Computation of Positions Error Signals in Augmented INS/GPS Syetem Based on Wavelet Analysis

Dr. Salam A. Ismaeel*, Prof. Dr. Waladin K. Sa’id*, and Ahmed M. Hassan*

Abstract

Integrated systems based on Inertial Navigation System (INS) and the Global Positioning System (GPS) generated increased interest in the airborne survey and remote sensing community over the past few years. Where, with full operational GPS capability, it has been recognized that an optimal combination of GPS with inertial navigation brings a number of advantages over stand-alone inertial or GPS navigation.

This paper aims at introducing GPS and INS system integration approach for fusing data utilizing wavelet analysis. Where the wavelet used to compare the sensers outputs at different resolution levels, for several types of errors in INS and GPS, to smooth and predect the INS erros.

Keywords:

Vehicular naviagation, inertial navigation, GPS, Wavelet Analysis

1. Introduction

Strapdown Inertial Navigation System (SDINS) technologies are based on the principle of integrating specific forces and rates measured by accelerometers and rate gyros of an Inertial Measurement Unit (IMU) fixed to the navigating body. Given the initial conditions of position, velocity, and attitude, accurate real time integration of IMU output will produce position and attitude information in some given navigation coordinate system [1]. On the other hand, the Global Positioning System (GPS) relies on the technique of comparing signals from orbiting satellites to calculate position (and possibly attitude) at regular time intervals [2, 3], but being dependent on the satellites signals makes GPS less reliable than self contained INS due to the possibility of drop-outs or jamming [4].

Typically, the dynamic error model for a terrestrial INS algorithm need for three position errors, three velocity errors and three attitude errors in an INS (i.e. the system error states). These errors are also augmented by some sensor error states such as accelerometer biases and gyroscope drifts, which are modeled as stochastic processes. In fact, there are several random errors associated with each inertial sensor. Therefore, it is usually difficult to set a certain stochastic model for each inertial sensor that works efficiently in all environments and reflects the long-term behavior of sensor errors [5]. Hence the wavelet algorithm can perform the self-following of the vehicle under all-conditions maneuvering is thus required.

In other words, this work concerned with a wavelet analysis method to estimate INS errors and also to analyze and compare the INS and GPS outputs at different resolution levels.

A comperhensive survey and description of the integrated INS/GPS is presented in refernce [4]. This paper is concerned with the error estimation in the low cost INS sytem based on INS/GPS

[1]integration using different types of simulated data at different resultution. Unlike the algorithm described in [5], where the real data from an experimental work for a special type of INS and GPS with special types of error sources.

2. GPS-INS Integration

The reasons for utilizing GPS-aided-INS approach are [4]:

1. An inertial system has almost no high frequency noise, but it can have large low frequency errors and bias errors that grow with time. GPS, on the other hand, has frequency noise but good long-term accuracy (i.e., small bias errors), exploitation of such complementary characteristics is possible in the aided architecture.
2. With the incorporation of GPS much cheaper INS units could be used for a high degree of accuracy application.
3. The INS provides navigation solutions in real time at rate higher than could be achievable from a GPS receiver, so the INS achieves a higher system bandwidth than would be possible in a system based solely on the aiding signal.
4. Optimal mixing of INS and GPS information reduces the effect of INS errors, and allows estimation and correction of errors in the INS state.

In other words, combined INS-GPS systems are capable of making up for the weaknesses inherent in each.

4. The Global Positoning System (GPS)

Most of the present vehicle navigation instruments rely on the Global Positioning System (GPS) as the primary source of information to provide the vehicle's position.

4.1 GPS System Overview

Global Positioning System (GPS) is a satellite-based navigation system that allows a user with the proper equipment access to useful and accurate positioning information anywhere on the globe. Position and time determination is accomplished by the reception of GPS signals to obtain ranging information as well as messages transmitted by the satellites [3].

