On Geometry of Viasman-Gray Manifold

Habeeb M. Abood Fatin A.Rejab

Department of Mathematic, College of Education for Pure Sciences, Basra University

Basra, Iraq

e-mail ***iraqsafwan@yahoo.com***

Abstract. In this paper, we studied the geometric structure of one important class of almost Hermitian manifold which is called Viasman-Gray manifold. This manifold is a generalization of the classes nearly Kahler manifold and the locally conformal Kahler manifold. We provedthat, if $M$is Viasman-Gray manifold with flat conformal curvature tensor, then M is a manifold of class R_1if and only if,$M$is a manifold of flat Ricci tensor. The necessary condition that M is of zero scalar curvature tensor has been found. Finally, we proved that, if$M$ is VG-manifold of class W_1 and of flat Ricci tensor then $M$is Kahler manifold.

Keywords: Almost Hermitian manifold, Viasman-Graymanifold,conformalcurvature tensor.

2010 AMS Classification: 53C55, 53B35.

1 Introduction.

The studyof conformal invariant properties of Riemannian manifold with other structure playanimportant place in modern geometry. This study began by the work of Gray and Hervalla[2] which were classified the almost Hermitian manifold into sixteen classes, between these classes there are eight invariant under the conformal transformation metric. One of these classes is Viasman-Gray manifold that is denoted by $W_1 \oplus W_4$which is represents a generalization of the classes W_1andW_4. The class $W_1$is called a nearly Kahler manifold. This class defined by Tachibana [9] under the name K-space which is not invariant under conformal transformation metric but it belongs to the class locally conformal nearly Kahler manifold which is invariant under conformal transformation. The class W_4 is called a locally conformal Kahler manifold which is invariant with respect to the transformation metric.

The class $W_1 \oplus W_4$ has rich differentialgeometric properties and it is represents interesting study. So there are many researchers are studied this class, in particular Gray and Vanheke[3], Viasman[11] and Kirichenko [6], [7].

2Preliminaries.

Let M be a smooth manifold of dimension$2n (n > 1)$,$C^{\infty}(X)$ be an algebra of smooth functions on M, $X(M)$be a Lie algebra of vector fields on M. An almost Hermitian structure (AH −structure) on M is a pair of tensors $\{J, g =<>\}$, where J is an almost complex structure, $g =<.,.>$ is a Riemannian metric, such that $< JX, JY >=< X, Y >$; $X, Y \in X(M)$.A smooth manifold M with AH −structure is called an almost Hermitian manifold (AH −manifold).

In the tangent space $T_P(M)$there exist a basis of the form $\{\varepsilon_1, \dots, \epsilon_n, \bar{\varepsilon}_1, \dots, \bar{\varepsilon}_n\}$.Its corresponding frame is $\{p, \varepsilon_1, \dots, \varepsilon_n, \bar{\varepsilon}_1 \dots, \bar{\varepsilon}_n\}$. Suppose that the indexes i, j, k, l in the range $1, 2, \dots, 2n$ and the indices a, b, c, d, e, f, g, h in the range $1, 2, \dots, n$.Denote$\hat{a} = a + n$.

It is known [5] that the setting an AH −structure on M is equivalent to the setting of an$G$-structure in the principle fiber bundle of all complex frames of manifold M which contains G-structure group that is the unitary group $U(n)$which is called an adjoined G-structure. In the space of the adjoined G-structure, the following forms define matrices which give components of tensor fields g and J:

$$(g_{ij}) = \begin{bmatrix} 0 & I_n \\ I_n & 0 \end{bmatrix}$$

$$,(J_j^i) = \begin{bmatrix} \sqrt{-1} I_n & 0 \\ 0 & -\sqrt{-1} I_n \end{bmatrix}, \quad (2.1)$$

where I_n is the unit matrix of order n.

