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Cascade result of Input-Output Stability for Index One Descriptor Control Systems

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Abstract

We study the recently introduced notion of input-output stability, which is a robust variant of the minimum-phase property for general smooth nonlinear control systems. The purpose of this paper is to investigate how the input-output stability property behaves under series connections of several subsystems. We will prove two lemmas. The first one says that the cascade of a 0-detectable descriptor system with an input-output stable descriptor system is weakly 0-detectable, which is a result of independent interest. The second lemma is a direct generalization of the first one.

Key words: input-output stability , minimum-phase , weakly 0- detectable , descriptor system

1. Introduction

Descriptor systems (also called singular systems or generalized state-space systems) have attracted much attention in recent years, because of their ability to capture the dynamical behaviour of many natural phenomena [l]. It is well-established that the DAE descriptor systems (differential algebraic equations systems) may have fundamentally different characteristics from the ordinary differential equation (ODE) system [2]. A concept that is commonly used to provide a measure of the differences between DAE and ODE systems is that of the (differential) index [3]. Loosely speaking, the index of DAE system is the minimum number of differentiations required to obtain an equivalent ODE system. DAE systems with index more than one are referred to as high-index systems.

Stability theory plays an important role in systems control engineering and practice. Many different kinds of control problems, arising in the study of dynamical systems, have direct connection with stability problems [4]. The stability theory for nonlinear descriptor systems was also studied [5-7], the main results were stability criteria for special nonlinear descriptor systems in gradient, Hamiltonian, and Lur'e type forms. These are immediately used in justifying power-system analysis methods, e.g., applying the Hamiltonian result to give a clear Lyapunov stability statement for undamped power systems with nonlinear loads, which provided a basis for using energy function method to determine transient stability region. Furthermore, geometric properties (transversality, well posedness, passivity) in nonlinear descriptor system models were related to circuit structural properties [8] and [9]. By reviewing the differential geometric theory [10], for nonlinear control systems, few literature on feedback control and stability of nonlinear descriptor systems or implicit differential equations could be found. Control systems described by a class of nonlinear descriptor systems are introduced in McClamroch [11]. Existence and uniqueness of solutions, and the feedback stabilization problem for the systems are studied. The concept of an equivalent state for the class of nonlinear descriptor systems is introduced. Analysis and control design for a class of linear differential algebraic equations, having the same structure as equation above, is demonstrated in detail in [12] and [13]. Tracking problem associated with the class of nonlinear descriptor systems, under the assumption that the reference input varies sufficiently slowly, is considered in [14]. A novel feedback linearization approach attracted extensive interest in power systems area [15]. The feedback linearization includes a static state feedback and a special nonlinear coordinate transformation such that the closed-loop system is fully linearized. Hence, a variety of proven linear control design techniques can be applied. The design of a feedback control law for a class of nonlinear descriptor systems is considered in [16], so that the desired output keeps track of the given reference inputs. The nonlinear descriptor systems considered is not in state variable form. A nonlinear feedback control law is then proposed which ensures, under appropriate assumptions, the tracking error in the closed loop nonlinear descriptor systems approaches zero exponentially. Nonlinear descriptor system models are not standard because they are not in state-space, well posedness of the models has to be studied carefully.

The zero dynamics of a control system are the dynamics describing the "internal" behavior of the system when input and initial conditions have been chosen in such a way as to constrain the output to remain identically zero. Some research on the zero dynamics of nonlinear control systems has been proceeded. The relationship between the zero dynamics of the system and the stability of the system has been found; The problems of reproducing the reference output have also been considered with the aid of the zero dynamics of the system [17]. Those results show that the zero dynamics of a system is an important property of the system. The concept of the zero dynamics studies special class of control systems (systems that are affine in control), therefore; many studies for the stability of the systems that nonlinear in general as input-output stability in Khalil [18], and M. Vidyasagar input-output stability in [19] studied the relation between the input and the output of the systems. The notion of input-output-to-state stability was studied in [20] and [21]. The new definition of input-output stability such a concept was proposed in the recent paper [22]. Its definition requires the state and the input of the system to be bound by a suitable function of the output and derivatives of the output, modulo a decaying term depending on initial conditions. Input-output stability can be investigated with the help of the tools that have been developed over the years to study ISS (input-to-state stability), OSS (output-to-state stability), and related notions which introduced in [23]and[24]. In [25]and [26] Liberzon is continuing to study the input-output stability property for multi-input, multi-output (MIMO) systems. Liberzon shows the relevance of the nonlinear structure algorithm in establishing input-output stability.