The NAVSTAR Global positioning system is a space-based radio positioning system which provides suitably equipped users with a relatively highly accurate position, velocity and time data. This service is provided globally, continuously, and under all weather conditions to users near the surface of the earth [2]. GPS receivers operate passively, thus allowing an unlimited number of simultaneous users. The GPS has features, which can provide accurate service to unauthorized

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[١] *University of Technology

users, prevent spoofing and reduce-receiver susceptibility to jamming [6].

Basically the GPS comprises of three separate elements: the space segment, the control segment, and the user segment.

4.2 GPS Mathematical Model [4]

In all, the standard GPS positioning problem poses four variables that can be solved from the following system of equations, representing measurements from four different satellites.

$$p_1 = \sqrt{(X_1 - x)^2 + (Y_1 - y)^2 + (Z_1 - z)^2} + c\Delta t \quad \ldots(1)$$
$$p_2 = \sqrt{(X_2 - x)^2 + (Y_2 - y)^2 + (Z_2 - z)^2} + c\Delta t \quad \ldots(2)$$
$$p_3 = \sqrt{(X_3 - x)^2 + (Y_3 - y)^2 + (Z_3 - z)^2} + c\Delta t \quad \ldots(3)$$
$$p_4 = \sqrt{(X_4 - x)^2 + (Y_4 - y)^2 + (Z_4 - z)^2} + c\Delta t \quad \ldots(4)$$

where

p_1, p_2, p_3, p_4 : Noise less pseudorange

$[X_j, Y_j, Z_j]^T$: Cartesian position coordinates of satellite j

$[x, y, z]^T$: Cartesian position coordinates of observer

Δt : Receiver offset from the satellite system time

c : Speed of light

The observer position $[x, y, z]^T$ is " slaved" to the coordinate frame of reference used by the satellite system.

The pseudorange measurement can be summarily represented by the following equation:

$$\tilde{p} = p + B_p + v_p \quad \ldots(5)$$

where

$\tilde{p}$: Measured pseudorange

B_P : Time correlated errors associated with pseudorange

v_p : Pseudorange measurement noise

Thus, equations (1)-(4) can be written as:

$$\tilde{p}_1 = \sqrt{(X_1 - x)^2 + (Y_1 - y)^2 + (Z_1 - z)^2} + c\Delta t + B_{p_1} + v_{p_1} \quad \ldots(6)$$
$$\tilde{p}_2 = \sqrt{(X_2 - x)^2 + (Y_2 - y)^2 + (Z_2 - z)^2} + c\Delta t + B_{p_2} + v_{p_2} \quad \ldots(7)$$
$$\tilde{p}_3 = \sqrt{(X_3 - x)^2 + (Y_3 - y)^2 + (Z_3 - z)^2} + c\Delta t + B_{p_3} + v_{p_3} \quad \ldots(8)$$
$$\tilde{p}_4 = \sqrt{(X_4 - x)^2 + (Y_4 - y)^2 + (Z_4 - z)^2} + c\Delta t + B_{p_4} + v_{p_4} \quad \ldots(9)$$

4.3 GPS Error Sources

Equations (6)-(9) indicate that the measurement of the satellite-to-receiver range in the GPS system is corrupted by several forms of errors, which can be summmurized as [6, 7]:

1. Receiver clock bias
2. Satellite clock bias

3. Atmospheric delay
4. Ephermeris errors
5. Multipath
6. Recevier noise.

Table (1), lists typical standard deviations for the various soures of noise computing the GPS observables. Figure (1) shows the noise data added to each poition vector in our work. The standered deviation of the total error was (7.2816 m).

5. Stapdown Terrestrial INS System

Strapdown system algorithms are the mathematical definition of processes which convert the measured outputs of inertial sensors that are fixed to a vehicle body axis into quantities which can be used to control the vehicle. There are two SDINS algorithms (Celestial and Terrestrial) [8].

