alpha-1)\sum_{i=1}^{r}\frac{t_i^2e^{-\beta t_i^2}}{1-e^{-\beta t_i^2}}-\frac{(n-r)\alpha T_r^2e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}}{1-(1-e^{-\beta T_r^2})^{\alpha}}\quad\text{.............(12)}$$

The Jacobean matrix J_k is the first derivative for each function of $f_1(\alpha)$ and $f_2(\beta)$ are with reference to α and β or it is the second derivative of the log-likelihood function to the two – parameters .

$$J_k=\begin{bmatrix}\frac{\partial f_1(\alpha)}{\partial\alpha} & \frac{\partial f_1(\alpha)}{\partial\beta}\\ \frac{\partial f_2(\beta)}{\partial\alpha} & \frac{\partial f_2(\beta)}{\partial\beta}\end{bmatrix}=\begin{bmatrix}\frac{\partial^2\ln L}{\partial\alpha^2} & \frac{\partial^2\ln Lf_1(\alpha)}{\partial\alpha\partial\beta}\\ \frac{\partial^2\ln L}{\partial\beta\partial\alpha} & \frac{\partial^2\ln L}{\partial\beta^2}\end{bmatrix}\quad\text{....................(13)}$$

Then :

$$\frac{\partial f_1(\alpha)}{\partial\alpha}=-\frac{r}{\alpha^2}-\frac{(n-r)(1-e^{-\beta T_r^2})^{\alpha}[\ln(1-e^{-\beta T_r^2})]^2}{[1-(1-e^{-\beta t_r^2})^{\alpha}]^2}\quad\text{....(14)}$$

$$\frac{\partial f_1(\alpha)}{\partial\beta}=\sum_{i=1}^{r}\frac{t_i^2e^{-\beta t_i^2}}{1-e^{-\beta t_i^2}}-\frac{(n-r)T_r^2e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}[1+\alpha\ln(1-e^{-\beta T_r^2})-(1-e^{-\beta T_r^2})^{\alpha}]}{[1-(1-e^{-\beta T_r^2})^{\alpha}]^2}\quad\text{...(15)}$$

$$\frac{\partial f_2(\beta)}{\partial\alpha}=\sum_{i=1}^{r}\frac{t_i^2e^{-\beta t_i^2}}{1-e^{-\beta t_i^2}}-\frac{(n-r)T_r^2e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}[1+\alpha\ln(1-e^{-\beta T_r^2})-(1-e^{-\beta T_r^2})^{\alpha}]}{[1-(1-e^{-\beta T_r^2})^{\alpha}]^2}\quad\text{...(16)}$$

$$\frac{\partial f_2(\beta)}{\partial\beta}=-\frac{r}{\beta^2}-(\alpha-1)\sum_{i=1}^{r}\frac{t_i^2e^{-\beta t_i^2}}{(1-e^{-\beta t_i^2})^2}-[(n-r)\alpha T_r^4e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}[\frac{(\alpha-1)e^{-\beta T_r^2}}{1-e^{-\beta T_r^2}}-1+(1-e^{-\beta T_r^2})^{\alpha}+e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}]/[1-(1-e^{-\beta T_r^2})^{\alpha}]^2\quad\text{..................(17)}$$

the Jacobean matrix must be a non-singular symmetric matrix in this technique because depending upon the first derivatives , so it's inverse can be founded .

The absolute value for the difference between the new founded values with the initial value is the fault term , it must be a symbol by ε , which is a very small value and assumed .

Then , fault term is formulated as :

$$\begin{bmatrix}\varepsilon_{k+1}(\alpha)\\ \varepsilon_{k+1}(\beta)\end{bmatrix}=\left|\begin{bmatrix}\alpha_{k+1}\\ \beta_{k+1}\end{bmatrix}-\begin{bmatrix}\alpha_k\\ \beta_k\end{bmatrix}\right|\quad\text{....................(18)}$$

3: Lindley approximation estimator method

Lindley procedure was suggested in (1980) first time to approximate the ratio of the integrals of the form:

$$\frac{\int w(\theta)e^{L(\theta)}\ d\theta}{\int v(\theta)e^{L(\theta)}\ d\theta}\quad\text{.......................(19)}$$

Where $\theta=(\theta_1,\theta_2,.....,\theta_n)$ are parameters , $L(\theta)$ is the logarithm of the likelihood function , $w(\theta)$ and $v(\theta)$ are any arbitrary functions for parameters.

