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Some Classes in Ideal Topological Spaces

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Abstract

This paper introduces new classes of open sets defined in ideal topological space. These classes, namely: $i - I -$open, weakly $i - I -$open, $ii - I - open$, and weakly $ii - I -$open. In addition to that, the current study gives new concepts of continuity of mapping between ideal topological spaces using these classes. It compares them with other classes of nearly open sets in ideal topological spaces. We prove that all open sets $\alpha - I -$ open, $semi - I - open$, $ii - I - open$, $weakly\ semi - I - open$ and weakly $ii - I -$open sets are weakly $i - I -$open for any ideal topological space. Additionally, we show that all $\alpha - I -$ continuous, semi $-I -$ continuous, and $ii - I -$ continuous mappings are $i - I -$ continuous. Finally, for ideal topological space $(\mathcal{M}, \mathcal{L}, I)$ and $D \subset \mathcal{M}$ satisfying $Int(D)^{\#} = Int(D)$, we show that the following statements are equal.:

1) D is open
2) D is $i - I - open$ and $D \cap H = Int(D)$ for some $H \in \mathcal{L} \setminus \{\mathcal{M}, \emptyset\}$
3) D is $semi - I - open$

Similarly, we show that the following statements are equal.:

1) D is a closed set
2) $(D \cap F) = cl(D)$ for some $F \in \mathcal{L}^c$
3) D is $semi - I - closed$

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Introduction

Kuratowski [6] and Vaidyanathaswamy [9] were the first to study the notion of ideal topological spaces. Jankovic and Hamlett [5] investigated further characteristics of perfect topological spaces in 1990. The classes $\boldsymbol{i - open}$ *and* $\boldsymbol{ii - open}$ *sets which introduced recently by Mohammed –Askandar and Abdullah –Mohammed in [2] and [1], respectively. Given the ideal topological space* $(\boldsymbol{\mathcal{M}}, \boldsymbol{\mathcal{L}}, \boldsymbol{I})$, *we define* $\boldsymbol{i - I - open}$ *and* $\boldsymbol{ii - I - open}$ *sets via* $\boldsymbol{i - open}$ *and* $\boldsymbol{ii - open}$. *The most important result is that for ideal Topological spaces, denoted by* $(\boldsymbol{\mathcal{M}}, \boldsymbol{\mathcal{L}}, \boldsymbol{I})$ *all* $\boldsymbol{\alpha - I - open}$ *sets are* $\boldsymbol{i - I - open}$ *and* $\boldsymbol{ii - I - open}$. *Further, all* $\boldsymbol{semi - I - open}$ *sets are* $\boldsymbol{i - I - open}$ *and* $\boldsymbol{ii - I - open}$, *and again, we prove that for ideal Topological spaces* $(\boldsymbol{\mathcal{M}}, \boldsymbol{\mathcal{L}}, \boldsymbol{I})$ *all* $\boldsymbol{i - I - open\ set}$ $\boldsymbol{are\ weakly\ i - I - open\ and\ ii - I - open\ set}$ *is* $\boldsymbol{weakly\ ii - I - open}$. *With a discussion of* $\boldsymbol{i - I - continuity}$ *and* $\boldsymbol{weakly\ i - I - continuity}$.

Definition 1.1. [**6**]

An ideal I is a nonempty set of a topological space $(\boldsymbol{\mathcal{M}}, \boldsymbol{\mathcal{L}})$ *is a collection of a subset of* $\boldsymbol{\mathcal{M}}$ *which fulfills*

1) $D \in I$ *and* $B \subseteq D$ *implies* $B \in I$,
2) $D \in I$ *and* $B \in I$ *implies* $D \cup B \in I$.

Definition 1.2.

Let $(\mathcal{M}, \mathcal{L}, I)$ *be ideal Topological spaces and let* $m \epsilon \mathcal{M}$. *Then a local function is a mapping from a family of a subset of* $\mathcal{M}$ *into itself, denoted by* $(D)^{\#}(I, \mathcal{L}) = \{m \in \mathcal{M}: U \cap D \notin I \text{ for each } U \in \mathcal{L}(m)\}$ *where* $D \subseteq \mathcal{M}$, *and* $\mathcal{L}(m) = \{U \in \mathcal{L}: m \in U\}$. *A Kuratowski closure operator* $cl^{\#}$ *is defined by* $cl^{\#}(D) = D \cup D^{\#}\ (I, \mathcal{L})$ *Note that we will write* $D^{\#}$ *for* $D^{\#}(I, \mathcal{L})$.