Recall that [4] an AH- structure $(J, g =<>)$ is called astructure of class $W_1 \oplus W_4$or Viasman– Gray structure if,

$$\nabla(F)(X, Y) = \frac{-1}{2(n-1)}\{< X, Y > \delta F(Y) - < X, Y > \delta F(X) - < JX, JY > \delta F(JX)\}, \quad (2.2)$$

where ∇ is the Riemannian connection of g, $F(X, Y) =< JX, Y >$ is the Kahler form, δ is acoderivativeand $X, Y \in X(M)$. An AH- structure $(J, g =<>)$ is called a structure of class W_1 or nearly Kahler if its Kahler form is aKilling form, or equivalently,

$$\nabla_X(J) = 0 \;, X \in X(M) \quad (2.3)$$

An AH- structure $(J, g =<.,.>)$ is called a structure of class W_4 if

$\nabla_X(F)(Y,Z) = \frac{-1}{2(n-1)}\{< X,Y > \delta F(Z) -< X,Z > \delta F(Y) -< X,JY > \delta F(JZ) +< X,JZ > F(JY)\}$(2.4)

A manifold *M* with Viasman-Gray structure is called a Viasman-Gray manifold (*VG*-manifold). For each *AH*- manifold, in particular for *VG*- manifold defined a Lie form by the formula

$$\alpha = \frac{1}{n-1}\delta F \circ J$$

It is well known [4] that the structure equations of Riemannian connection of the *VG*-structure on adjoined *G*-structure space which are called the structure equations of VG- structure have the forms

1) $d\omega^a = \omega^a_b \Lambda \omega^b + \mathrm{B}^{ab}{}_c \omega^c \Lambda \omega_b + \mathrm{B}^{abc}\omega_b \Lambda \omega_c$;

2) $d\omega_a = -\omega^b_a \Lambda \omega_b + \mathrm{B}_{ab}{}^c \omega_c \Lambda \omega^b + \mathrm{B}_{abc}\omega^b \Lambda \omega^c$;

3) $d\omega^a_b = \omega^a_c \Lambda \omega^c_b + \left(2\mathrm{B}^{adh}\mathrm{B}_{hbc} + \mathrm{A}^{ad}_{bc}\right)\omega^c \Lambda \omega_d + \left(\mathrm{B}^{ah}{}_{[c}\mathrm{B}_{d]bh} + \mathrm{A}^a_{bcd}\right)\omega^c \Lambda \omega^d + \left(\mathrm{B}_{bh}{}^{[c}\mathrm{B}^{d]ah} + \mathrm{A}^{acd}_b\right)\omega_c \Lambda \omega_d$,

where $\{\omega^i\}$ ar the components of the solder form, $\{\omega^i_j\}$are the components of the connection form for Riemannian metric, $\omega_a = \omega^{\hat{a}}$and $\{\mathrm{A}^{ad}_{bc}, \mathrm{A}^a_{bcd}, \mathrm{A}^{acd}_b\}$are some functions on adjoined G −structure space.The functions $\{\mathrm{A}^{ad}_{bc}\}$ defined a tensor field on the manifold M, this tensor field is called a tensor of holomorphic sectional curvature. It is known that $\overline{\mathrm{A}}^{ad}_{bc} = \mathrm{A}^{bc}_{ad}$.

The tensors $\{\mathrm{B}^{abc}\}$ and $\{\mathrm{B}_{abc}\}$ are called the structure tensors and the tensors $\{\mathrm{B}^{ab}{}_c\}$ and $\{\mathrm{B}_{ab}{}^c\}$ are called the virtual tensors. It is obvious that $\overline{\mathrm{B}}^{abc} = \mathrm{B}_{abc}$ and $\overline{\mathrm{B}}^{ab}{}_c = \mathrm{B}_{ab}{}^c$

Remark 2.1. By the Banaru's classification of AH −manifold [1], the *VG*- manifold satisfies the following properties

$$\mathrm{B}^{[abc]} = \mathrm{B}^{abc};\ \mathrm{B}_{[abc]} = \mathrm{B}_{abc}; \mathrm{B}^{ab}{}_c = \alpha^{[a}\delta^{b]}_c; \mathrm{B}_{ab}{}^c = \alpha_{[a}\delta^c_{b]},$$

where $\{\alpha_a, \alpha^a \equiv \alpha_{\hat{a}}\}$are the components of theLie form α.