Let $v(\theta)$ be the prior distribution of θ and $w(\theta)=u(\theta)v(\theta)$. From (19) we can get posterior expectation which is as follow :

$$E[u(\theta)\,|\,\underline{x}]=\frac{\int u(\theta)e^{L(\theta)+P(\theta)}\ d\theta}{\int e^{L(\theta)+P(\theta)}\ d\theta}\quad\text{....................(20)}$$

Where $P(\theta) = \log[v(\theta)]$

Lindley approximation method is used to obtain Bayes estimators which can be written as follows for generalized Rayleigh distribution :

$$\hat{R} = R(\hat{\alpha}, \hat{\beta}) + \frac{1}{2}[A + l_{30}B_{12} + l_{03}B_{21} + l_{21}C_{12} + l_{12}C_{21}] + p_1A_{12} + p_2A_{12} \quad \text{.................(21)}$$

Where

$$A = \sum_{i=1}^{2} \sum_{j=1}^{2} w_{ij} T_{ij} \quad ; \quad i, j = 1,2 \quad \text{.............(22)}$$

And $l_{ij} = \dfrac{\partial^{i+j} L(\alpha, \beta)}{\partial \alpha^i \partial \beta^j}$ ....(23)

$$p_i = \frac{\partial p}{\partial \beta_i} \quad , \quad p = \ln \Pi(\alpha, \beta) \quad \text{................(24)}$$

$$w_1 = \frac{\partial R}{\partial \alpha} \quad , \quad w_2 = \frac{\partial R}{\partial \beta} \quad , \quad w_{ij} = \frac{\partial^2 R}{\partial \alpha_i \partial \beta_j} \quad \text{...........(25)}$$

$$A_{ij} = w_i T_{ij} + w_i T_{ji} \quad \text{..................(26)}$$

$$B_{ij} = (w_i T_{ij} + w_j T_{ij}) T_{ii} \quad \text{................(27)}$$

$$C_{ij} = 3w_i T_{ii} T_{ij} + w_i (T_{ii} T_{jj} + 2T_{ij}^2) \quad \text{....................(28)}$$

The p.d.f of generalized Rayleigh distribution is :

$$f(t_i; \alpha, \beta) = \begin{cases} 2\alpha\beta t_i e^{-\beta t_i^2} (1 - e^{-\beta t_i^2})^{\alpha - 1} & t_i \geq 0 \\ 0 & o.w \end{cases}$$

We assumed that α , β have the following Gamma conjugate prior distribution such that :

$$\alpha \sim \Gamma(n, a) \quad ; \quad \beta \sim \Gamma(n, b)$$

$$f(\alpha) = \begin{cases} \dfrac{a^n}{\Gamma(n)} \alpha^{n-1} e^{-a\alpha} & a > 0 \ , \ n > 0 \ , \ \alpha > 0 \\ 0 & o.w \end{cases} \quad \text{.......(29)}$$

$$f(\beta) = \begin{cases} \dfrac{b^n}{\Gamma(n)} \beta^{n-1} e^{-b\beta} & b > 0 \ , \ n > 0 \ , \ \beta > 0 \\ 0 & o.w \end{cases} \quad \text{........(30)}$$

The equations (29) and (30) are prior distribution for α and β .

The likelihood function of the marked data is :

$$L(t_1, t_2, ..., t_n; \alpha, \beta) = a2^r \alpha^r \beta^r \prod_{i=1}^{r} t_i e^{-\beta \sum_{i=1}^{r} t_i^2} \prod_{i=1}^{r} (1 - e^{-\beta t_i^2})^{\alpha - 1} [1 - (1 - e^{-\beta T_r^2})^{\alpha}]^{n-r} \quad \text{........(31)}$$

The joint p.d.f of α and β is :

$$J(t_1, t_2, ..., t_n; \alpha, \beta) = L(t_1, t_2, ..., t_n; \alpha, \beta) f(\alpha) f(\beta) \quad \text{.......(32)}$$

$$J(t_1, t_2, ..., t_n; \alpha, \beta) = a2^r \alpha^r \beta^r \prod_{i=1}^{r} t_i e^{-\beta \sum_{i=1}^{r} t_i^2} \prod_{i=1}^{r} (1 - e^{-\beta t_i^2})^{\alpha - 1}$$
$$[1 - (1 - e^{-\beta T_r^2})^{\alpha}]^{n-r} \frac{a^n}{\Gamma(n)} \alpha^{n-1} e^{-a\alpha} \frac{b^n}{\Gamma(n)} \beta^{n-1} e^{-b\beta} \quad \text{.....(33)}$$

$$p(\alpha, \beta \mid t_1, t_2, ..., t_n) = \frac{L(t_1, t_2, ..., t_n; \alpha, \beta) f(\alpha) f(\beta)}{\int_0^{\infty} \int_0^{\infty} L(t_1, t_2, ..., t_n; \alpha, \beta) f(\alpha) f(\beta) \, d\alpha \, d\beta} \quad \text{....(34)}$$