The celestial strapdown inertial navigation system is mechanized in inertial frame. This frame is widely used for spacecraft applications in which geographical information in not required. But, for terrestrial navigation, the inherent time-varying relationship between the inertial and geographic frames complicates the space-stable system design [8].

Thus, the inertial frame implementation results in the most straight-forward navigation-state differential equations, which is not commonly used. The reasons for this lack of use are the difficulty in calculating gravitational forces, and the terrestrial navigation has the same coordinate used by the GPS system, for GPS aiding navigation system [4]. References [4, 9] gives the ditails of the terrestrial algorithm.

5.1 INS Error Souces

Knowledge of the error sources enables the system to cancel their effects as it navigates. In a strapdwon system, however, only few of the sensor errors can be calibrated. Errors that cannot be calibrated will propagate into navigation errors when the system begins to navigate.

There are several sources of these errors in SDINS are discussed in references [6, 9]. The most effected errors have been summarized for one degree INS system in figure (2), which shows the acceleration for an arbitrary moving vehicle. Figure (3) shows the INS sensor’s errors effect on the position. These errors are desccribed in table (2) [4].

6. Wavelet Transform

Wavelet Transform (WT) is a relatively new technique for signal decomposition. Unlike Fourier technique (FT) is extremely useful because the signal’s frequency content is of great importance. However, Fourier analysis has a serious drawback. In transforming to the frequency domain, time domain information is lost. When looking at a

Fourier transform of a signal, it is impossible to tell when a particular event took place. In an effort to overcome this deficiency in FT, the same transform was adapted to analyze only a small window of the signal at a time. This technique is presently known as the short-time Fourier transform (STFT), which maps a signal onto a two-dimensional function of time and frequency. Apparently, a narrow window width leads to good time localization but poor frequency resolution, while a wide window width leads to poor time localization but good frequency resolution. Therefore, it is necessary to have multiple resolutions in time and frequency. The major drawback of the STFT is that the window width is fixed, which means that their will usually is a time–frequency tradeoff corresponding to the choice of the window function’s width. Wavelet analysis represents the next logical step, which is based on a windowing technique with variable-sized windows. Wavelet transform (WT) allows the use of long time intervals where precise low frequency information is needed, and shorter intervals where high frequency information is considered [10, 11].

In general, the major advantage offered by wavelets is the ability to perform local analysis; that is, to analyze a localized area of a larger signal. Therefore, it will be adopted in this study to analyze both the INS and GPS position components.

6.1 Discrete Wavelet Transform (DWT) [10]

The DWT of a discrete time sequence *x(n)* is given as:

$$C_{j,k} = 2^{(-j/2)}\sum_{n} x(n)\Psi(2^{-j}n - k \qquad \ldots(10)$$

Where $\Psi(n)$ is the wavelet function (the basis function utilized in the wavelet transform) and $2^{(-j/2)}\Psi(2^{-j}n-k)$ are scaled and shifted versions of $\Psi(n)$ based on the values of *j* (scaling coefficient) and *k* (shifting coefficient) and are usually written as $\Psi_{j,k}(n)$. The *j* and *k* coefficients take integer values for different scaling and shifted versions of $\Psi(n)$ and $C_{j,k}$ are the corresponding wavelet coefficients. The original signal *x(n)* can be generated from the corresponding wavelet function using the following equation: …(11)

$$x(n) = \sum_{j}\sum_{k} C_{j,k}\Psi_{j,k}(n)$$

The basis functions $\Psi_{j,k}(n)$ are not limited to exponential functions as in the case of FT or STFT. The only restriction on $\Psi_{j,k}(n)$ is that it must be short and oscillatory (it must have zero average and decay quickly at both ends). This restriction ensures that the summation in the DWT transform equation is finite. The function $\Psi_{j,k}(n)$ has given the name ‘wavelet’ or ‘smallwave’ to the transform and is referred to as the ‘mother wavelet’ and it’s dilates and translates are simply referred to as ‘wavelets’ or ‘daughter wavelets’. Therefore, the wavelet transformation of a time-domain signal can be defined in terms of the projections of this signal onto a family of functions that are all normalized dilations and translations of a wavelet function. The mathematical procedure of wavelet analysis is described in references [5, 11].