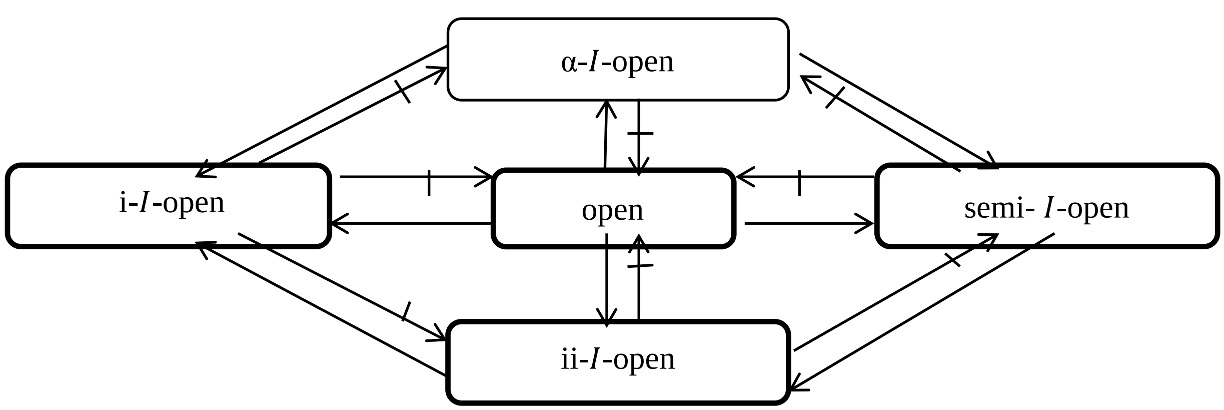
Definition 1.3.

A subset D of Topological spaces $(\mathcal{M}, \mathcal{L})$ *is called to be*
$a)$ $\alpha - open$ [8] $if\ D \subset Int(cl(Int(D)))$
$b)$ $semi - open$ [7] $is\ D \subset cl(Int(D))$
For a subset D of I.T.S. $(\mathcal{M}, \mathcal{L}, I)$ *is called to be*
$c)$ $\alpha - I - open$ [4] $if\ D \subset Int(cl^{\#}(Int(D)))$
$d)$ $semi - I - open$ [4] $is\ D \subset cl^{\#}(Int(D))$

Definition 1.4.

A subset D of ideal Topological spaces $(\mathcal{M}, \mathcal{L}, I)$ *is* $i - I - open$ *if there exists* $H \in \mathcal{L} \setminus \{\mathcal{M}, \emptyset\}$ *such that* $D \subset cl^{\#}(D \cap H)$. *The complement of* $i - I - open\ set$ *is* $i - I - closed$.
DEFINITION 1.5. A subset D of ideal Topological spaces $(\mathcal{M}, \mathcal{L}, I)$ *is* $ii - I - open$ *if there exist* $H \in \mathcal{L} \setminus \{\mathcal{M}, \emptyset\}$ *such that* D *is* $i - I - open$ *and* $Int(D) = H$. *The complement of* $ii - I - open$ *is* $ii - I - closed$.
Let $(\mathcal{M}, \mathcal{L}, I)$ *be ideal Topological spaces Then*
Theorem 1.1. Every open set is $i - I - open$.
Proof. For any open set H, *we have* $H \subset cl^{\#}(H \cap H) = cl^{\#}(H)$.
$This\ implies\ that$ H *is* $i - I - open$. ■
Theorem 1.2. Let $(\mathcal{M}, \mathcal{L}, I)$ *ideal Topological spaces If* $D \subset H \in \mathcal{L}$. *Then* D *is* $i - I -$*open.*
Proof. Let $D \subset H \in \mathcal{L}. \rightarrow D \cap H = D$. *Then* $D \subset cl^{\#}(D \cap H)$. *Therefore* D *is* $i - I -$*open.* ■

Theorem 1.3. Each open set is $ii - I - open$.
Proof. For any open set H, *we have* $H \subset cl^{\#}(H \cap H) = \subset cl^{\#}(H)$, *and* $Int(H) = H$. *This means that* H *is* $ii - I - open$.
Theorem 1.4. Every $\alpha - I - open$ *set is* $ii - I - open$.
Proof. Let $D \subset \mathcal{M}$ *be* $\alpha - I - open$. *We have* $D \subset Int\left(cl^{\#}(Int(D))\right)$. *This implies that* $D \subset cl^{\#}(Int(D))$. *Now if* $Int(D) = \emptyset$. *There is nothing to prove, so* $Int(D) \neq \emptyset$. *Therefore, put* $Int(D) = H$ *for some* $H \in \mathcal{L}$. *Hence* $D \subset cl^{\#}(Int(D)) \subset cl^{\#}(D \cap H)$. $That\ is\ D\ ii - I - open$. ■
Theorem 1.5. Each $semi - I - open$ *set is* $ii - I - open$
Proof. Direct. ■
From the above, we get the following Diagram:



2. Weakly Semi-I-open, Weakly i-I-open and Weakly ii-I-open

Definition 2.1.[3]