Lemma2.2[4]. The components of Riemaniancurvature tensor R of VG-Manifold in the adjoint G −structure space are given as the following forms:

1) $R_{\mathrm{abcd}} = 2(\mathrm{B}_{\mathrm{ab[cd]}} + \alpha_{\mathrm{[a}}\mathrm{B}_{\mathrm{b]cd}})$;

2) $R_{\hat{a}bcd} = 2\mathrm{A}^a_{bcd}$;

3) $R_{\hat{a}\hat{b}\mathrm{cd}} = 2(-\mathrm{B}^{\mathrm{abh}}\mathrm{B}_{\mathrm{hcd}} + \alpha^{[\mathrm{a}}_{[\mathrm{c}}\delta^{\mathrm{b}]}_{\mathrm{d}]})$;

4) $R_{\hat{a}bc\hat{d}} = \mathrm{A}^{\mathrm{ad}}_{\mathrm{bc}} + \mathrm{B}^{\mathrm{adh}}\mathrm{B}_{\mathrm{hbc}} - \mathrm{B}^{\mathrm{ah}}{}_{\mathrm{c}}\mathrm{B}_{\mathrm{hb}}{}^{\mathrm{d}}$,

where $\{\alpha^a{}_b, \alpha_a{}^b, \alpha_{ab}, \alpha^{ab}\}$ are some functions on adjoined G −structure space such that

$$d\alpha_{\mathrm{a}} + \alpha_b\omega^b_a = \alpha^b_a\omega_b + \alpha_{ab}\omega^b$$

and

$$d\alpha^a - \alpha^b \omega_b^a = \alpha_b^a \omega^b + \alpha^{ab}\omega_b$$

The others components of Riemannian curvature tensor R can be obtained by the property of symmetry for R

Lemma2.3[4]. The components of Ricci tensor of VG-manifold in adjoined G-structures space are given as the following forms:

1) $r_{ab} = \frac{1-n}{2}(\alpha_{ab} + \alpha_{ba} + \alpha_a \alpha_b)$
2) $r_{\hat{a}b} = 3\mathrm{B}^{cah}\mathrm{B}_{cbh} - \mathrm{A}_{bc}^{ca} + \frac{n-1}{2}(\alpha^a \alpha_b - \alpha^h \alpha_h) - \frac{1}{2}\alpha^h{}_h \delta_b^a + (n-2)\alpha^a{}_b$

Definition 2.4. Ascalar curvature tensor of an *AH*-manifold is denoted by *K* and defined as:

$K = g^{ij} r_{ij}$ (2.5)

Lemma2.5. For any almost Hermitian manifold.In the adjoined*G*-structure space, the scalar curvature tensor satis-fies the following equation

$$K = 2r_a^a$$

Proof. In the adjoined*G*-structure space, the equation (2.5) becomes:

$$K = g^{ab}r_{ab} + g^{\hat{a}b}r_{\hat{a}b} + g^{a\hat{b}}r_{a\hat{b}} + g^{\hat{a}\hat{b}}r_{\hat{a}\hat{b}}$$

By using equation (2.1) we get:

$$K = 2g^{\hat{a}b}r_{\hat{a}b} = 2\delta_a^b r_{\hat{a}b} = 2r_{\hat{a}a}$$

Therefore, $= 2r_a^a$. ■

3Main results.

Definition3.1[8]. As for as the Riemannian space, the conformal curvature tensor or Weyl's tensor $\{W = W_{jkl}^i\}$ of type (3,1) is defined by the form:

$$W_{ijkl} = R_{ijkl} + \frac{1}{m-2}\left(r_{ik}g_{jl} + r_{jl}g_{ik} - r_{il}g_{jk} - r_{jk}g_{il}\right) + \frac{K(g_{il}g_{jk} - g_{ik}g_{jl})}{(m-2)(m-1)} \quad (3.1)$$

where $R_{ijkl}$are the components of the Riemannian curvature tensor, r_{ij} are the components of Ricci tensor, g_{ij} are components of the Riemannian metric g and K is the scalar curvature tensor. This tensor is invariant under conformal transformation metric.

According to our case, the AH −manifold which we have, $dim\, M = 2n$, then the conformal curvature tensor is redefinedby the following form:

$$W_{ijkl} = R_{ijkl} + \frac{1}{2(n-1)}\left(r_{ik}g_{jl} + r_{jl}g_{ik} - r_{il}g_{jk} - r_{jk}g_{il}\right) \frac{K(g_{il}g_{jk} - g_{ik}g_{jl})}{2(\mathrm{n}-1)(2\mathrm{n}-1)} \quad (3.2)$$

This tensor has similar properties to those of the Riemannian curvature tensor.