7. <u>Proposed System Evaluation and Discussion</u>

Based on six degree of freedom (6DoF) equations of motion block shown in figure (4), designed in MatLab, the output of GPS receiver are directly comparable with the INS output (i.e. position outputs in North, East, and vertical frame) with errors shown is sections 4.3 and 5.1, respectively.

First we assumed that the GPS and INS provide position in every one second, figure (5) shows the position of the vehicle in (North, East, and vertical) frame without noise.

For three level decompositions, the error was smoother and the altitude errors are more than the North and East errors, as shown in figure (6). Where this is the optimal error estimation using thresholding technique described in [10]. Where the slandered deviation of the estimated error for X, Y, and Z axis in earth centered-earth fixed frame (ECEF) reduced to 1.6765 m, 1.6992 m, and 4.5394m, respectively.

On the other hand, figure (7) shows the simulation performance for high-speed dynamic system, where we assumed the GPS provide position in every one second, while the INS data is available in every 0.1 second and the spline method (described in [12]) used to predict the required values (positions) between instants of the GPS receiver. The slandered deviation of the estimated error for X, Y, and Z axis in ECEF-frame become 10.1635 m, 10.4999 m, and 10.8429m, respectively.

Form figure (7), the error was increased and modeling of such error becomes more complicated.

8. <u>Conclusions</u>

This paper offers a new method for error estimation in an INS/GPS augmented system. Where, the wavelet analysis was beneficial in filtering out some of the noise components and disturbances that may exist at the INS and GPS outputs. In addition, it provides the advantage of comparing the INS and GPS position components at different levels of resolution.

The spline algorithm was improved to implemented for exterpolation (rather than interpolation) and give relatively minimum error compared with linear and standered least square interpolation polynomials.

9. References

1. Lukman, N., “Low Cost INS-GPS Integration for Navigation (INS Software Implementation),” Commonwealth of Australia, 1996.
2. Wellenhof, B. H., H. Lichtenegger, and J. Collins, *GPS Theory and Practice*, Springer-Verlag/Vienna, 2001.
3. Leick, A., *GPS Satellite Surveying*, Wiley Interscience, 1992.
4. Ismaeel S. A., “ Design of Kalman Filter of Augmenting GPS to INS Systems,” Ph.D. Thesis, Computer Engineering Dept., College of Engineering, Al-Nahrain University, 2003.
5. Noureldin A., Osman A., and El-Sheimy N., “A Neuro-Wavelet Method for Multi-Sensor System Integration for Vehicular Navigation,” Measurement Science and Technology, 15, pp. 404-412, 2004.
6. Farrell J. and Barth M., *The Global Positioning System and Inertial Navigation*, McGraw-Hill Companies, Inc. 1999.
7. Park M.,” Error Analysis and Stochastic Modeling of MEMS based Inertial Sensors for Land Vehicle Navigation Applications,” M.Sc. Thesis, Department of Geometric Engineering, University of Calgary, April 2004.
8. Ismaeel S. A., and Aziez S. A.,”Development of a Real-Time Software for Strapdownm Celestial INS Algorithm,” Iraqi Journal of Informatics, 27 Annum,pp.36-44, 2004.
9. Khadim A. A., “Application of Analytical Methods to Improve the Accuracy of Strapdown Navigation System,” M.Sc. Thesis, College of Engineering, University of Baghdad, 2000.
10. M. Misite, Y. Misite, G. Oppenheim,and J.M. Poggi, “Wavelet Toolbox:Computation, Visualization, and Programming”,User’s Guide, version 2, Mathworks, Inc.,2002.
11. Mahmoud W. A. and Hassasn A. M.,” Gray-Scale Image Denoising Using Wavelet Network,” Iraqi Journal of Informatics, 27 Annum,pp.18-26, 2004.
12. Gerald C. F. and Eheatly P. O., *Applied Numerical Analysis*, 3rd Edition, Addison-Wesly Publishing Company, Inc. 1988.