A subset D of I.T.S. $(\mathcal{M},\mathcal{L},I)$ *is said to be a weakly* $\boldsymbol{semi - I - open}$ *set if* $D \subset cl^{\#}(Int(cl(D)))$.
Remark 2.1.[3] Each semi-I-open set is weakly semi-I-open, but the converse is not true in general as shown in the following example
Example 1 Let $\mathcal{M} = \{a, b, c\}$, $\mathcal{L} = \{\mathcal{M}, \emptyset, \{a, b\}\}$ *and* $I = \{\emptyset, \{c\}\}$. *Then* $D = \{a\}$ *is weakly* $\boldsymbol{mi - I - open}$ *, but not* $\boldsymbol{semi - I - open}$.
DEFINITION 2.2. Let D $\in \mathcal{M}$ *of I.T.S.* $(\mathcal{M},\mathcal{L},I)$ *be said to be* $\boldsymbol{weakly\ i - I - open}$ *if there exists an open set* $H \neq \{\mathcal{M}, \emptyset\}$ *such that* $H \subset cl^{\#}\big((cl(D \cap H)\big)$.
DEFINITION 2.3. Let D $\in \mathcal{M}$ *of I.T.S.* $(\mathcal{M},\mathcal{L},I)$ *be said to be* $\boldsymbol{weakly\ ii - I - open}$ *if there exists an open set* $H \neq \{\mathcal{M}, \emptyset\}$ *where* $\boldsymbol{D\ is\ weakly - i - I - open\ \ and\ Int(D}$
$) = H$.
Theorem 2.1. Let $(\mathcal{M},\mathcal{L},I)$ *ideal Topological spaces Then every* $\boldsymbol{open}$ *set is* $\boldsymbol{akly\ i - I - open}$ *, but the converse is not true.*
Proof. Let $H \in \mathcal{L} \setminus \{\mathcal{M}, \emptyset\}$, *we have* $H \subset cl^{\#}\big((H \cap H)\big) \subset cl^{\#}\big((cl(H)\big)$. *This means* H *is* $\boldsymbol{weakly\ i - I - open}$. ■

Theorem 2.2. Let $(\mathcal{M},\mathcal{L},I)$ *ideal Topological spaces Then every* $\boldsymbol{i - I - open}$ *set is* $\boldsymbol{akly\ i - I - open}$ *, but the converse is not true*
Proof. let $D \subset \mathcal{M}$ *be* $\boldsymbol{i - I - open}$ *set.* $\boldsymbol{For\ some}$ $H \in \mathcal{L} \setminus \{\mathcal{M}, \emptyset\}$, *we have* $D \subset cl^{\#}\big((D \cap H)\big) \subset cl^{\#}\big((cl(D \cap H)\big)$. *This means* D *is* $\boldsymbol{weakly\ i - I - open}$. ■
Example 2
Let $\mathcal{M} = \{a, b, c\}$, $\mathcal{L} = \{\mathcal{M}, \emptyset, \{a\}, \{a, b\}\}$ *and* $I = \{\emptyset, \{b\}\}$. *Then* $D = \{b, c\}$ *is* $\boldsymbol{weakly\ i - I - open}$, *however, not* $\boldsymbol{i - I - open}$.
Theorem 2.3. Let $(\mathcal{M},\mathcal{L},I)$ *be ideal Topological spaces Then every* $\boldsymbol{ii - I - open}$ *set is* $\boldsymbol{weakly\ ii - I - open}$, *but not conversely.*
Proof. Direct. ■
Example 3
$\mathcal{M} = \{a, b, c\}$, $\mathcal{L} = \{\mathcal{M}, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ *and* $I = \{\emptyset, \{a\}, \{c\}, \{a, c\}\}$. *Then* $D = \{a, c\}$ *is* $\boldsymbol{weakly\ ii - I - open}$, *but is not* $\boldsymbol{ii - I - open}$.
Corollary 2.1. Let $(\mathcal{M},\mathcal{L},I)$ *be ideal Topological spaces Then every* $\boldsymbol{weakly\ ii - I - open}$ *set is* $\boldsymbol{weakly\ i - I - open}$ *, but not conversely.*
Proof. Direct. ■
Corollary 2.2. Let $(\mathcal{M},\mathcal{L},I)$ *be ideal Topological spaces Then each* $\boldsymbol{open}$ *set is* $\boldsymbol{weakly\ ii - I - open}$ *& i* $\boldsymbol{- I - open}$ *, but not conversely.*
Proof. Direct. ■
DEFINITION 2.4. Let $(\mathcal{M},\mathcal{L},I)$ *be ideal Topological spaces and* (Y, σ) *be T.S. A map* $f: (\mathcal{M},\mathcal{L},I) \to (Y, \sigma)$ *is to be* $\boldsymbol{i - I - continuous}$ *if* $f^{-1}(V)$ *is* $\boldsymbol{i - I - open}$ *in* $(\mathcal{M},\mathcal{L},I)$ *for every open set* V *of* (Y, σ).
Theorem 2.4. Let $(\mathcal{M},\mathcal{L},I)$ *be ideal Topological spaces and* (Y, σ) *be T.S. Then every continuous mapping from* $(\mathcal{M},\mathcal{L},I)$ *to* (Y, σ) *is* $\boldsymbol{i - I - continuous}$ *but the converse is not true.*
Proof. Let $U \in \sigma$. *Since* f *is continuous mapping, then* $f^{-1}(V)$ *is an open set in* $\mathcal{M}$, *by Theorem 1.1.* $f^{-1}(V)$ *is an* $\boldsymbol{i - I - open}$ *in* $\mathcal{M}$. *Hence* f *is* $\boldsymbol{i - I - continuous}$. ■
Example 4
Let $\mathcal{M} = \{a, b, c\}$, $\mathcal{L} = \{\mathcal{M}, \emptyset, \{a\}, \{a, b\}\}$ *and* $I = \{\emptyset, \{a\}\}$. *Then* $Y = \{1, 2, 3\}$, $\sigma = \big\{Y, \emptyset, \{1\}, \{2\}, \{1, 2\}\big\}$ *And Let f:* $(\mathcal{M},\mathcal{L},I) \to (Y, \sigma)$ *such that* $f(a) = 1, f(b) = 2, f(c) = 3$. *Then,* $\boldsymbol{f\ is\ i - I - continuous}$, *but* f *is not* $\boldsymbol{continuous}$ *mapping.*
DEFINITION 2.5. Let $(\mathcal{M},\mathcal{L},I)$ *be ideal Topological spaces and* (Y, σ) *be T.S. Then A map* $f: (\mathcal{M},\mathcal{L},I) \to (Y, \sigma)$ *is to be* $\boldsymbol{weakly\ i - I - continuous}$ *if* $f^{-1}(V)$ *is* $\boldsymbol{weakly\ i - I - open}$ *in* $(\mathcal{M},\mathcal{L},I)$ *for every open set* V *of* (Y, σ).
Theorem 2.5. Let $(\mathcal{M},\mathcal{L},I)$ *be ideal Topological spaces and* (Y, σ) *be T.S. Then every continuous mapping from* $(\mathcal{M},\mathcal{L},I)$ *to* (Y, σ) *is* $\boldsymbol{weakly\ i - I - continuous}$ *but the converse is not true.*