For nonlinear descriptor control systems that are affine in controls, a major contribution of J. Wang, C. Chen [27-30] was to use the ideas of differential geometric control theory to define *M* derivative and *M* bracket in order to study the index one nonlinear descriptor control systems and proposed a systematic state feedback linearization strategy to produce stabilization and adaptive stabilization control laws. They define the minimum-phase property in terms of the concept of zero dynamics. The zero dynamics are the internal dynamics of the descriptor system under the action of an input that holds the output constantly at zero. The descriptor system is called minimum-phase if the zero dynamics are (globally) asymptotically stable. In the SISO case, a unique input capable of producing the zero output is guaranteed to exist if the descriptor system has *M* relative degree, which is now defined to be the number of times one has to differentiate the output until the input appears [28]. Extensions to MIMO systems are discussed in [27] and [29].

In [31] we introduced the notion of *input-output stability* for nonlinear control descriptor systems in general form. In view of the need to work with the zero dynamics, the definition of a minimum-phase nonlinear descriptor control system prompts one to look for a change of coordinates that transforms the descriptor system into a certain normal form. It has also been recognized that just asymptotic stability of the zero dynamics is sometimes insufficient for control design purposes, so that additional requirements need to be placed on the internal dynamics of the descriptor system. In this paper we continue to study the result of input-output stability property by studying cascade descriptor control systems.

2.Background

Consider the descriptor system

$$\dot{x} = f(x,z,u)$$
$$0 = \sigma_i(x,z) \quad \forall \; i = 1,\dots,m$$
$$y = h(x,z) \qquad (1)$$

where the dynamic state $x \in \Re^n$, the algebraic state $z \in \Re^m$, the input $u \in \Re^p$, and the output $y \in \Re^q$. Whenever possible and convenient, these equations will be rewritten in the more condensed form

$$\dot{x} = f(x,z,u)$$
$$0 = \sigma(x,z)$$
$$y = h(x,z)$$

where $\sigma(x,z) = (\sigma_1(x,z),\dots,\sigma_m(x,z))^T$. We assume that f, σ, and h are all smooth mapping, and the Jacobian matrix of $\sigma(x,z)$ with respect to z is nonsingular at all $x \in \Re^n$ and $z \in \Re^m$, we restrict admissible input (or "control") signals to be at least continuous, and for every initial condition $(x(0), z(0))$ and every input $u(.)$, there is a solution $(x(.), z(.))$ of (1) defined on a maximal interval $\left[0, T_{\max}\right)$ and the corresponding output $y(.)$. We write C^k for the space of k times continuously differentiable functions $u : [0,\infty) \to \Re^p$, where k is some nonnegative integer. Whenever the input u is in C^k, the derivatives $\dot{y}, \ddot{y}, \dots, y^{(k+1)}$ exist and are continuous; they are given by

$$y^{(i)}(t) = H_i(x(t), z(t), u(t), \dots, u^{(i-1)}(t)) \;,\; i = 1,\dots,k+1, \quad t \in [0, T_{\max})$$

where for $i = 0,1,\dots$ the functions $H_i : \Re^{n+m} \times \Re^{pi} \to \Re^q$ are defined recursively via $H_0 := h(x,z)$ and

$$H_i(x,z,u,...,u^{(i-1)}) = \left(\frac{\partial H_{i-1}}{\partial x} - \frac{\partial H_{i-1}}{\partial z} (\frac{\partial \sigma}{\partial z})^{-1} (\frac{\partial \sigma}{\partial x}) \right) f(x,z,u) + \sum_{j=0}^{i-2} \frac{\partial H_{i-1}}{\partial u^{(j)}} u^{(j+1)}$$

where the arguments of H_i are $(u, \dot{u}, ..., u^{(i-1)}) \in \Re^{ip}$ and $(x,z) \in \Re^{n+m}$.

We will let $\|.\|_{[a,b]}$ denote the supremum norm of a signal restricted to an interval $[a,b]$,i.e.,

$$\|z\|_{[a,b]} := \sup\{|z(s)| : a \le s \le b\} \ ,$$

where $|\ .\ |$ is the standard Euclidean norm.

The minimum number of times that all or part of the constraint equation must be differentiated with respect to time in order to solve for $\dot{z}$ as a continuous function of x , and z is the index of the descriptor system (1), [4].