Table (1): GPS Errors due to noise sources

Error	***Standared deviation (m)***
Common mode	
Ionosphere	0.7
Clock and ephemers	3.6
Troposphere	0.7
Non common mode	
Receiver noise	0.1-0.7
Multipath	0.1-3.0

Table (2): Gyros and Accelerometers errors

Gyro Errors	***Accelerometer Errors***
Constant = $12^0/h$	Bias = $0.001\times g$
g dependent = $2^0/h/g$	Scale factor = 5%
Random = $10^0/h$	Random = $2g\times 10^{-5}$

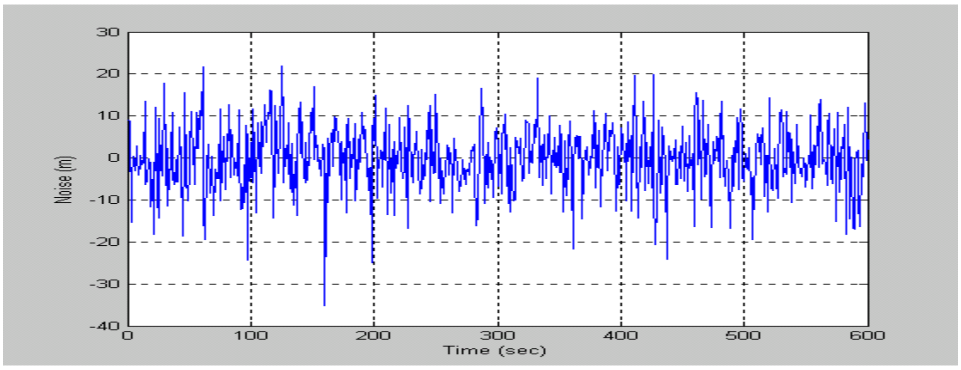


Figure (1): GPS noise errors

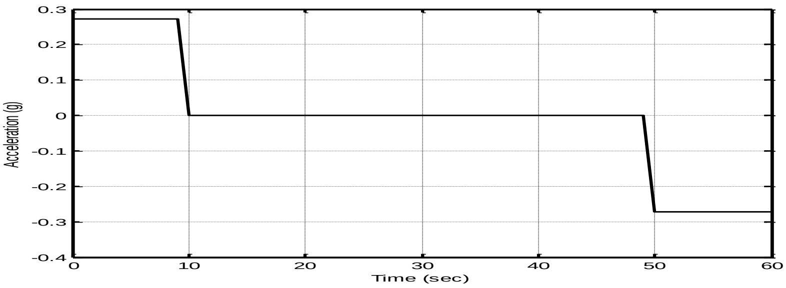


Figure (2): Acceleration for a moving vehicle

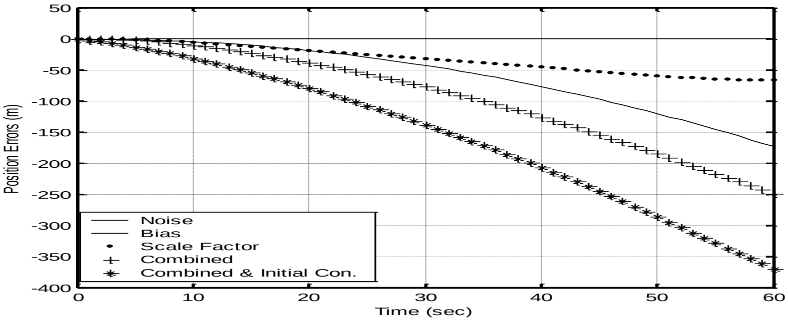


Figure (3): Effect of one-degree INS sensor errors on the position of the moving vehicle