$$p(\alpha, \beta; t_1, t_2, ..., t_n) = a2^r \alpha^r \beta^r \prod_{i=1}^{r} t_i e^{-\beta \sum_{i=1}^{r} t_i^2} \prod_{i=1}^{r} (1 - e^{-\beta t_i^2})^{\alpha - 1} [1 - (1 - e^{-\beta T_r^2})^{\alpha}]^{n-r}$$
$$\frac{a^n}{\Gamma(n)} \alpha^{n-1} e^{-a\alpha} \frac{b^n}{\Gamma(n)} \beta^{n-1} e^{-a\beta} / \int_0^{\infty} \int_0^{\infty} a2^r \alpha^r \beta^r \prod_{i=1}^{r} t_i e^{-\beta \sum_{i=1}^{r} t_i^2} \prod_{i=1}^{r} (1 - e^{-\beta t_i^2})^{\alpha - 1}$$
$$[1 - (1 - e^{-\beta T_r^2})^{\alpha}]^{n-r} \frac{a^n}{\Gamma(n)} \alpha^{n-1} e^{-a\alpha} \frac{b^n}{\Gamma(n)} \beta^{n-1} e^{-b\beta} \, d\alpha \, d\beta \quad \text{............(35)}$$

The Bayesian estimator for α and β by using squared error loss function is :

$$\hat{R} = E[R(\alpha, \beta)]$$

Where $R(\alpha, \beta)$ be any function for α and β

$$\hat{R} = E[R(\alpha, \beta)] = \frac{\int_0^{\infty} \int_0^{\infty} R(\alpha, \beta) L(t_1, t_2, ..., t_n; \alpha, \beta) f(\alpha) f(\beta) \, d\alpha \, d\beta}{\int_0^{\infty} \int_0^{\infty} L(t_1, t_2, ..., t_n; \alpha, \beta) f(\alpha) f(\beta) \, d\alpha \, d\beta} \quad \text{................(36)}$$

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$$\hat{R}=\int_0^\infty\int_0^\infty R(\alpha,\beta)a2^r\alpha^r\beta^r\prod_{i=1}^r t_i e^{-\beta\sum_{i=1}^r t_i^2}\prod_{i=1}^r(1-e^{-\beta t_i^2})^{\alpha-1}$$

$$[1-(1-e^{-\beta T_r^2})^\alpha)]^{n-r}\frac{a^n}{\Gamma(n)}\alpha^{n-1}e^{-a\alpha}\frac{b^n}{\Gamma(n)}\beta^{n-1}e^{-b\beta}\,d\alpha\,d\beta\,/$$

$$\int_0^\infty\int_0^\infty a2^r\alpha^r\beta^r\prod_{i=1}^r t_i e^{-\beta\sum_{i=1}^r t_i^2}\prod_{i=1}^r(1-e^{-\beta t_i^2})^{\alpha-1}[1-(1-e^{-\beta T_r^2})^\alpha)]^{n-r}$$

$$\frac{a^n}{\Gamma(n)}\alpha^{n-1}e^{-a\alpha}\frac{b^n}{\Gamma(n)}\beta^{n-1}e^{-b\beta}\,d\alpha\,d\beta\;\ldots\ldots\ldots\ldots..(37)$$

We used Lindley's approximate $\hat{R}$ which approximate the ratio of the two integrals to obtain Bayes estimators approximation that can be written as follows :

$$\ln L(\alpha,\beta;t_i)=\ln a+r\ln 2+r\ln\alpha+r\ln\beta+\sum_{i=1}^r\ln t_i-\beta\sum_{i=1}^r t_i^2$$

$$+(\alpha-1)\sum_{i=1}^r\ln(1-e^{-\beta t_i^2})+(n-r)\ln[1-(1-e^{-\beta T_r^2})^\alpha)]\ldots(38)$$

$$l_{12}=\frac{\partial^3\ln L(\alpha,\beta)}{\partial\alpha\partial\beta^2}\;\ldots\ldots\ldots\ldots\ldots..(39)$$

$$\frac{\partial\ln L(\alpha,\beta)}{\partial\alpha}=\frac{r}{\alpha}+\sum_{i=1}^r\ln(1-e^{-\beta t_i^2})-\frac{(n-r)(1-e^{-\beta T_r^2})^\alpha\ln(1-e^{-\beta T_r^2})}{1-(1-e^{-\beta T_r^2})^\alpha}\;\ldots.(40)$$

$$\frac{\partial^2\ln L(\alpha,\beta)}{\partial\alpha\partial\beta}=\sum_{i=1}^r\frac{t_i^2e^{-\beta t_i^2}}{(1-e^{-\beta t_i^2})}-[(n-r)T_r^2e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}$$

$$[1+\alpha\ln(1-e^{-\beta T_r^2})-(1-e^{-\beta T_r^2})^\alpha]/[1-(1-e^{-\beta T_r^2})^\alpha]^2\ldots.(41)$$