Carrying out the differentiation procedure once gives $0 = \frac{\partial \sigma}{\partial x} f + \frac{\partial \sigma}{\partial z} \dot{z}$. It is apperent that if the Jacobian $(\frac{\partial \sigma}{\partial z})$ is nonsingular, then it is possible to solve for $\dot{z}$ and the system has index of one. For high index systems (index >1) the Jacobian is not invertible and the constraint equations are identically singular with respect to z. If the descriptor system (1) has index one then two nonlinear control systems can be found as in the following two cases, which needed in the next speaking. The first case is taken from the assumption that (1) has index one, then the Jacobian matrix $(\frac{\partial \sigma}{\partial z})$ is nonsingular and from the Implicit Function Theorem, the algebraic equation of (1) determine unique smooth mapping $p(x) : \Re^n \to \Re^m$ defined on a neighborhood of the origin, such that $z = p(x)$. If there is an analytic expression of $p(x)$, then substituting $p(x)$ into (1) results in

$$\begin{aligned} \dot{x} &= \bar{f}(x,u) \\ y &= \bar{h}(x) \end{aligned} \qquad (2)$$

where $\bar{f}(x,u) = f(x, p(x))$,and $\bar{h}(x) = h(x, p(x))$ which is standard state space system.

The second case is taken where σ is nonlinear in general, so it is impossible to get analytic expression of $p(x)$; therefore, It is meaningful to discuss the problem under the form of differential algebraic systems, and can be used the following notation

$F(x,z) = \begin{bmatrix} I_n \\ -(\frac{\partial \sigma}{\partial z})^{-1} \frac{\partial \sigma}{\partial x} \end{bmatrix}$, then nonlinear control system

$$\begin{aligned} \begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} &= F(x,z) f(x,z,u) \\ y &= h(x,z) \end{aligned} \qquad (3)$$

is found.

According to definition (4.3) of [31], the descriptor system (1) is called input-output stable if there exist a positive integer *N*, a class KL function[1] β , and a class K_∞ function γ such that

[1] Recall that a function $\alpha : [0,\infty) \to [0,\infty)$ is said to be of *class K* if it is continuous, strictly increasing, and $\alpha(0) = 0$. If it is unbounded, then it is said to be of *class* K_∞. A function $\beta : [0,\infty) \times [0,\infty) \to [0,\infty)$ is said to be of *class KL* if $\beta(.,t)$ is of class K for each fixed $t \ge 0$ and $\beta(s,t)$ decreases to 0 as $t \to 0$ for each fixed $s \ge 0$.

for every initial state $(x(0), z(0))$ and every $N-1$ times continuously differentiable input u the inequality

$$\left\| \begin{pmatrix} u(t) \\ x(t) \\ z(t) \end{pmatrix} \right\| \leq \beta(\left\| \begin{pmatrix} x(0) \\ z(0) \end{pmatrix} \right\|, t) + \gamma(\left\| Y^N \right\|_{[0,t]}) \tag{4}$$

holds for all t in the domain of the corresponding solution of (1), where $Y^N = (y, \dot{y}, ..., y^{(N)})^T$. (The assumption that u belongs to C^{N-1} is made to guarantee that $y^{(N)}$ is well defined, and can be weakened if the function H_N is independent of $u^{(N-1)}$).

It is perhaps best to interpret input-output stability as a combination of three separate properties of the descriptor control system. The first property is that if the output and its derivatives are small, then the dynamic state becomes small. This property is expressed by the following inequality

$$|x(t)| \leq \beta(\left\| \begin{pmatrix} x(0) \\ z(0) \end{pmatrix} \right\|, t) + \gamma(\left\| Y^N \right\|_{[0,t]}) \tag{5}$$

The second property is that if the output and its derivatives are small, then the algebraic state becomes small. This property is expressed by the following inequality

$$|z(t)| \leq \beta(\left\| \begin{pmatrix} x(0) \\ z(0) \end{pmatrix} \right\|, t) + \gamma(\left\| Y^N \right\|_{[0,t]}) \tag{6}$$

The results of [21] imply that the inequalities (5) and (6) hold with k if there exists a smooth, positive definite, radially unbounded function $V : \Re^{n+m} \rightarrow \Re$ that satisfies

$$\nabla V(x,z) F(x,z) f(x,z,u) \leq -\alpha(\left\| \begin{pmatrix} x \\ z \end{pmatrix} \right\|) + \chi(\left\| Y^N \right\|) \tag{7}$$

where $F(x,z) = \begin{bmatrix} I_n \\ -(\frac{\partial \sigma}{\partial z})^{-1} \frac{\partial \sigma}{\partial x} \end{bmatrix}$, for all $(x,z) \in \Re^{n+m}, u \in \Re^p$ for some functions $\alpha, \chi \in K_\infty$.

The inequality (7) is equivalent to weak uniform 0-detectability of order N for equivalent control system
(3) according to [22] (is equivalent to holding the following inequality

$$\left\| \begin{pmatrix} x(t) \\ z(t) \end{pmatrix} \right\| \leq \beta_1(\left\| \begin{pmatrix} x(0) \\ z(0) \end{pmatrix} \right\|, t) + \gamma_1(\left\| Y^N \right\|_{[0,t]}) \tag{8}$$

class KL function β_1 , and a class K_∞ function γ_1). In general, however, this property needs be understood better, which is precisely the goal of the present paper. In the next section we formulate and study a useful definiation which needs in the results of the paper.