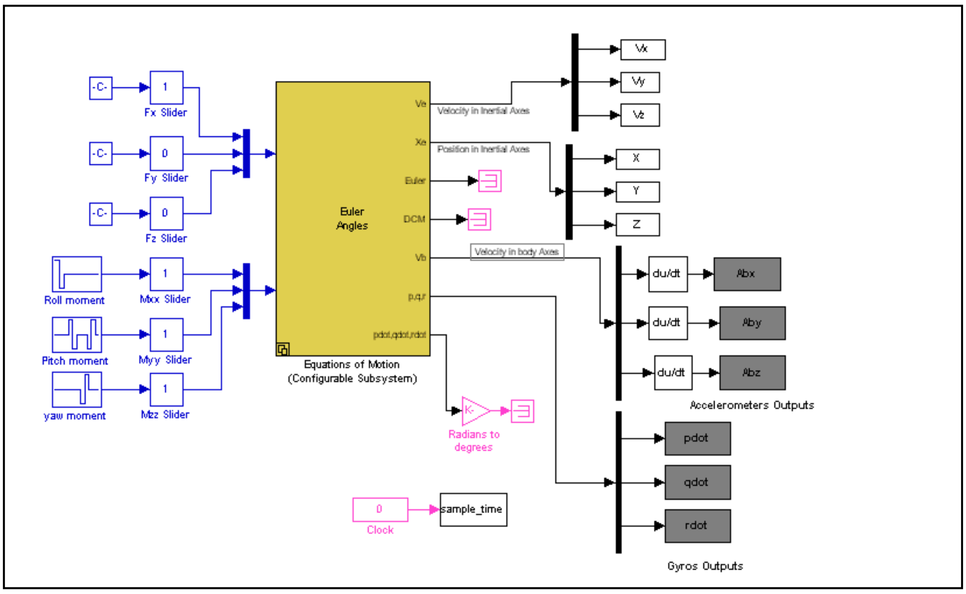
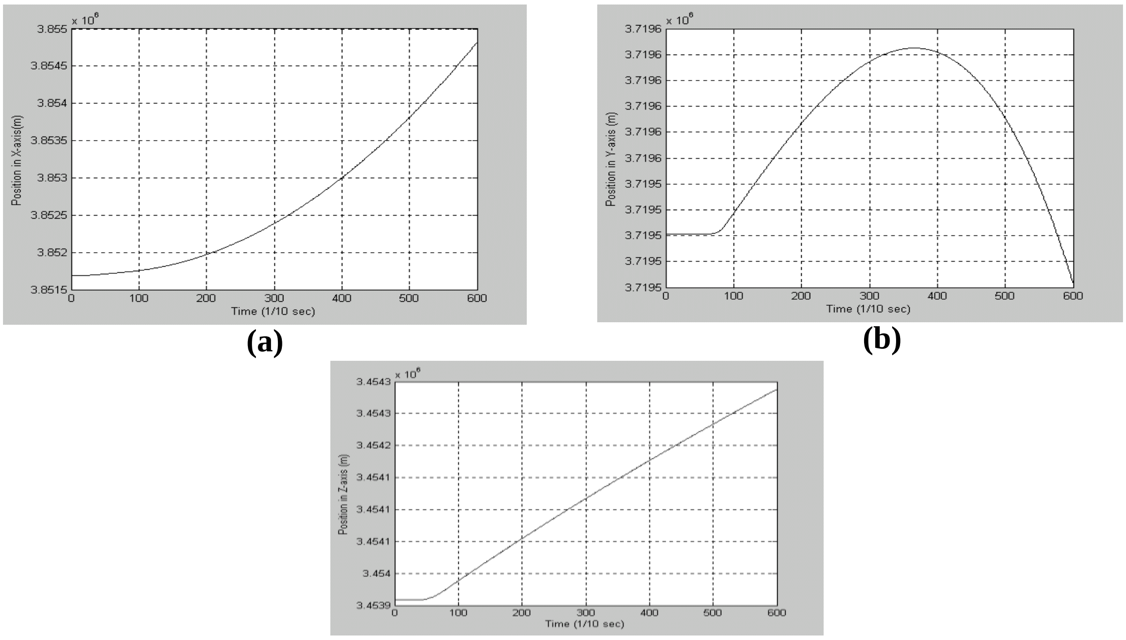


