T- SEQUENTIALLY ABSOLUTELY CLOSED SPACES

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Abstract

In this work, we introduce the concept of T- sequentially absolutely closed spaces which is a generalization of the concept of sequentially absolutely closed spaces [1]. Several properties of this new concept are proved.

1. Introduction :

A topological space X is called sequentially compact iff every sequence in X cantains a convergent sub sequence [2]. A topological space (X,τ) said to be sequentially absolutely closed (S.A.C) iff each sequence cantains a weakly convergent sub sequence [1] Now, Let (X,τ) be a topological space , let T be an operator associated with the topology τ on X [5] In the work, we introduce and study the concept of T-sequentially absolutely closed space (T-S.A.C.S) when T is the closure operator, we get the concept of sequentially absolutely closed space.

2. Basic definitions:

In this section , we recall and introduce the basic definitions needed in this work.

2.1. Definitions: [1]

Let (X,τ) be a topological space and $A\subseteq X$, $p\in X$. p is a weak limit point of A means that for each open set U containing P, we have $(\overline{U}-\{p\})\cup A\neq\phi$.

Now we give the following generalization of the above definition.

2.2 .Definition [5]:

Let (X,τ) be a topological space and let T : P(X) $\rightarrow$ P(X) be a function, where P(X) is the power set of X. We say that T is an operator associated with the topology τ on X, if for each open set . $w\subseteq T(w)$ The Triple (X, T, τ) is called an operator topological space.

2.3 .Definition:

Let (X,τ) be a topological space and let T be an operator associated with the topology τ on X, Let $A\subseteq X$, $p\in X$. , we say that p is a T-limit point of A if for each open set U containing P, we have $(T(U)-\{p\})\cap A\neq\phi$.

If T is the closure operator then we get definition (2.1).

2.4. Eamples :

i) Let X = R , τ = the cofinite topology on R, T = The closre operator that is T(A) = cl(A) , and let A = { 1, 2, 3, ...}, p = 0. Now p is a T-limit point of A because T(U) = cl(U) = R for each open set U, hence

$$(T(U)-\{p\})\cap A=(R-\{p\})\cap A=\phi$$

ii) Let X = { *a* , b , c } , τ = { X, Ø , {*a*} , {b,c} }The cofinite topology on R, T = The closre operator then the sequence b , b , b , . . . , *a* , b, b , b . . . . does not T-converge to *a* , because : If U = {*a*} then T(U)= cl(U) = cl({*a*}) ={*a*} which contans only *a*.

2.5. Definition [1]:

A Sequence (x_n) in a topological space (X,τ) is said to converge weakly to p iff the closure of each neighborhood of p contains all but a finite number of the x_n's. Now we give the following generalization of the above definition.

2.6. Definition:

Let (X,τ) be a topological space and let T be an operator associated with the topology τ on X. A Sequence (x_n) in X is T – converges to p iff T(U) contains all but a finite number of the x_n (U is a neighborhood of p)

2.7. Eamples :

i) Let X = { a , b , c } , τ = { X, Ø, {a} , {b,c} }The cofinite topology on R, T : P(X) $\to$ P(X) is defineed as follows : T(A) = Intcl(A), consider the sequence b , b , b , . . . , a , b, b , b . . . . let U = {a}. Now T(U)= Intcl(U) = cl({a}) = Int X = X which means that the seqence T-converges to a.

ii) Let X = R , τ = The closure operator Let A = [3 , 5] , p = 2 $\in$ R. Now 2 is not a T-limit point of A because if U = [1 , 2.5] then

$$(T(U)-\{p\})\cap A=([1,2.30]-\{2\})\cap A=\phi$$

2.8. Definition [1, 3]:

A topological space (X,τ) is said to be sequential absolutely closed (S.A.C) iff each sequence contains a weakly convergent sub sequence, Now we introduce our definition.