The third property is that if the output and its derivatives are small, then the input becomes small. This ingredient of the input-output stability property is described by the following inequality

$$|u(t)| \leq \beta(\left\| \begin{pmatrix} x(0) \\ z(0) \end{pmatrix} \right\|, t) + \gamma(\left\| Y^N \right\|_{[0,t]}) \tag{9}$$

Which says that the input should become small if the output and its derivative are small. Loosely speaking, this suggests that the descriptor system has a stable left inverse in the input-output sense. Unlike uniform detectability (see [22]), this property does not seen to admit a Lyapunove-like characterization. In the

SISO case it is closely related to the existence of a M uniform relative degree see ([27],[28],[29], and [30]).

3.Detectability and Related Notions

Consider a general descriptor control system of the form

$$\dot{x} = f(x,z,u) \qquad (10)$$
$$0 = \sigma(x,z)$$

We recall from [23] that this descriptor system is called input-to-state stable if there exist some functions $\beta \in KL$ and $\gamma \in K_\infty$ such that for every initial state $(x(0),z(0))$ and every input u the corresponding solution satisfies the inequality

$$\left|\begin{pmatrix} x(t) \\ z(t) \end{pmatrix}\right| \le \beta\left(\left|\begin{pmatrix} x(0) \\ z(0) \end{pmatrix}\right|,t\right) + \gamma\left(\|u(t)\|_{[0,t]}\right)$$

for all $t > 0$. Intuitively, this means that the state eventually becomes small when the input is small.

Given a descriptor system with both inputs and outputs

$$\begin{aligned} \dot{x} &= f(x,z,u) \\ 0 &= \sigma(x,z) \qquad (11) \\ y &= h(x,z,u) \end{aligned}$$

According to [22], we will say that it is *0-detectable* if there exist some functions $\beta \in KL$ and $\gamma_1, \gamma_2 \in K_\infty$ such that for every $(x(0),z(0))$ and every u the corresponding solution satisfies the inequality

$$\left|\begin{pmatrix} x(t) \\ z(t) \end{pmatrix}\right| \le \beta\left(\left|\begin{pmatrix} x(0) \\ z(0) \end{pmatrix}\right|,t\right) + \gamma_1\left(\|u(t)\|_{[0,t]}\right) + \gamma_2\left(\|y\|_{[0,t]}\right) \qquad (12)$$

as long as it exists.

In particular, adescriptor system without inputs given by

$$\begin{aligned} \dot{x} &= f(x,z) \\ 0 &= \sigma(x,z) \\ y &= h(x,z) \end{aligned}$$

is called *0-detectable* if there exist some functions $\beta \in KL$ and $\gamma \in K_\infty$ such that for every initial state $(x(0),z(0))$ the corresponding solution satisfies the following inequality as long as it exists:

$$\left|\begin{pmatrix} x(t) \\ z(t) \end{pmatrix}\right| \le \beta\left(\left|\begin{pmatrix} x(0) \\ z(0) \end{pmatrix}\right|,t\right) + \gamma\left(\|y\|_{[0,t]}\right) \qquad (13)$$

Let us call the descriptor system (11) uniformly 0-detectable if there exist some functions $\beta \in KL$ and $\gamma \in K_\infty$ such that for every initial state $(x(0),z(0))$ and every input the inequality (13) holds along the corresponding solution. As the name suggests, uniform 0-detectability amounts to 0-detectability that is uniform with respect to inputs.

These concepts were studied in [24] for control systems under the names of input-output-to-state stability and output-to-state stability, respectively. This property was studied for ordinary control systems in [20],[22],[24],[25] we formulate above definition by transformation descriptor control system into ordinary control system (see section 2) and we apply the concepts in [20],[22][24],[25].

4.Cascade Results

The purpose of this paper is to investigate how the input-output stability property behaves under series connections of several subsystems of differential algebraic equations with index one. We will prove two lemmas. The first one says that the cascade of a 0-detectable descriptor system with an input-output descriptor stable system is weakly 0-detectable (i.e., 0-detectable through the output and its derivatives), which is a result of independent interest. The second lemma is a direct generalization of the first one.

Suppose that we are given two systems

$$\begin{aligned} &\sum 1: \\ &\dot{x}_1 = f_1(x_1, z_1, u_1) \\ &0 = \sigma_1(x_1, z_1) \\ &y_1 = h_1(x_1, z_1) \end{aligned} \tag{14}$$

and

$$\begin{aligned} &\sum 2: \\ &\dot{x}_2 = f_2(x_2, z_2, u_2) \\ &0 = \sigma_2(x_2, z_2) \\ &y_2 = h_2(x_2, z_2) \end{aligned} \tag{15}$$

Upon setting $u_2 = y_1$, we obtain a cascade descriptor system with input u_1 and output y_2, which we denote by $\sum c$ (see Fig. 1).