$$\frac{\partial^3\ln L(\alpha,\beta)}{\partial\alpha\partial\beta^2}=-\sum_{i=1}^r\frac{t_i^4e^{-\beta t_i^2}}{(1-e^{-\beta t_i^2})^2}-(n-r)T_r^2[2\{(-T_r^2e^{-\beta T_r^2}$$

$$(1-e^{-\beta T_r^2})^{\alpha-1}\{1+\alpha\ln(1-e^{-\beta T_r^2})\})+T_r^2e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{2\alpha-1}$$

$$\{3+2\alpha\ln(1-e^{-\beta T_r^2})\})+(T_r^2(e^{-\beta T_r^2})^2(1-e^{-\beta T_r^2})^{\alpha-2}$$

$$\{2\alpha-1+\alpha(\alpha-1)\ln(1-e^{-\beta T_r^2})\})+(T_r^2(e^{-\beta T_r^2})^2$$

$$(1-e^{-\beta T_r^2})^{2(\alpha-1)}\{-5\alpha+3-\alpha(\alpha-2)\ln(1-e^{-\beta T_r^2})\})$$

$$+(T_r^2e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{3\alpha-1}\{-3-\alpha\ln(1-e^{-\beta T_r^2})\})$$

$$+(T_r^2(e^{-\beta T_r^2})^2(1-e^{-\beta T_r^2})^{3\alpha-2}\{4\alpha-3-\alpha\ln(1-e^{-\beta T_r^2})\})$$

$$+(T_r^2e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{4\alpha-1}-((\alpha-1)T_r^2(e^{-\beta T_r^2})^2$$

$$(1-e^{-\beta T_r^2})^{2(2\alpha-1)})\}]\;/\;[1-(1-e^{-\beta T_r^2})^\alpha]^4\ldots\ldots.(42)$$

$$l_{21}=\frac{\partial^3\ln L(\alpha,\beta)}{\partial\alpha^2\partial\beta}\;\ldots\ldots\ldots\ldots\ldots..(43)$$

$$\frac{\partial^2\ln L(\alpha,\beta)}{\partial\alpha^2}=-\frac{r}{\alpha^2}-\frac{(n-r)(1-e^{-\beta T_r^2})^\alpha\{\ln(1-e^{-\beta T_r^2})\}^2}{[1-(1-e^{-\beta T_r^2})^\alpha]^2}\;\ldots\ldots\ldots..(44)$$

$$\frac{\partial^3\ln L(\alpha,\beta)}{\partial\alpha^2\partial\beta}=-(n-r)T_r^2e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}\ln(1-e^{-\beta T_r^2})$$

$$[2+\alpha\ln(1-e^{-\beta T_r^2})+2(1-e^{-\beta T_r^2})^\alpha\{-2+(1-e^{-\beta T_r^2})^\alpha\}$$

$$-\alpha(1-e^{-\beta t_r^2})^{2\alpha}\ln(1-e^{-\beta t_r^2})]\;/\;[1-(1-e^{-\beta T_r^2})^\alpha]^4\ldots\ldots(45)$$

$$l_{03}=\frac{\partial^3\ln L(\alpha,\beta)}{\partial\beta^3}\;\ldots\ldots\ldots\ldots\ldots..(46)$$

$$\frac{\partial\ln L(\alpha,\beta)}{\partial\beta}=\frac{r}{\beta}-\sum_{i=1}^r t_i^2+(\alpha-1)\sum_{i=1}^r\frac{t_i^2e^{-\beta t_i^2}}{(1-e^{-\beta t_i^2})}-\frac{(n-r)\alpha T_r^2e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}}{1-(1-e^{-\beta T_r^2})^\alpha}\;\ldots\ldots\ldots\ldots..(47)$$

$$\frac{\partial^2\ln L(\alpha,\beta)}{\partial\beta^2}=-\frac{r}{\beta^2}-(\alpha-1)\sum_{i=1}^r\frac{t_i^4e^{-\beta t_i^2}}{(1-e^{-\beta t_i^2})^2}-(n-r)$$

$$\alpha T_r^4e^{-\beta t_i^2}(1-e^{-\beta T_r^2})^{\alpha-1}[\frac{(\alpha-1)e^{-\beta T_r^2}}{(1-e^{-\beta T_r^2})}-1+e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}$$

$$+(1-e^{-\beta T_r^2})^\alpha\;/\;[1-(1-e^{-\beta T_r^2})^\alpha]^2\ldots\ldots\ldots\ldots..(48)$$

$$\frac{\partial^3 \ln L(\alpha,\beta)}{\partial\beta^3} = \frac{2r}{\beta^3} + (\alpha-1)\sum_{i=1}^{r}\frac{t_i^6 e^{-\beta t_i^2}(1+e^{-\beta t_i^2})}{(1-e^{-\beta t_i^2})^3}$$