Figure (4): Six degree of freedom (6DoF) Simulation



(a) **(b)**

(c)

Figrue (5): Position of the vehicle in **(a)** North, **(b)** East, and **(c)** vertical frame

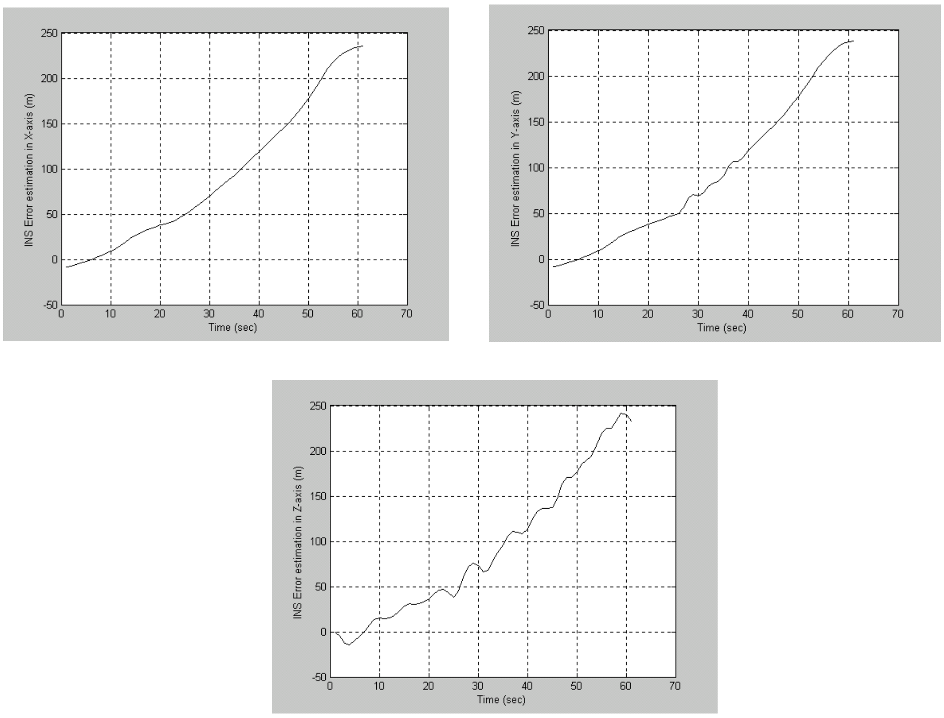
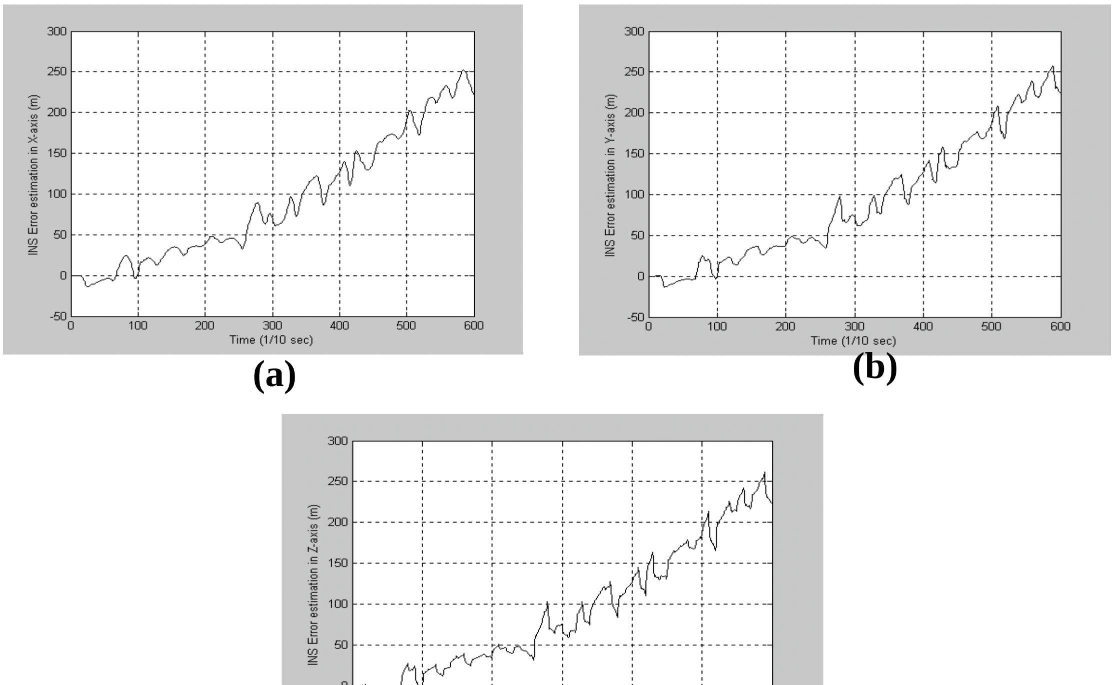