Lemma 3.2. In the adjoined*G*-structure space, the components of the conformal curvature tensor of the*VG*-manifold are given by the following forms:

1) $W_{abcd} = R_{abcd}$;

2) $W_{\hat{a}bcd} = R_{\hat{a}bcd} + \frac{1}{2(n-1)}(r_{bd}\delta_c^a - r_{bc}\delta_d^a)$;

3) $W_{\hat{a}\hat{b}cd} = R_{\hat{a}\hat{b}cd} + \frac{2}{(n-1)} r_{[c}^{[a}\delta_{d]}^{b]} - \frac{K\delta_{cd}^{ab}}{2(n-1)(2n-1)}$;

4) $W_{\hat{a}bc\hat{d}} = R_{\hat{a}bc\hat{d}} + \frac{1}{2(n-1)}\left(r_c^a\delta_b^d + r_b^d\delta_c^a\right) - \frac{K\delta_c^a\delta_b^d}{2(n-1)(2n-1)}$,

where $\delta_{cd}^{ab} = \delta_c^a\delta_d^b - \delta_d^a\delta_c^b$.

Proof. 1) For $= a$, $j = b$, $k = c$, and $l = d$, the equation(3.2) becomes:

$$W_{abcd} = R_{abcd} + \frac{1}{2(n-1)}(r_{ac}g_{bd} + r_{bd}g_{ac} - r_{ad}g_{bc} - r_{bc}g_{ad}) + \frac{K(g_{bc}g_{ad} - g_{bd}g_{ac})}{2(n-1)(2n-1)}$$

According to the equation (2.1), we get that

$$W_{abcd} = R_{abcd}$$

2)For $i = \hat{a}$, $j = b$, $k = c$ and $l = d$,
we have

$$W_{\hat{a}bcd} = R_{\hat{a}bcd} + \frac{1}{2(n-1)}(r_{\hat{a}c}g_{bd} + r_{bd}g_{\hat{a}c} - r_{\hat{a}d}g_{bc} - r_{bc}g_{\hat{a}d}) + \frac{K(g_{bc}g_{\hat{a}d} - g_{bd}g_{\hat{a}c})}{2(n-1)(2n-1)}$$

$$W_{\hat{a}bcd} = R_{\hat{a}bcd} + \frac{1}{2(n-1)}(r_{bd}\delta_c^a - r_{bc}\delta_d^a)$$

3) For $i = \hat{a}$, $j = \hat{b}$, $k = c$ and $l = d$,
we have

$$W_{\hat{a}\hat{b}cd} = R_{\hat{a}\hat{b}cd} + \frac{1}{2(n-1)}(r_{\hat{a}c}g_{\hat{b}d} + r_{\hat{b}d}g_{\hat{a}c} - r_{\hat{a}d}g_{\hat{b}c} - r_{\hat{b}c}g_{\hat{a}d}) + \frac{K(g_{\hat{b}c}g_{\hat{a}d} - g_{\hat{b}d}g_{\hat{a}c})}{(2n-1)(2n-2)}$$

$$= R_{\hat{a}\hat{b}cd} + \frac{2}{4(n-1)}(r_c^a\delta_d^b + \delta_d^b\delta_c^a - r_d^a\delta_c^b - r_c^b\delta_d^a) + \frac{K}{(2n-1)(2n-2)}(\delta_c^b\delta_d^a - \delta_d^b\delta_c^a)$$

$$= R_{\hat{a}\hat{b}cd} + \frac{2}{(n-1)} r_{[c}^{[a}\delta_{d]}^{b]} + \frac{K\delta_{cd}^{ab}}{2(n-1)(2n-1)}$$

4) For $i = \hat{a}$, $j = b$, $k = c$, and $l = \hat{d}$,
we have

$$W_{\hat{a}bc\hat{d}} = R_{\hat{a}bc\hat{d}} + \frac{1}{2(n-1)}(r_{\hat{a}c}g_{b\hat{d}} + r_{b\hat{d}}g_{\hat{a}c} - r_{\hat{a}\hat{d}}g_{bc} - r_{bc}g_{\hat{a}\hat{d}}) + \frac{K(g_{bc}g_{\hat{a}\hat{d}} - g_{b\hat{d}}g_{\hat{a}c})}{2(n-1)(2n-1)}$$

$$= R_{\hat{a}bc\hat{d}} + \frac{1}{2(n-1)}\left[r_c^a\delta_b^d + r_b^d\delta_c^a\right] - \frac{K\delta_c^a\delta_b^d}{(2n-1)(2n-2)}.■$$

Lemma 3.3 [10]. In the adjoined G-structure space, an *AH*- manifold is manifold of class:

R_1 if and only if, $R_{abcd} = R_{\hat{a}bcd} = R_{\hat{a}\hat{b}cd} = 0$,

R_2 if and only if, $R_{abcd} = R_{\hat{a}bcd} = 0$,

R_3(*RK*-manifold) if and only if, $R_{\hat{a}bcd} = 0$.