$$-[[(n-r)\alpha T_r^4[2(\alpha-1)^2 T_r^2 (e^{-\beta T_r^2})^3 (1-e^{-\beta T_r^2})^{\alpha-3}$$

$$-2(\alpha-1)T_r^2(e^{-\beta T_r^2})^2(1-e^{-\beta T_r^2})^{\alpha-3}+(5-3\alpha)T_r^2$$

$$(e^{-\beta T_r^2})^3(1-e^{-\beta T_r^2})^{3(\alpha-1)}-2(\alpha-1)T_r^2(e^{-\beta T_r^2})^2$$

$$(1-e^{-\beta T_r^2})^{3(\alpha-1)}+(9\alpha-12)T_r^2(e^{-\beta T_r^2})^2(1-e^{-\beta T_r^2})^{2(\alpha-1)}$$

$$-(\alpha-1)T_r^4(e^{-\beta T_r^2})^2(1-e^{-\beta T_r^2})^{2(\alpha-1)}-4(\alpha-1)T_r^2(e^{-\beta T_r^2})^2$$

$$(1-e^{-\beta T_r^2})^{(\alpha-2)}+(-2\alpha^2+8\alpha-6)T_r^2(e^{-\beta T_r^2})^3$$

$$(1-e^{-\beta T_r^2})^{2\alpha-3}+4(\alpha-1)T_r^2(e^{-\beta T_r^2})^2(1-e^{-\beta T_r^2})^{2\alpha-3}$$

$$+2T_r^2e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}-5T_r^2e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{2\alpha-1}$$

$$+(\alpha-1)^2T_r^4(e^{-\beta T_r^2})^4(1-e^{-\beta T_r^2})^{2(\alpha-2)}-2\alpha T_r^2(e^{-\beta T_r^2})^3$$

$$(1-e^{-\beta T_r^2})^{4\alpha-3}+(12-6\alpha)T_r^2(e^{-\beta T_r^2})^2(1-e^{-\beta T_r^2})^{3\alpha-2}$$

$$+2(\alpha-1)T_r^4(e^{-\beta T_r^2})^2(1-e^{-\beta T_r^2})^{3\alpha-2}+4T_r^2e^{-\beta T_r^2}$$

$$(1-e^{-\beta T_r^2})^{3\alpha-1}-2(\alpha-1)^2T_r^4(e^{-\beta T_r^2})^4(1-e^{-\beta T_r^2})^{3\alpha-4}$$

$$+(\alpha-1)T_r^2(e^{-\beta T_r^2})^3(1-e^{-\beta T_r^2})^{5\alpha-3}+(\alpha-4)T_r^2(e^{-\beta T_r^2})^2$$

$$(1-e^{-\beta T_r^2})^{2(2\alpha-1)}-(\alpha-1)T_r^4(e^{-\beta T_r^2})^2(1-e^{-\beta T_r^2})^{2(2\alpha-1)}$$

$$-T_r^2e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{4\alpha-1}+(\alpha-1)^2T_r^4(e^{-\beta T_r^2})^4$$

$$(1-e^{-\beta T_r^2})^{2(2\alpha-2)}]]\ /\ [1-(1-e^{-\beta T_r^2})^\alpha]^4 \text{..............(49)}$$

$$l_{30} = \frac{\partial^3 \ln L(\alpha,\beta)}{\partial\alpha^3} \quad \text{.................(50)}$$

$$\frac{\partial^3 \ln L(\alpha,\beta)}{\partial\alpha^3} = \frac{2r}{\alpha^3} - (n-r)\frac{[1-(1-e^{-\beta T_r^2})^\alpha](1-e^{-\beta T_r^2})^\alpha\{\ln(1-e^{-\beta T_r^2})\}^3[1+(1-e^{-\beta T_r^2})^\alpha]}{[1-(1-e^{-\beta T_r^2})^\alpha]^4} \quad \text{.....(51)}$$

Now when $R(\alpha,\beta)=\alpha$ then ,

$$w_1 = \frac{\partial R}{\partial\alpha} = 1 \ ,\quad w_2 = \frac{\partial R}{\partial\beta} = 0 \ ,\quad w_{12} = w_{21} = \frac{\partial^2 R}{\partial\alpha_i\partial\beta_j} = 0 \text{ ....(52)}$$

$$D = \frac{\partial^2 \ln L(\alpha,\beta)}{\partial\alpha^2} = -\frac{r}{\alpha^2} - (n-r)\frac{(1-e^{-\beta T_r^2})^\alpha\{\ln(1-e^{-\beta T_r^2})\}^2}{[1-(1-e^{-\beta T_r^2})^\alpha]^2} \quad \text{........(53)}$$