Proof. Let $\boldsymbol{f}: (\boldsymbol{\mathcal{M}}, \boldsymbol{\mathcal{L}}, \boldsymbol{I}) \to (\boldsymbol{Y}, \boldsymbol{\sigma})$ a continuous mapping and let $\boldsymbol{V}$ an open set in $\boldsymbol{Y}$**.** Since $\boldsymbol{f}$ continuous, then $\boldsymbol{f^{-1}}(\boldsymbol{V})$ is an open set in $\boldsymbol{\mathcal{M}}$ from theorem1.1 and theorem2.1 $\boldsymbol{f}$ **is weakly** $\boldsymbol{i-I}$ −continuous. ■

Example 5

Let $\boldsymbol{\mathcal{M}} = \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$, $\boldsymbol{\mathcal{L}} = \{\boldsymbol{\mathcal{M}}, \emptyset, \{\boldsymbol{a}, \boldsymbol{b}\}\}$ and $\boldsymbol{I} = \{\emptyset, \{\boldsymbol{a}\}\}$, $\boldsymbol{Y} = \{\boldsymbol{1}, \boldsymbol{2}, \boldsymbol{3}\}$, $\boldsymbol{\sigma} = \{\boldsymbol{Y}, \emptyset, \{\boldsymbol{1}\}, \{\boldsymbol{1}, \boldsymbol{2}\}\}$, $\boldsymbol{f}: (\boldsymbol{\mathcal{M}}, \boldsymbol{\mathcal{L}}, \boldsymbol{I}) \to (\boldsymbol{Y}, \boldsymbol{\sigma})$, such that $\boldsymbol{f}(\boldsymbol{a}) = \boldsymbol{1}$, $\boldsymbol{f}(\boldsymbol{b}) = \boldsymbol{2}$, $\boldsymbol{f}(\boldsymbol{c}) = \boldsymbol{3}$, $\boldsymbol{f^{-1}}(\boldsymbol{1}) = \{\boldsymbol{a}\} \notin \boldsymbol{\mathcal{L}}$**.** Then $\boldsymbol{f}$ is not continuous, but $\{\boldsymbol{a}\} \cap \{\boldsymbol{a}, \boldsymbol{b}\} = \{\boldsymbol{a}\}$ **and** $\boldsymbol{cl}(\{\boldsymbol{a}\}) = \boldsymbol{\mathcal{M}}$, $\{\boldsymbol{a}\} \subset \boldsymbol{cl^*}(\boldsymbol{cl}(\{\boldsymbol{a}\} \cap \{\boldsymbol{a}, \boldsymbol{b}\})$ **and** $\boldsymbol{f^{-1}}(\{\boldsymbol{1}, \boldsymbol{2}\}) = \{\boldsymbol{a}, \boldsymbol{b}\} \in \boldsymbol{\mathcal{L}}$. Then $\boldsymbol{f}$ **is weakly** $\boldsymbol{i-I-}$**continuous**, but $\boldsymbol{f}$ is not **continuous** mapping.

Theorem 2.6. Let $(\boldsymbol{\mathcal{M}}, \boldsymbol{\mathcal{L}}, \boldsymbol{I})$ be ideal Topological spaces and $(\boldsymbol{Y}, \boldsymbol{\sigma})$ be T.S. $f: (\boldsymbol{\mathcal{M}}, \boldsymbol{\mathcal{L}}, \boldsymbol{I}) \to (\boldsymbol{Y}, \boldsymbol{\sigma})$ is $\boldsymbol{i-I-}$**continuous**. **Then** $\boldsymbol{f}$ **is weakly** $\boldsymbol{i-I-}$**continuous**, but the converse is not true.