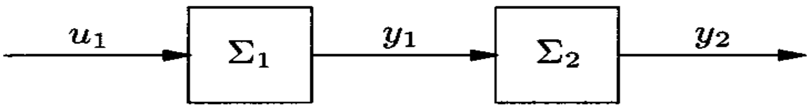

Fig. 1. The cascade descriptor system.

Assume that $\sum 2$ has a uniform M relative degree according to [31]. Consider the r – output extension of $\sum c$. As following (explained in [31])

$$h_r(x_1, x_2, z_1, z_2) = (y_2, \dot{y}_2, \ddot{y}_2, ..., y_2^{(r-1)}, y_2^{(r)})$$

$$h_r(x_1, x_2, z_1, z_2) = (h_2(x_2, z_2), H_1(x_2, z_2), ..., H_{r-1}(x_2, z_2), H_r(x_2, z_2, h_1(x_1, z_1)))$$

which is independent of u_1. In particular, no differentiability assumptions on the inputs are needed. Thus, the r - output extension of $\sum c$ is a descriptor system with input u_1 and output $Y_2^r = (y_2, \dot{y}_2, ..., y_2^{(r)})^T$. The following result says that this descriptor system is 0-detectable.

Lemma 6: If $\sum 1$ is 0-detectable and $\sum 2$ has uniform M relative degree r, and is weakly uniformly 0-detectable (i.e., 0-detectable through the output and its derivatives) of order r-1 i.e., satisfy the following inequality

$$\left\| \begin{pmatrix} x_2(t) \\ z_2(t) \end{pmatrix} \right\| \le \beta\left(\left\| \begin{pmatrix} x_2(0) \\ z_2(0) \end{pmatrix} \right\|, t\right) + \gamma\left(\left\| Y_2^{r-1} \right\|_{[0,t]} \right) \tag{16}$$

then the cascade system $\sum c$ is weakly 0-detectable of order r.

Proof: In the proofs of this lemma and the next one, β with various subscripts will be used to denote class KL functions, and γ and ρ with various subscripts will be used to denote class K_∞ functions. For $t \geq t_0 \geq 0$, the 0-detectable of $\sum 1$ can be expressed by the inequality

$$\left|\begin{pmatrix} x_1(t) \\ z_1(t) \end{pmatrix}\right| \leq \beta_1(\left|\begin{pmatrix} x_1(t_o) \\ z_1(t_0) \end{pmatrix}\right|, t-t_0) + \gamma_0(\|u_1(t)\|_{[t_0,t]}) + \gamma_1(\|y_1\|_{[t_0,t]})$$

while the uniform *M* relative degree *r*, and is weakly uniformly 0-detectable of $\sum 2$ leads to the inequalities

$$\left|\begin{pmatrix} x_2(t) \\ z_2(t) \end{pmatrix}\right| \leq \beta_2(\left|\begin{pmatrix} x_2(t_0) \\ z_2(t_0) \end{pmatrix}\right|, t-t_0) + \gamma_2(\|Y_2^{r-1}\|_{[t_0,t]}) \tag{17}$$

and

$$|y_1(t)| \leq \rho_1(\left|\begin{pmatrix} x_2(t) \\ z_2(t) \end{pmatrix}\right|) + \rho_2(|y_2^{(r)}(t)|)$$

Since the system $\sum c$ has a cascade structure, we employ the trick of breaking a time interval under consideration into several parts in order to derive the result (as done in [23]). Straightforward but lengthy calculations yield

$$\left|\begin{pmatrix} x_1(t) \\ z_1(t) \end{pmatrix}\right| \leq \beta_1(\left|\begin{pmatrix} x_1(t/2) \\ z_1(t/2) \end{pmatrix}\right|, t/2) + \gamma_0(\|u_1\|_{[t/2,t]}) + \gamma_1(\|y_1\|_{[t/2,t]})$$

$$\leq \overline{\beta}_1(\left|\begin{pmatrix} x_1(0) \\ z_1(0) \end{pmatrix}\right|, t) + \overline{\beta}_2(\left|\begin{pmatrix} x_2(0) \\ z_2(0) \end{pmatrix}\right|, t) + \overline{\gamma}_0(\|u_1\|_{[0,t]}) + \overline{\gamma}_1(\|Y_2^r\|_{[0,t]})$$

where

$\overline{\beta}_1(s,t) := \beta_1(3\beta_1(s,t/2),t/2)$

$\overline{\beta}_2(s,t) := \beta_1(9\gamma_1(3\rho_1(2\beta_2(s,0))),t/2) + \gamma_1(3\rho_1(2\beta_2(s,t/2)))$

$\overline{\gamma}_0(s) := \gamma_0(s) + \beta_1(3\gamma_0(s),0)$

$\overline{\gamma}_1(s) := \beta_1(9\gamma_1(3\rho_1(2\gamma_2(s))),0) + \beta_1(9\gamma_1(3\rho_2(s)),0) + \gamma_1(3\rho_1(2\gamma_2(s))) + \gamma_1(3\rho_2(s))$.