Figure (6): INS/GPS error signal with three level decomposition in (a) North, (b) East, and (c) vertical frame



(a) **(b)**

عنوان البحث: حساب خطأ المواقع في المنظومات المدمجة **INS/GPS** مستند على تحليلِ **Wavelet**

م. د. سلام عبد الرزاق اسماعيل*   أ. د. ولاء الدين خيري سعيد*   م. أحمد مظهر حسن*

الخلاصة:

المنظومات المتكاملة اعتمادا على منظومتي تحديد الموقع العالمي (GPS) ومنظومة الملاحة من النوع الثابت (SDINS) ولدت أهتماما واسعا خلال السنوات الأخيرة. حيث بلاستغلال الكلي لعمل الـ (GPS) أمكن بناء افضل ترابط بين منظومتي الملاحة والتي لها عدة فوائد على عمل كل منظومة ملاحة بشكل منفصل.

هذه البحث يهدف الى تَقديم نظام متكامل لدَمْج البيانات من (GPS) و(SDINS) بأستخدام الـ wavelet. حيث أستخدم الـ wavelet لمُقَارَنَة نواتجِ sensers في مستويات القرارِ المختلفةِ، لعِدّة أنواع من الأخطاءِ في النظامين، لصَقْل و التنبوء بمقدار الخطأ في منظومة (INS).

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*الجامعة التكنولوجية

Dr. Salam A. Ismaeel
Informatics Institute for Postgraduate Studies
University of Technology
Baghdad, Iraq
Email: Salam_ismaeel@Yahoo.com

Prof. Dr. Waladin K. Sa'id
Control and System Dept.
University of Technology
Baghdad, Iraq
Email: UOT_CONTROL@hotmail.com

Ahmed M. Hassan
Control and System Dept.
University of Technology
Baghdad, Iraq
Email: ahmedmudher@Yahoo.com

الباحثين

١. م. د. سلام عبد الرزاق اسماعيل / دكتوراه هندسة حاسبات / مدرس/ معهد المعلوماتية للدراسات العليا/ الجامعة التكنولوجية / معاون العميد

٢. أ. د. ولاء الدين خيري سعيد / دكتوراه هندسة سيطرة / أستاذ / قسم هندسة السيطرة والنظم/الجامعة التكنولوجية/ رئيس قسم هندسة السيطرة والنظم

٣. م. أحمد مظهر حسن / بكلوريوس هندسة حاسبات/مهندس/ قسم هندسة السيطرة والنظم/الجامعة التكنولوجية