Proof. Let $\boldsymbol{f}: (\boldsymbol{\mathcal{M}}, \boldsymbol{\mathcal{L}}, \boldsymbol{I}) \to (\boldsymbol{Y}, \boldsymbol{\sigma})$ be an $\boldsymbol{i-I-}$**continuous** mapping and let $\boldsymbol{V}$ be an open set in $\boldsymbol{Y}$**.** Since $\boldsymbol{f}$ is $\boldsymbol{i-I-}$**continuous**, then $\boldsymbol{f^{-1}}(\boldsymbol{V})$ is $\boldsymbol{i-I-}$**open** in $\boldsymbol{\mathcal{M}}$**.** From theorem 2.1 $\boldsymbol{f^{-1}}(\boldsymbol{V})$ **weakly** $\boldsymbol{i-I-}$**open**. So $\boldsymbol{f}$ is **weakly** $\boldsymbol{i-I-}$**continuous**. ■

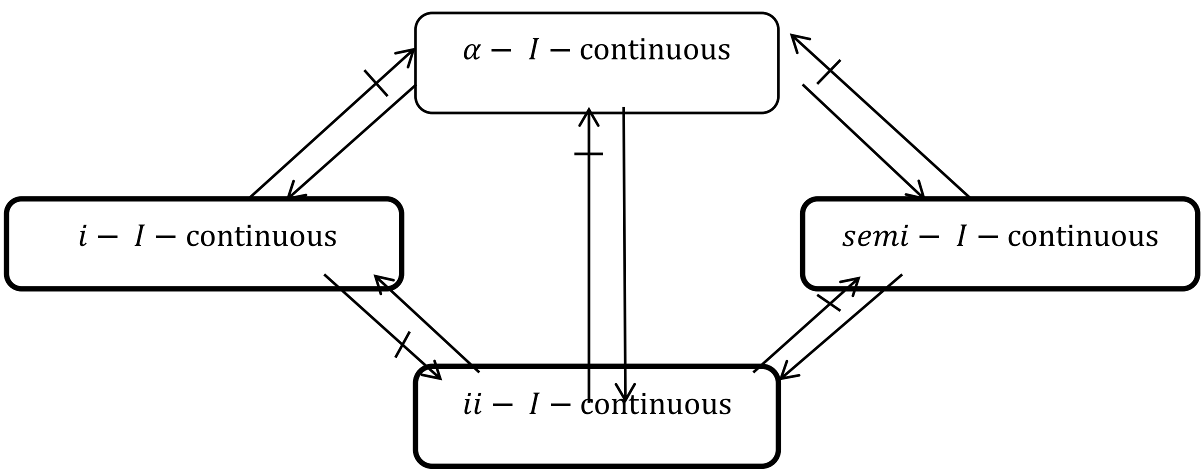
Example 6

Let $\boldsymbol{\mathcal{M}} = \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$, $\boldsymbol{\mathcal{L}} = \{\boldsymbol{\mathcal{M}}, \emptyset, \{\boldsymbol{a}\}\}$ and $\boldsymbol{I} = \{\emptyset, \{\boldsymbol{a}\}\}$. $\boldsymbol{Y} = \{\boldsymbol{1}, \boldsymbol{2}, \boldsymbol{3}\}$, $\boldsymbol{\sigma} = \{\boldsymbol{Y}, \emptyset, \{\boldsymbol{1}\}, \{\boldsymbol{1}, \boldsymbol{2}\}\}$ $\boldsymbol{\sigma}$, $\boldsymbol{f}: (\boldsymbol{\mathcal{M}}, \boldsymbol{\mathcal{L}}, \boldsymbol{I}) \to (\boldsymbol{Y}, \boldsymbol{\sigma})$, such that $\boldsymbol{f}(\boldsymbol{a}) = \boldsymbol{1}, \boldsymbol{f}(\boldsymbol{b}) = \boldsymbol{2}, \boldsymbol{f}(\boldsymbol{c}) = \boldsymbol{3}$. Then $\boldsymbol{f^{-1}}(\boldsymbol{1}) = \{\boldsymbol{a}\} \in \boldsymbol{\mathcal{L}}$ **and** $\boldsymbol{f^{-1}}(\{\boldsymbol{1}, \boldsymbol{2}\}) = \{\boldsymbol{a}, \boldsymbol{b}\} \notin \boldsymbol{\mathcal{L}}$, $\{a\}\cap\{a,b\}=\{a\}$, then $\boldsymbol{cl^*}(\{\boldsymbol{a}\}) = \{\boldsymbol{a}\} \cup \{\boldsymbol{a}\}^* = \{\boldsymbol{a}\}$ (since $\{\boldsymbol{a}\}^* = \emptyset$). Therefore $\{\boldsymbol{a}, \boldsymbol{b}\} \not\subset \boldsymbol{cl^*}(\{\boldsymbol{a}\} \cap \{\boldsymbol{a}, \boldsymbol{b}\})$, but $\boldsymbol{cl}(\{\boldsymbol{a}\}) = \boldsymbol{X}$, **then** $\{\boldsymbol{a}, \boldsymbol{b}\} \subset \boldsymbol{cl^*}(\boldsymbol{cl}(\{\boldsymbol{a}\} \cap \{\boldsymbol{a}, \boldsymbol{b}\}))$. Then $\boldsymbol{f}$ **is weakly** $\boldsymbol{i-I-}$**continuous**, but $\boldsymbol{f}$ is not $\boldsymbol{i-I-}$**continuous**.