Combining this with (17), we arrive at the desired result. □

Next, suppose that the descriptor system $\sum 1$ has another output $y_3 = h_3(x_1, z_1)$. Letting $u_2 = y_1$ as before, and defining the output $y_4 := y_3 - y_2$, we obtain a cascade-feedforward descriptor system $\sum cf$ with input u_1 and output y_4 , shown in Fig. 2.

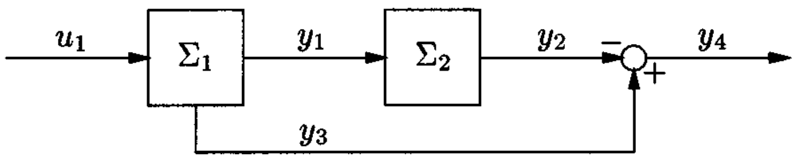

Fig. 2. The cascade-feedforward descriptor system.

Assume that the input u_1 is in C^{r-1} , where *r* is the uniform *M* relative degree of $\sum 2$ as before. We can then consider the descriptor system $\overline{\sum 1}$ whose input is $U_1^{r-1} = (u_1, \dot{u}_1, ..., u_1^{(r-1)})^T$ and whose output is $Y_3^r = (y_3, \dot{y}_3, ..., y_3^{(r)})^T$. Indeed, for each the $i \in \{1,...,r\}$ *i*th derivative y_3 of exists and can be written as

$y^{(i)}(t) = H_i(x(t), z(t), u_1(t), ..., u_1^{(i-1)}(t))$, for a suitable function H_i. Moreover, since y_2 is *r* times differentiable almost everywhere, we can consider the *r*-output extension of $\sum cf$, whose input is U_1^{r-1} and whose output is Y_4^r. The next result says that this last descriptor system is 0-detectable.

Lemma 2: Suppose that $\sum 1$ is 0-detectable (with respect to its input u_1 and both its outputs, y_1 and y_3), $\sum 2$ has uniform *M* relative degree *r*, and is weakly uniformly 0-detectable (i.e., 0-detectable through the output and its derivatives) of order *r*-1, and the system $\overline{\sum 1}$ with input U_1^{r-1} and output Y_3^r is input-to-output stable. Then the cascade-feedforward system $\sum cf$ is weakly 0-detectable of order r.

Proof: For $t \geq t_0 \geq 0$, the hypotheses of the lemma lead to the following inequalities:

$$\left|\begin{pmatrix} x_1(t) \\ z_1(t) \end{pmatrix}\right| \leq \beta_1(\left|\begin{pmatrix} x_1(t_o) \\ z_1(t_0) \end{pmatrix}\right|, t-t_0) + \gamma_0(\|u_1(t)\|_{[t_0,t]}) + \gamma_1(\|y_1\|_{[t_0,t]}) + \gamma_3(\|y_3\|_{[t_0,t]})$$

$$\left|\begin{pmatrix} x_2(t) \\ z_2(t) \end{pmatrix}\right| \leq \beta_1(\left|\begin{pmatrix} x_2(t_o) \\ z_2(t_0) \end{pmatrix}\right|, t-t_0) + \gamma_2(\|Y_2^{r-1}(t)\|_{[t_0,t]})$$

$$|y_1(t)| \leq \rho_1(\left|\begin{pmatrix} x_2(t) \\ z_2(t) \end{pmatrix}\right|) + \rho_2(|y_2^{(r)}(t)|)$$

$$|Y_3^r| \leq \beta_3(\left|\begin{pmatrix} x_1(t_o) \\ z_1(t_0) \end{pmatrix}\right|, t-t_0) + \gamma_4(\|U_1^{r-1}(t)\|_{[t_0,t]})$$