$$U = \frac{\partial^2 \ln L(\alpha,\beta)}{\partial\beta^2} = -\frac{r}{\beta^2} - (\alpha-1)\sum_{i=1}^{r}\frac{t_i^4 e^{-\beta t_i^2}}{(1-e^{-\beta t_i^2})^2} - (n-r)$$

$$\alpha T_r^4 e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}[\frac{(\alpha-1)e^{-\beta T_r^2}}{(1-e^{-\beta t_i^2})} - 1 + (1-e^{-\beta T_r^2})^\alpha$$

$$+e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}]\ /\ [1-(1-e^{-\beta T_r^2})^\alpha]^2 \text{..........(54)}$$

$$V = \frac{\partial^2 \ln L(\alpha,\beta)}{\partial\beta\partial\alpha} = \frac{\partial^2 \ln L(\alpha,\beta)}{\partial\alpha\partial\beta} = \sum_{i=1}^{r}\frac{t_i^2 e^{-\beta t_i^2}}{(1-e^{-\beta t_i^2})}$$

$$-(n-r)T_r^2 e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}[1+\alpha\ln(1-e^{-\beta T_r^2})$$

$$-(1-e^{-\beta T_r^2})^\alpha]\ /\ [1-(1-e^{-\beta T_r^2})^\alpha]^2 \text{ ....(55)}$$

$$T_{11} = \frac{U}{DU-V^2} = \frac{-\frac{r}{\beta^2}-(\alpha-1)\sum_{i=1}^{r}\frac{t_i^4e^{-\beta t_i^2}}{(1-e^{-\beta t_i^2})^2}-(n-r)\alpha T_r^4e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}[\frac{(\alpha-1)e^{-\beta T_r^2}}{(1-e^{-\beta t_i^2})} -1+(1-e^{-\beta T_r^2})^\alpha+e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}]\ /\ [1-(1-e^{-\beta T_r^2})^\alpha]^2}{[-\frac{r}{\alpha^2}-(n-r)\frac{(1-e^{-\beta T_r^2})^\alpha\{\ln(1-e^{-\beta T_r^2})\}^2}{[1-(1-e^{-\beta T_r^2})^\alpha]^2}][-\frac{r}{\beta^2}-(\alpha-1)\sum_{i=1}^{r}\frac{t_i^4e^{-\beta t_i^2}}{(1-e^{-\beta t_i^2})^2}-(n-r)\alpha T_r^4e^{-\beta T_r^2} (1-e^{-\beta T_r^2})^{\alpha-1}[\frac{(\alpha-1)e^{-\beta T_r^2}}{(1-e^{-\beta t_i^2})}-1+(1-e^{-\beta T_r^2})^\alpha+e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}]\ /\ [1-(1-e^{-\beta T_r^2})^\alpha]^2] -[\sum_{i=1}^{r}\frac{t_i^2e^{-\beta t_i^2}}{(1-e^{-\beta t_i^2})}-(n-r)T_r^2e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}[1+\alpha\ln(1-e^{-\beta T_r^2})-(1-e^{-\beta T_r^2})^\alpha]\ /\ [1- (1-e^{-\beta T_r^2})^\alpha]^2]^2} \text{..................................(56)}$$

$$T_{22} = \frac{D}{DU-V^2} = [-\frac{r}{\alpha^2}-(n-r)$$

$$\frac{(1-e^{-\beta T_r^2})^\alpha\{\ln(1-e^{-\beta T_r^2})\}^2}{[1-(1-e^{-\beta T_r^2})^\alpha]^2}]/[-\frac{r}{\alpha^2}-(n-r)$$

$$\frac{(1-e^{-\beta T_r^2})^\alpha\{\ln(1-e^{-\beta T_r^2})\}^2}{[1-(1-e^{-\beta T_r^2})^\alpha]^2}][-\frac{r}{\beta^2}-(\alpha-1)\sum_{i=1}^{r}\frac{t_i^4e^{-\beta t_i^2}}{(1-e^{-\beta t_i^2})^2}$$

$$-(n-r)\alpha T_r^4e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}[\frac{(\alpha-1)e^{-\beta T_r^2}}{(1-e^{-\beta t_i^2})}$$

$$-1+(1-e^{-\beta T_r^2})^\alpha+e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}]\ /\ [1-$$

$$(1-e^{-\beta T_r^2})^\alpha]^2]-[\sum_{i=1}^{r}\frac{t_i^2e^{-\beta t_i^2}}{(1-e^{-\beta t_i^2})}-(n-r)T_r^2e^{-\beta T_r^2}$$