2.9. Definition:

Let (X,τ) be a topological space and let T be an operator associated with the topology T on X, then the triple (X,τ,T) is called an operator topological space. (X,τ,T) space. (X,τ,T) is called T- sequentially absolutely closed (T-S.A.C) iff each sequence contains a T-convergent sub sequence subsequence.

2.10. Remarks and examples:

It is clear that if a space X is sequentially a compact then it is T-sequentially absolutely closed. The converse is not necessarily true.

Let X= R $\tau=[(-\infty,a)\ |a\in R]\cup[\phi]\cup[R]$

T = the closure operator that is

$$T(A)=\overline{A}, A\subseteq X$$

That is clear that this space is T-sequentially absolutely closed since that closure of every non-empty open set is all of R. Now let (x_n) = (n), this sequence has no convergent sub sequence, this prove that (X,τ) is not sequentially compact.

3. Main Results:

In this section , we prove several properties of T-sequentially absolutely closed space (T-S.A.C.S). First, we recall the following definition.

3.1. Definition[4, 5]:

Let (X,τ,T) be an operator topological space we say that X is T-Regular if for each $p\in X$ and W an open set containing p then there exists an open set V $P\in V\subseteq T(v)\subseteq W$ such that , That is X is T-Regular if : W is open in X iff W is T-open [5].

3.2 . Theorem:

Let (X,τ,T) be T-Regular operator topological space then (X,τ,T) is T-sequentially absolutely closed iff it is sequentially compact.

Proof :

Let ($x_{\mathbf{n}}$) be any sequence in X, X is T- sequentially absolutely closed, then there exists a sub sequence (x_{n_k}) which T-converges to a point say p. To show that (x_{n_k}) converges to p.Let υ be any open set containing p. X is T-Regular ,so there exists V open such that, $p \in V \subseteq T(v) \subseteq U$

Now (x_{n_k}) T- converges top. Therefore T(V) contains all but a finite number of (x_{n_k}) Accordingly U contains all but a finite number of (x_{n_k}) .thus (X, τ, T) is sequentially compact. The other direction is $W \subseteq T(w)$ clear because for every open set W . Before , we state the next result We recall the following definition.

3.3. Definition[4]:

An operator topological space (X, τ, T) is said to be T- countably absolutely closed iff each countable open cover {Gi / i=1,2,…} of X has a finite sub family {Gi,$Gi_{\mathbf{1}}$,…$Gi_{\mathbf{n}}$} such that

$$X = \cup_{k=1}^{n} T(Gi_K)$$

3.4. Theorem:

If (X, τ, T) is T- sequentially absolutely closed then it is T-countably absolutely closed space

Proof:

Assume X is infinite ,for otherwise the proof is trivial. Assume there exists acountable open cover {Gi / i = 1, 2, ... } of X with no finite sub family {Gi_1, Gi_2, . . . , Gi_n} such that

$$X = \cup_{k=1}^{n} T(Gi_K)$$

Define a sequence ($x_{\mathbf{n}}$) as follows, choose $x_1 \in T(G_1)$, $x_2 \in T(G_2) - T(G1)$ such a point always exists ,for otherwise T(G_1) covers X, continuing in this manner ,we obtain a sequence (x_n) with the property that

$$X_i \in T(G_i) \text{ and } X_i \notin \cup_{j=1}^{i-1} T(G_j)$$

Claim :

(x_n) has no T- convergent sub sequence Let

$$p \in X, \text{then there exists } G_k. \text{ such that } p \in T(G_k)$$

$$\text{but}, X_k \notin T(G_j) \text{ for } J < k$$

So no sequence of X has a T- convergent sub sequence and this contradicts the hypothesis, So (X, τ, T) must be T- countable absolutely closed.

References

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المستخلص:

في هذا العمل قدمنا مفهوم T− الفضاءات المغلقة تماما المتتابعة التي هي تعميم للفضاءات المغلقة تماما المتتابعة. برهنا مجموعة الخصائص الجديدة لهذه الفضاءات.