Theorem 2.7. Let $(\boldsymbol{\mathcal{M}}, \boldsymbol{\mathcal{L}}, \boldsymbol{I})$ be ideal Topological spaces and $(\boldsymbol{Y}, \boldsymbol{\sigma})$ be T.S. $f: (\boldsymbol{\mathcal{M}}, \boldsymbol{\mathcal{L}}, \boldsymbol{I}) \to (\boldsymbol{Y}, \boldsymbol{\sigma})$ is $\boldsymbol{\alpha-I-}$**continuous**. Then f is **semi** $\boldsymbol{-I-}$**continuous** , $\boldsymbol{ii-I-}$**continuous**, and $\boldsymbol{i-I-}$**continuous**.

Proof. Let $\boldsymbol{f}: (\boldsymbol{\mathcal{M}}, \boldsymbol{\mathcal{L}}, \boldsymbol{I}) \to (\boldsymbol{Y}, \boldsymbol{\sigma})$ be $\boldsymbol{\alpha-I-}$**continuous**. Then $\boldsymbol{f^{-1}}(\boldsymbol{V})$ **is** $\boldsymbol{\alpha-I-}$**open** for every open set $\boldsymbol{V}$ of $(\boldsymbol{Y}, \boldsymbol{\sigma})$). This mean $\boldsymbol{f^{-1}}(\boldsymbol{V})$ is $\boldsymbol{mi-I-}$**open** . Then $\boldsymbol{f}$ is **semi** $\boldsymbol{-I-}$**continuous**. By theorem.1.4. we have $\boldsymbol{f^{-1}}(\boldsymbol{V})$ is $\boldsymbol{ii-I-}$**open** set. Therefore $\boldsymbol{f}$ is $\boldsymbol{ii-I-}$**continuous**. Thus $\boldsymbol{f}$ $\boldsymbol{i-I-}$**continuous**. ■

From the previous theorem, we have



Definition 2.6.