$$(1-e^{-\beta T_r^2})^{\alpha-1}[1+\alpha\ln(1-e^{-\beta T_r^2})-$$

$$(1-e^{-\beta T_r^2})^\alpha]\ /\ [1-(1-e^{-\beta T_r^2})^\alpha]^2]^2 \text{............(57)}$$

$$T_{12}=T_{21}=\frac{-V}{DU-V^2}=-[\sum_{i=1}^{r}\frac{t_i^2e^{-\beta t_i^2}}{(1-e^{-\beta t_i^2})}-(n-r)T_r^2e^{-\beta T_r^2}$$

$$(1-e^{-\beta T_r^2})^{\alpha-1}[1+\alpha\ln(1-e^{-\beta T_r^2})-(1-e^{-\beta T_r^2})^{\alpha}]\;/\;[1-$$

$$(1-e^{-\beta T_r^2})^{\alpha}]^2\,/[-\frac{r}{\alpha^2}-(n-r)\frac{(1-e^{-\beta T_r^2})^{\alpha}\{\ln(1-e^{-\beta T_r^2})\}^2}{[1-(1-e^{-\beta T_r^2})^{\alpha}]^2}]$$

$$[-\frac{r}{\beta^2}-(\alpha-1)\sum_{i=1}^{r}\frac{t_i^4e^{-\beta t_i^2}}{(1-e^{-\beta t_i^2})^2}-(n-r)\alpha T_r^4e^{-\beta T_r^2}$$

$$(1-e^{-\beta T_r^2})^{\alpha-1}[\frac{(\alpha-1)e^{-\beta T_r^2}}{(1-e^{-\beta t_i^2})}-1+(1-e^{-\beta T_r^2})^{\alpha}+e^{-\beta T_r^2}$$

$$(1-e^{-\beta T_r^2})^{\alpha-1}]\;/\;[1-(1-e^{-\beta T_r^2})^{\alpha}]^2]-[\sum_{i=1}^{r}\frac{t_i^2e^{-\beta t_i^2}}{(1-e^{-\beta t_i^2})}$$

$$-(n-r)T_r^2e^{-\beta T_r^2}(1-e^{-\beta T_r^2})^{\alpha-1}[1+\alpha\ln(1-e^{-\beta T_r^2})$$

$$-(1-e^{-\beta T_r^2})^{\alpha}]\;/\;[1-(1-e^{-\beta T_r^2})^{\alpha}]^2]^2 \ldots\ldots\ldots\ldots(58)$$

$$p=\ln f(\alpha,\beta)=\ln[\frac{a^n}{\Gamma(n)}\alpha^{n-1}e^{-a\alpha}.\frac{b^n}{\Gamma(n)}\beta^{n-1}e^{-b\beta}] \ldots\ldots(59)$$

$$p=n\ln a-\ln\Gamma(n)+(n-1)\ln\alpha-a\alpha+n\ln b-\ln\Gamma(n)+$$
$$(n-1)\ln\beta-b\beta \ldots\ldots\ldots\ldots\ldots\ldots(60)$$

$$p_1=\frac{\partial p}{\partial\alpha}=\frac{n-1}{\alpha}-a \ldots\ldots\ldots\ldots\ldots(61)$$

$$p_2=\frac{\partial p}{\partial\beta}=\frac{n-1}{\beta}-b \ldots\ldots\ldots\ldots\ldots(62)$$

$$A=\sum_{i=1}^{2}\sum_{j=1}^{2}w_{ij}T_{ij}=w_{12}T_{12}+w_{21}T_{21}=0 \ldots.(63)$$

$$B_{12}=T_{12}T_{11}\quad;\quad B_{21}=T_{21}T_{22} \ldots\ldots\ldots\ldots\ldots(64)$$

$$C_{12}=3T_{11}T_{12}+T_{11}T_{22}+2T_{12}^2\quad;\quad C_{21}=0 \ldots.(65)$$

$$A_{12}=T_{12}\quad;\quad A_{21}=T_{21} \ldots\ldots\ldots\ldots\ldots(66)$$

$$\hat{\alpha}=\hat{\alpha}_{MLE}+\frac{1}{2}[l_{30}T_{12}T_{11}+l_{03}T_{21}T_{22}+l_{21}(3T_{11}T_{12}+T_{11}T_{22}+2T_{12}^2)]+$$
$$(\frac{n-1}{\alpha}-a)T_{12}+(\frac{n-1}{\beta}-b)T_{12} \ldots\ldots\ldots\ldots\ldots(67)$$