We have

$$\left|\begin{pmatrix} x_2(t) \\ z_2(t) \end{pmatrix}\right| \leq \hat{\beta}_1(\left|\begin{pmatrix} x_1(0) \\ z_1(0) \end{pmatrix}\right|, t) + \hat{\beta}_2(\left|\begin{pmatrix} x_2(0) \\ z_2(0) \end{pmatrix}\right|, t) + \hat{\gamma}_0(\|U_1^{r-1}\|_{[t0,t]}) + \hat{\gamma}_4(\|Y_4^r\|_{[0,t]})$$

where

$$\hat{\beta}_1(s,t) := \beta_2(6\gamma_2(4\beta_3(s,0), \frac{t}{2}) + \gamma_2(4\beta_3(s, \frac{t}{2}))$$

$$\hat{\beta}_2(s,t) := \beta_2(3\beta_2(s, \frac{t}{2}), \frac{t}{2})$$

$$\hat{\gamma}_0(s) := \beta_2(6\gamma_2(4\gamma_4(s),0) + \gamma_2(4\gamma_4(s))$$

$$\hat{\gamma}_4(s) := \gamma_2(2s) + \beta_2(3\gamma_2(2s),0)$$

Similarly

$$\left|\begin{pmatrix} x_1(t) \\ z_1(t) \end{pmatrix}\right| \leq \hat{\beta}_1(\left|\begin{pmatrix} x_1(0) \\ z_1(0) \end{pmatrix}\right|, t) + \hat{\beta}_2(\left|\begin{pmatrix} x_2(0) \\ z_2(0) \end{pmatrix}\right|, t) + \hat{\gamma}_0(\|U_1^{r-1}(t)\|_{[0,t]}) + \hat{\gamma}_4(\|Y_4^r\|_{[0,t]})$$

Where, for example

$$\hat{\beta}_2(s,t) := \beta_1(12\gamma_1(3\rho_1(2\beta_2(s,0))), \frac{t}{2}) + \gamma_1(4\rho_1(4\beta_2(2\beta_2(s, \frac{t}{4}), \frac{t}{4}))).$$

Combining the two estimates, we obtain the desired result. □

5.Conclusion

In this paper we introduced an implications of the input-output stability property for feedback stabilization, analysis of cascade descriptor systems.

References

[1] S. kawaji and E. Z. Taha, "Feedback Linearization of a Class of Nonlinear Descriptor Systems", Proceedings of the 33rd Conference on Decision and Control, Lake Buena Vista, p.p. 4035-4037, 1994.

[2] L. R. Petzold, " Differential/Algebraic Equations are not ODE's", SIAM Journal SCI. STAT. Comput., Vol. 3, No. 3, p.p. 367-384, 1982.

[3] A. Kumar, P. Daoutidis, "Control of Nonlinear Differential Algebraic Equation Systems with Applications to Chemical Processes", Research Notes in Mathematics Series, Chapman and HALL/CRC, 1999.

[4] He-Sheng Wang, Chee-Fai Yung and Fan-ren Cheng. "H_∞ Control for Nonlinear Descriptor Systems". Springer-Verlag London Limited. 2006.

[5] D. J. Hill and M. Y. Iven, "Stability Theory for Differential/Algebraic Systems with Application to Power Systems," IEEE Circuits Syst., Vol. 37, pp. 1416–1423, Nov. 1990.

[6] I. A. Hiskens and D. J. Hill, "Energy Functions, Transient Stability, and Voltage Behavior in Power Systems with Nonlinear Loads," IEEE Power Syst., Vol. 4, pp. 1525–1533, Nov. 1989.

[7] M. A. Pai, P. W. Sauer, and B. C. Lesieutre, "Static and Dynamic Nonlinear Loads and Structural Stability in Power Systems," Proc. IEEE, Vol.83, pp. 1562–1572, Dec. 1995.

[8] T. Matsumoto, L. O. Chua, H. Kawakami, and S. Ichivaku, "Geometric Properties of Dynamic Nonlinear Networks: Transversality, Local-Solvability and Eventual Passivity," IEEE Circuits Syst., Vol. CAS-28, pp. 406–452, May 1981.

[9] B. C. Haggman and P. R. Bryant, "Solutions of Singular Constrained Differential Equations: A Generalization of Circuits Containing Capacitor- only Loops and Inductor-Only Cutsets, " IEEE Circuits Syst., Vol. CAS-31, pp. 1015–1029, Dec. 1984.

[10] A. Isidori, "Nonlinear Control Systems". Berlin, Germany: Springer-Verlag, 1992.

[11] N. H. McClamroch, "Feedback Stabilization of Control Systems Described by a Class of Nonlinear Differential-Algebraic Equations," Systems Control Lett., Vol. 15, pp. 53–60, 1990.

[12] K. P. Janokowski and H. A. Elmaraghy, "Dynamic Decoupling for Hybrid Control of Rigid/Flexible Joint Robots Interacting with the Environment," IEEE Trans. Robot. Automat., Vol. 8, pp. 519–534, 1992.