A subset $\boldsymbol{D} \subset \boldsymbol{\mathcal{M}}$ *is called* $\# - \boldsymbol{Int} - \boldsymbol{perfect}$ *if* $(\boldsymbol{Int}(\boldsymbol{D}))^{\#} = \boldsymbol{Int}(\boldsymbol{D})$
Theorem 2.8. Let $\boldsymbol{D} \subset \boldsymbol{\mathcal{M}}$ *of ideal Topological spaces* $(\boldsymbol{\mathcal{M}}, \boldsymbol{\mathcal{L}}, \boldsymbol{I})$ *and* $\boldsymbol{D}$ $\boldsymbol{be}$ $\# - \boldsymbol{Int} - \boldsymbol{perfect}$. *Then, the latter statement is equivalent.*
1) $\boldsymbol{D}$ *is open*
2) $\boldsymbol{D}$ $\boldsymbol{is}$ $\boldsymbol{i} - \boldsymbol{I} - \boldsymbol{open}$ $\boldsymbol{and}$ $\boldsymbol{D} \cap \boldsymbol{H} = \boldsymbol{Int}(\boldsymbol{D})$ $\boldsymbol{for}$ $\boldsymbol{some}$ $\boldsymbol{H} \in \boldsymbol{\mathcal{L}} \setminus \{\boldsymbol{\mathcal{M}}, \emptyset\}$
3) $\boldsymbol{D}$ *is* $\boldsymbol{semi} - \boldsymbol{I} - \boldsymbol{open}$
Proof. $[\mathbf{1} \Rightarrow \mathbf{2}]$. *Direct*
$[\mathbf{2} \Rightarrow \mathbf{3}]$
Let $\boldsymbol{D}$ *be* $\boldsymbol{i} - \boldsymbol{I} - \boldsymbol{open}$ *and* $\boldsymbol{D} \cap \boldsymbol{H} = \boldsymbol{Int}(\boldsymbol{D})$. *We have* $\boldsymbol{D} \subset \boldsymbol{cl}^{\#}(\boldsymbol{D} \cap \boldsymbol{H}) = \boldsymbol{cl}^{\#}\big(\boldsymbol{Int}(\boldsymbol{D})\big)$. *Therefore* $\boldsymbol{D}$ $\boldsymbol{is}$ $\boldsymbol{semi} - \boldsymbol{I} - \boldsymbol{open}$ $\boldsymbol{set}$.
$[\mathbf{3} \Rightarrow \mathbf{1}]$
Let $\boldsymbol{D}$ $\boldsymbol{be}$ $\boldsymbol{semi} - \boldsymbol{I} - \boldsymbol{open}$. *We have* $\boldsymbol{D} \subset \boldsymbol{cl}^{\#}\big(\boldsymbol{Int}(\boldsymbol{D})\big) = \boldsymbol{Int}(\boldsymbol{D}) \cup \big(\boldsymbol{Int}(\boldsymbol{D})\big)^{\#} = \boldsymbol{Int}(\boldsymbol{D}) \cup \boldsymbol{Int}(\boldsymbol{D}) = \boldsymbol{Int}(\boldsymbol{D})$. $\boldsymbol{Therefore}$ $\boldsymbol{D}$ $\boldsymbol{is}$ $\boldsymbol{an}$ $\boldsymbol{open}$ $\boldsymbol{set}$. ■
Example 7
Let $\boldsymbol{\mathcal{M}} = \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$, $\boldsymbol{\mathcal{L}} = \{\boldsymbol{\mathcal{M}}, \emptyset, \{\boldsymbol{a}\}\}$ *and* $\boldsymbol{I} = \{\emptyset, \{\boldsymbol{a}\}\}$.
$\boldsymbol{D} = \{\boldsymbol{a}\}$ *is an open set. Then* $\boldsymbol{D}$ *is* $\boldsymbol{i} - \boldsymbol{I} - \boldsymbol{open}$ *and* $\boldsymbol{D} \cap \boldsymbol{D} = \boldsymbol{D} = \boldsymbol{Int}(\boldsymbol{D})$. *Therefore* $\boldsymbol{D}$ *is* $\boldsymbol{semi} - \boldsymbol{I} - \boldsymbol{open}$.
Theorem 2.9. Let $(\boldsymbol{\mathcal{M}}, \boldsymbol{\mathcal{L}}, \boldsymbol{I})$ *be ideal Topological spaces and* $(\boldsymbol{Y}, \boldsymbol{\sigma})$ *be T.S. let* $\boldsymbol{f}: (\boldsymbol{\mathcal{M}}, \boldsymbol{\mathcal{L}}, \boldsymbol{I}) \rightarrow (\boldsymbol{Y}, \boldsymbol{\sigma})$ *and* $\boldsymbol{D}$ $\boldsymbol{are}$ $\# - \boldsymbol{Int} - \boldsymbol{perfect}$ *, the following conditions are equivalent.*
1) $\boldsymbol{f}$ *is* $\boldsymbol{continuous}$
2) $\boldsymbol{f}$ $\boldsymbol{is}$ $\boldsymbol{i} - \boldsymbol{I} - \boldsymbol{continuous}$ *and* $\boldsymbol{f}$ *is satisfy* $\boldsymbol{f}^{-1}(\boldsymbol{V}) \cap \boldsymbol{H} = \boldsymbol{Int}\big(\boldsymbol{f}^{-1}(\boldsymbol{V})\big)$ *for some* $\boldsymbol{H} \in \boldsymbol{\mathcal{L}} \setminus \{\boldsymbol{\mathcal{M}}, \emptyset\}$
3) $\boldsymbol{f}$ $\boldsymbol{is}$ $\boldsymbol{semi} - \boldsymbol{I} - \boldsymbol{continuous}$
Proof. $[\mathbf{1} \Rightarrow \mathbf{2}]$. *Clear*
$[\mathbf{2} \Rightarrow \mathbf{3}]$
Let $\boldsymbol{V} \subset \boldsymbol{Y}$ *since* $\boldsymbol{f}$ *is* $\boldsymbol{i} - \boldsymbol{I} - \boldsymbol{continuous}$ *it follows that* $\boldsymbol{f}^{-1}(\boldsymbol{V})$ *is* $\boldsymbol{i} - \boldsymbol{I} - \boldsymbol{open}$, $\boldsymbol{f}^{-1}(\boldsymbol{V}) \subset \boldsymbol{cl}^{\#}(\boldsymbol{f}^{-1}(\boldsymbol{V}) \cap \boldsymbol{H}) = \boldsymbol{cl}^{\#}(\boldsymbol{Int}(\boldsymbol{f}^{-1}(\boldsymbol{V})))$ *then* $\boldsymbol{f}^{-1}(\boldsymbol{V})$ *is* $\boldsymbol{semi} - \boldsymbol{I} - \boldsymbol{open}$. *Therefore* $\boldsymbol{f}$ $\boldsymbol{is}$ $\boldsymbol{semi} - \boldsymbol{I} - \boldsymbol{continuous}$.
$[\mathbf{3} \Rightarrow \mathbf{1}]$
Let $\boldsymbol{U} \in \boldsymbol{\sigma}$. *Since* $\boldsymbol{f}$ *is* $\boldsymbol{semi} - \boldsymbol{I} - \boldsymbol{continuous}$, *it follows that* $\boldsymbol{f}^{-1}(\boldsymbol{V})$ *is* $\boldsymbol{semi} - \boldsymbol{I} - \boldsymbol{open}$ *, so,* $\boldsymbol{f}^{-1}(\boldsymbol{V}) \subset \boldsymbol{cl}^{\#}\Big(\boldsymbol{Int}\big(\boldsymbol{f}^{-1}(\boldsymbol{V})\big)\Big) = \boldsymbol{Int}\big(\boldsymbol{f}^{-1}(\boldsymbol{V})\big)$. *Then* $\boldsymbol{f}^{-1}(\boldsymbol{V})$ *is open.* $\boldsymbol{Therefore}$ $\boldsymbol{f}$ *is* $\boldsymbol{continuous}$. ■
Theorem 2.10. For a subset $\boldsymbol{D}$ *of space* $(\boldsymbol{\mathcal{M}}, \boldsymbol{\mathcal{L}}, \boldsymbol{I})$ *and* $(\boldsymbol{Int}(\boldsymbol{D}))^{\#} = \boldsymbol{Int}(\boldsymbol{D})$
, the following conditions are equivalent.
1) $\boldsymbol{D}$ $\boldsymbol{is}$ $\boldsymbol{a}$ $\boldsymbol{closed}$ $\boldsymbol{set}$
2) $(\boldsymbol{D} \cap \boldsymbol{F}) = \boldsymbol{cl}(\boldsymbol{D})$ $\boldsymbol{for}$ $\boldsymbol{some}$ $\boldsymbol{F} \in \boldsymbol{\mathcal{L}}^{\boldsymbol{c}}$
3) $\boldsymbol{D}$ $\boldsymbol{is}$ $\boldsymbol{semi} - \boldsymbol{I} - \boldsymbol{closed}$
Proof. $[\mathbf{1} \Rightarrow \mathbf{2}]$. $\boldsymbol{Clear}$
$[\mathbf{2} \Rightarrow \mathbf{3}]$ $\boldsymbol{cl}^{\#}\big(\boldsymbol{Int}(\boldsymbol{D})\big) = \boldsymbol{Int}(\boldsymbol{D}) \subset \boldsymbol{D}$. $\boldsymbol{Thus}$ $\boldsymbol{D}$ $\boldsymbol{is}$ $\boldsymbol{semi} - \boldsymbol{I} - \boldsymbol{closed}$
$[\mathbf{3} \Rightarrow \mathbf{1}]$
$\boldsymbol{D}$ $\boldsymbol{is}$ $\boldsymbol{semi} - \boldsymbol{I} - \boldsymbol{closed}$. *Then* $\boldsymbol{D}^{\boldsymbol{c}}$ $\boldsymbol{is}$ $\boldsymbol{semi} - \boldsymbol{I} - \boldsymbol{open}$. *We have* $\boldsymbol{D}^{\boldsymbol{c}} \subset \boldsymbol{cl}^{\#}\big(\boldsymbol{Int}(\boldsymbol{D}^{\boldsymbol{c}})\big) = \boldsymbol{Int}(\boldsymbol{D}^{\boldsymbol{c}})$. *Therefore* $\boldsymbol{D}^{\boldsymbol{c}}$ *is an open set. It follows* $\boldsymbol{D}$ *is closed.* ■