When $R(\alpha,\beta)=\beta$ then ,

$$w_1=\frac{\partial R}{\partial\alpha}=0\quad,\quad w_2=\frac{\partial R}{\partial\beta}=1\quad,\quad w_{12}=w_{21}=\frac{\partial^2R}{\partial\alpha_i\partial\beta_j}=0 \ldots\ldots\ldots(68)$$

$$A=\sum_{i=1}^{2}\sum_{j=1}^{2}w_{ij}T_{ij}=w_{12}T_{12}+w_{21}T_{21}=0 \ldots\ldots\ldots\ldots\ldots(69)$$

$$B_{12}=T_{12}T_{11}\quad;\quad B_{21}=T_{21}T_{22} \ldots\ldots\ldots\ldots\ldots(70)$$

$$C_{12}=0\quad;\quad C_{21}=3T_{22}T_{21}+T_{22}T_{11}+2T_{21}^2 \ldots\ldots\ldots\ldots(71)$$

$$A_{12}=T_{21}\quad;\quad A_{21}=T_{21} \ldots\ldots\ldots\ldots\ldots(72)$$

$$\hat{\beta}=\hat{\beta}_{MLE}+\frac{1}{2}[l_{30}T_{12}T_{11}+l_{03}T_{21}T_{22}+l_{12}(3T_{22}T_{21}+T_{22}T_{11}+2T_{21}^2)]+$$
$$(\frac{n-1}{\alpha}-a)T_{21}+(\frac{n-1}{\beta}-b)T_{21} \ldots\ldots\ldots(73)$$

4: results and discussion

This research , based on real data for the Bladder cancer disease , selecting this type of cancer because it is widespread and deadly in time in Iraq and this type of diseases has failure time (death time) occurs which is interesting phenomenon in this paper .

To collect data for the Bladder cancer disease , returning the educational hospital in Diwaniya .

The time of study point in this paper determined from 1-1-2015 until 31-6-2015 , that means the duration time of this study is constant and fixed for (6) months .

The number of sick people in the experiment for the above duration time is (15) . nine patients were dead and six patients remain alive .

We found from the our samples that the Male formed (9) and Female formed (6) from my study .

When applying the test statistic (Kolmogorov-Smirnov) in order to fit generalized Rayleigh distribution data , it is discovered that the calculated value is (0.11397) , this means data is distributed according to generalized Rayleigh distribution .

The null and alternative hypotheses are as follows :

H_0 : The survival time data is distributed as generalized Rayleigh pdf .

H_1 : The survival time data is not distributed as generalized Rayleigh pdf .

By using program of MATLAB (R2009b) , we've got the following estimated parameters values :

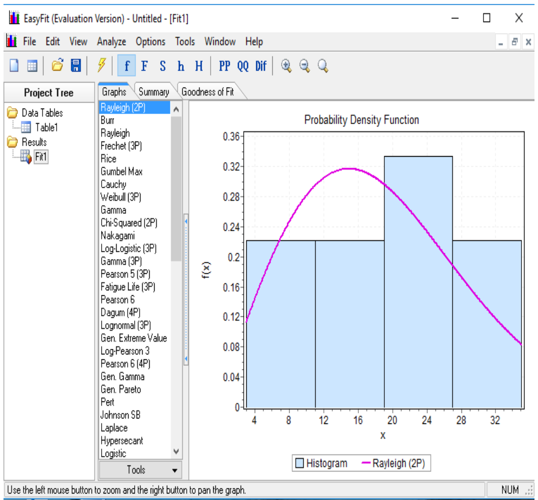

Figure (1)
Fit the data for Bladder cancer from Educational Hospital in Diwaniya

The assumed initial values for two-parameters are as follows:

$$\alpha_0 = 0.98 \qquad \beta_0 = 0.0003$$

Table (1)

Estimated values for the parameters by MLE method for censored data type two

Estimate values	**Number of iteration**	**Errors for all parameters**
$\hat{\alpha} = 1.5373$	4	2.1552
$\hat{\beta} = 0.0023$		0.0010

The calculated estimated values for the two-parameters used Bayes estimator by using Lindley approximation Method which depending on the values of estimate parameters in maximum likelihood method are as follows :

$$\hat{\alpha} = 399.7118287$$

$$\hat{\beta} = 397.7464541$$

The estimated values of a and b are respectively as follows :

$$a = 0.1 \qquad b = 0.3$$

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تقدير دالة البقاء لعينة مراقبة من النوع الثاني لمرضى سرطان المثانة

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المستخلص :

هذا البحث يهتم بتقدير المعلمات غير المعلومة لنموذج توزيع رالي المعمم لعينات منفردة تحت المراقبة من النوع الثاني . استخدمت طريقة تقدير الإمكان الأعظم لاشتقاق التقدير النقطي لجميع المعلمات غير المعلومة بالاعتماد على طرائق التكرار ومنها طريقة نيوتن – رافسون، ثم اشتقاق طريقة بيز المعتمدة على تقريب Lindely . وأخيراً تم اختبار مدى ملائمة النموذج الحالي (نموذج توزيع رالي المعمم) لمجموعة بيانات حقيقية.