Theorem 3.4. If *M* is *VG*- manifold with flat conformal curvature tensor, then *M* is a manifold of class R_1 if and only if, *M* is a manifold of flat Ricci tensor.

Proof. Suppose that *M* is *VG*-manifold with flat conformal curvature tensor.

Making use of Lemma 3.2 we get:

$$R_{\hat{a}bcd} + \frac{1}{2(n-1)}\left(r_{bd}\delta_c^a - r_{bc}\delta_d^a\right) = 0 \quad (3.3)$$

Since *M* is manifold of class R_1, So by the Lemma 3.3 we have

$$\frac{1}{2(n-1)}\left(r_{bd}\delta_c^a - r_{bc}\delta_d^a\right) = 0 \quad (3.4)$$

Contracting the equation (3.4) by the indices a and c, we obtain

$$\frac{1}{2(n-1)}\left(r_{bd}\delta_a^a - r_{ba}\delta_d^a\right) = 0$$

Or equivalently,

$$\frac{1}{2(n-1)}(n-1)r_{bd} = 0$$

Therefore, $r_{bd} = 0$ and this complete the proof. ■

Theorem 3.5. Suppose that *M* is flat *VG*-manifold with flat conformal curvature tensor, then *M* is of zero scalar curvature tensor.

Proof. By using Lemma 3.2 we have

$$W_{\hat{a}\hat{b}cd} = \mathrm{R}_{\hat{a}\hat{b}cd} + \frac{2}{(n-1)} r_{[c}^{[a}\delta_{d]}^{b]} + \frac{k\delta_{cd}^{ab}}{2(n-1)(2n-1)} \quad (3.5)$$

Suppose that *M* is flat *VG*-manifold with flat conformal curvature tensor. This means that the Riemannian and conformal curvature tensors are vanishing. Thus equation (3.5) becomes

$$\frac{2}{(n-1)}\left(r_c^a\delta_d^b + r_d^b\delta_c^a - r_c^b\delta_d^a - r_d^a\delta_c^b\right) + \frac{K(\delta_c^a\delta_d^b - \delta_c^b\delta_d^a)}{2(n-1)(2n-1)} = 0 \quad (3.6)$$

Contracting (3.6) by the indexes (b, d) and (a, c) we get:

$$\frac{2}{n-1}\left(r_a^a\delta_b^b + r_b^b\delta_a^a - r_a^b\delta_b^a - r_b^a\delta_a^b\right) + \frac{K(\delta_a^a\delta_b^b - \delta_a^b\delta_b^a)}{2(n-1)(2n-1)} = 0$$

$$4r_a^a + \frac{nK}{2(2n-1)} = 0$$

By using Lemma2.5 we get

$$\left(2 + \frac{n}{2(2n-1)}\right)K = 0$$

Hence, $K = 0$

Therefore, M is of zero scalar curvature tensor. ■

Similarly to the Lemma 3.3 we can construct the three special classes of *AH*-manifold depend on conformal curvature tensor, which are embodied in the following Lemma.

Lemma 3.6. In the adjoined G −structure space, an *AH*- manifold is manifold of class:

W_1 if and only if,$W_{abcd} = W_{\hat{a}bcd} = W_{\hat{a}\hat{b}cd} = 0$,

W_2 if and only if,$W_{abcd} = W_{\hat{a}bcd} = 0$,

$W_3(WRK$ −manifold) if and only if,$W_{\hat{a}bcd} = 0$.

Theorem 3.7. Suppose that*M* is *VG*-manifold of class W_1 and of flat Ricci tensor then *M*is Kahler manifold.