Conclusions

This work concludes that each $\boldsymbol{open}$ *set is* $\boldsymbol{\alpha} - \boldsymbol{I} - \boldsymbol{open}$, $\boldsymbol{semi} - \boldsymbol{I} - \boldsymbol{open}$, $\boldsymbol{ii} - \boldsymbol{I} - \boldsymbol{open}$, *and* $\boldsymbol{i} - \boldsymbol{I} - \boldsymbol{open}$ *set. Furthermore, each* $\boldsymbol{\alpha} - \boldsymbol{I} - \boldsymbol{continuous}$ *mapping is* $\boldsymbol{semi} - \boldsymbol{I} - \boldsymbol{continuous}$ *,* $\boldsymbol{ii} - \boldsymbol{I} - \boldsymbol{continuous}$, *and* $\boldsymbol{i} - \boldsymbol{I} - \boldsymbol{continuous}$ *mapping.*

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Conflict of interest

The author has no conflict of interest.

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بعض الاصناف في الفضاءات التبولوجية المثالية

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الخلاصة

في هذه الدراسة , قدمنا اصناف جديدة من المجاميع المفتوحة في الفضاء التبولوجي المثالي تحديدا : مفتوحة من النمط $i-I$, مفتوحة من النمط (الضعيفة) $i-I$, مفتوحة من النمط $ii-I$ و مفتوحة من النمط (الضعيفة $ii-I$). وكذلك, اعطينا مصطلحات جديدة لاستمرارية التطبيق بين الفضاءات التبولوجية المثالية باستخدام تلك الأصناف ونقارن مع اصناف اخرى من المجموعات الشبه المفتوحة في الفضاءات التبولوجية المثالية.
اثبتنا ان كل مجموعة, مفتوحة من النمط $I\alpha$ - , (ضعيفة) مفتوحة من النمط شبه $-I$ و (ضعيفة) مفتوحة من النمط $ii-I$ هي (ضعيفة) مفتوحة من النمط $i-I$ لأي فضاء تبولوجي مثالي. اضافة الى ذلك اثبتنا ان كل التطبيقات المستمرة من النمط $I\alpha$ - , المستمرة من النمط شبه $-I$ و المستمرة من النمط $ii-I$ بين الفضاءات التبولوجية المثالية هي مستمرة من النمط $i-I$. اخيرا, لأي فضاء تبولوجي مثالي $(\mathcal{M},\mathcal{L},I)$ $\mathcal{M}D \subset$ تحقق $Int(D)^{\#} = Int(D)$. اثبتنا ان الجمل الاتية تكون متكافئة :
1) D هي مفتوحة 2) D هي مفتوحة من النمط $i-I$ و $D \cap H = Int(D)$ لبعض $H \in \mathcal{L} \setminus \{\mathcal{M},\emptyset\}$ 3) D هي مفتوحة من النمط شبه $-I$. على نفس المنوال اثبتنا ان الجمل الاتية تكون متكافئة :
1) D هي مغلقة 2) D هي مغلقة من النمط $i-I$ و $cl(D) = (D \cap F)$ لبعض $F \in \mathcal{L}^c$ 3) D هي مغلقة من النمط شبه $-I$.