[13] H. Krishnan and N. H. McClamroch, "Computation of State Realizations for Control Systems Described by a Class of Linear Differential-Algebraic Equations," Int. J. Control, Vol. 55, pp. 1425–1441, 1992.

[14] H. Krishnan and N. H.Mc Clamroch, "Tracking Reference Inputs in Control Systems Described by a Class of Nonlinear Differential-Algebraic Equations," in Proc. IEEE Decision and Control Conf. , Brighton, U.K, pp. 1796–1801. , 1991

[15] T. L. Liao, "Design of an Adaptive Nonlinear Controller to Improve Stabilization of a Power System," Elect. Power Eng. Syst., Vol. 21, pp. 433–441, 1999.

[16] H. Krishnan and N. H. Mcclamroch, "Tracking in Nonlinear Differential-Algebraic Control Systems with Applications to Constrained Robot Systems," Automatica, Vol. 30, no. 12, pp. 1885–1897, 1994.

[17] A. Isidori, "Nonlinear Control Systems II". Springer, London,1995.

[18] Hassan K. Khalil, "Nonlinear Systems" Second Edition, Prentice Hall, Upper Saddle River, NJ 07458., 1996.
[19] M. Vidyasagar "Nonlinear Systems Analysis". Prentice Hall, Englewood Cliffs, New Jersey 07632. Second Edition, 1993.
[20] M. Krichman and E. D. Sontag, “A version of a Converse Lyapunov Theorem for Input–Output to State Stability,” in Proc. 37th IEEE Conf. Decision Control, pp. 4121–4126. , 1998.
[21] M. Krichman, E. D. Sontag, and Y. Wang, “Input–Output-to-State Stability,” SIAM J. Control Optim., Vol. 39, pp. 1874–1928, 2001.
[22] D. Liberzon. A. S. Morse, and E. D. Sontag, “Output-Input Stability and Minimum-Phase Nonlinear Systems”. IEEE Trans. Automat. Control, 47-3422-436. 2002.
[23] E. D. Sontag, “Smooth Stabilization Implies Coprime Factorization”.
IEEE Trans. Automat. Control, 3443543. 1989.
[24] E. D. Sontag and Y. Wang, “Output-to-State Stability and Detectability of Nonlinear Systems”. Systems Control Lett. ,29279-290. 1997.
[25] D. Liberzon, “Output-Input Stability Implies Feedback Stabilization”. Systems and Control Letters p.p. 53. 237-248. 2004.
[26] Daniel Liberzon, "Output-Input Stability and Feedback Stabilization of Multivariable Nonlinear Control Systems" IEEE. 2005.
[27] J. Wang and C. Chen, "Parametric Adaptive Control Of Multimachine Power Systems With Nonlinear Loads", IEEE Transactions on Circuits and Systems, Vol. 51, No. 2, p.p. 91-100, 2004.
[28] J. Wang and C. Chen, "Exact Linearization of Nonlinear Differential Algebraic Systems", Proceedings 2001 International Conference on Information Technology and Information Networks, Beijing, Vol.4, p.p. 284 – 290, 2001.
[29] J. Su, C. Chen, "Static Var Compensator Control for Power Systems with Nonlinear Loads", IEE, proceeding Generation, Transmission and Distribution, Vol. 151, No.1, p.p. 78-82, 2004.
[30] G.H.Xu, C.Chen, J. Wang , and D.D.Li, "Nonlinear Control for AC/DC Power System with Nonlinear Loads.IEEE 2004.
[31]- Wasen I. khalil “Input-Output Stability For Index One Descriptor Control partial fulfillment of the Requirements for the Degree of Master of Science In Mathematics.Bsrah university 2009.

نتيجة تداخل أستقرارية المدخلات والمخرجات لأنظمة السيطرة الوصفية بدليل واحد

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الخلاصة

قدمنا في هذا البحث دراسة للمصطلح الحديث أستقرارية المدخلات والمخرجات الذي يمثل تعميم لخاصية (minimum–phase) لجميع أنظمة السيطرة العامة. الهدف من هذا البحث هو لتحقيق كيف يكون سلوك خاصية أستقرارية المدخلات والمخرجات تحت نظام مكون من تداخل سلسلة من الأنظمة الوصفية. سوف نبرهن نتيجتين النتيجة الاولى هي النظام وصفي الناتج من تداخل نظام وصفي (0–detectable) مع نظام يمتلك خاصية input–output (stable) هو نظام الوصفي له خاصية (weakly 0–detectable), والنتيجة الثانية هي تعميم مباشر للنتيجة الاولى.

الكلمات المفتاحية : الأستقرارية المدخلات والمخرجات، الأنظمة الوصفية.