Proof. In the adjoined *G*-structure space, the components ofconformal curvature tensorcan be written as follows

$$W_{\hat{a}\hat{b}cd} = W(\varepsilon_{\hat{a}}, \varepsilon_{\hat{b}}, \varepsilon_c, \varepsilon_d) = W(\varepsilon_{\hat{a}}, \varepsilon_{\hat{b}}, J\varepsilon_c, J\varepsilon_d)$$

$$W\left(\varepsilon_{\hat{a}}, \varepsilon_{\hat{b}}, \sqrt{-1}\varepsilon_c, \sqrt{-1}\varepsilon_d\right) = \left(\sqrt{-1}\right)\left(\sqrt{-1}\right) W(\varepsilon_{\hat{a}}, \varepsilon_{\hat{b}}, \varepsilon_c, \varepsilon_d)$$

$$-W(\varepsilon_{\hat{a}}, \varepsilon_{\hat{b}}, \varepsilon_c, \varepsilon_d) = -W_{\hat{a}\hat{b}cd}$$

Thus,

$$2W_{\hat{a}\hat{b}cd} = 0$$

Suppose that M is *VG*-manifold of class W_1.

By using Lemmas 2.2 and 3.2, it follows that

$$-4B^{abh}B_{hcd} + 4\alpha^{[a}_{[c}\delta^{b]}_{d]} + \frac{4}{n-1} r^{[a}_{[c}\delta^{b]}_{d]} + \frac{K\delta^{ab}_{cd}}{(n-1)(2n-1)} = 0 \quad (3.7)$$

Contracting (3.7) by indexes (a, c) and (b, d)we get:

$$-4B^{abh}B_{hab} + 4\alpha^{[a}_{[a}\delta^{b]}_{b]} + \frac{4}{n-1} r^{[a}_{[a}\delta^{b]}_{b]} + \frac{2K\delta^{ab}_{ab}}{2(n-1)(2n-1)} = 0$$

Or equivalently,

$$-4B^{abh}B_{hab} + 4n\alpha^a_a + \frac{4n}{n-1} r^a_a + \frac{2n\, r^a_a}{(2n-1)} = 0$$

Since *M* is manifold of flat Ricci tensor, then we get

$$-4B^{abh}B_{hab} + 4n\alpha^a_a = 0 \quad (3.8)$$

Symmetrizing (3.8) by the indexes (a, b), it follows that

$$4n\alpha^a_a = 0$$

Thus, $\alpha^a_a = 0$ (3.9)

Making use of the equations (3.9)and (3.8), it follows that

$$B^{abh}B_{hab} = 0 \Leftrightarrow \sum |B_{hab}|^2 = 0$$
$$\Leftrightarrow B_{hab} = 0$$

According to theBanaru's classification we get that *M* is Kahler manifold. ■

References

[1]Banaru M., A new characterization of the Gray-Hervella classes of almost Hermitianmanifold, 8th International conference on differential geometry and its applications, Opava-Czech Republic, 27-31, August 2001.

[2]Gray A ., Hervella L . M., Sixteen classes of almost Hermitianmanifold and their linearinvariants, Ann. Math. Pure and Appl., 123(1980), No.3, 35-58.

[3] Gray A., Vanhecke L., Almost Hermitian manifold with constant holomorphic sectional curvature, Cas. Pestov. Mat., V.104, 1979, p.170-179.

[4] Ignatochkina L., New aspects of geometry of Viasman-Gray manifold, Ph.D. thesis, Moscow State Pedagogical University, Moscow, 2001.

[5]Kirichenko V.F., New results of K–spaces theory, Ph.D. thesis, Moscow State University, Moscow, 1975.

[6] Kirichenko V. F., Locally conformal Kahler manifold of constant holomorphic sectional curvature, Math. Sbor., V. 182, No.3, 1991, p.354-363.

[7]Kirichenko V. F., Locally conformal flat of locall conformal Kahler manifold, Mathematical Notes, V.51, No.5, 1992, p.57-66.

[8]Rachevski P. K., Riemmaniangeome-try and tensor analysis,M. Nauka, 1964**.**

[9] Tachibana S. , On almost analytic vector in certain almost Hermitian manifold, Tohoku Math. J. V. 11, 1959, p.351-363.

[10] Terkiakova E. V. Curvature identities for almost kahler manifold, VINITE, Moscow, No. 208-B99, 1999.

[11] Viasman I., Some curvature properties of locally conformal Kahler manifold, Trans. Amer. Math. Soc., V.259, No.2, 1980, p.439-4476.