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Research Article

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Investigation of Polarization Preservation in Water Channel by Using Mueller Matrix Based on Single Photon Detection

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Abstract: Polarization encoding in quantum key distribution systems is widely used for distributing secret keys between a sender and a receiver. As the polarization-encoded photons pass through the quantum channel, their polarization states might be changed. A prior verification of the channel characteristic is required in order to investigate the effect of the channel on the polarization of the photons passing through it. In this work, an experimental investigation of polarization preservation of a pure water channel is presented. The investigation relies on the verification of the Mueller matrix in order to check the polarization preservation of the channel. The measurements were carried out using a single photon detection module. The results showed that the polarization of the optical pulses is preserved when passing through the pure water channel and it can be used as a quantum channel in the quantum key distribution systems.

Keywords: quantum key distribution, Mueller matrix, polarization encoded pulses, single photon detection.

1. Introduction

The implementation of quantum cryptography is based on quantum mechanics laws to send data with high security between the sender and receiver by quantum key distribution (QKD) [1]. Quantum cryptography depends on optical sources to generate photons as a carrier of information [2]. Single photons can be approximately by highly attenuated coherent source instead of single photon source because single photons are troublesome to be experimentally observed [3]. The security requirement of quantum cryptography is a highly polarization-maintaining channel called the quantum channel known as a medium to transmit and isolate single photons from interactivity with the environment [4]. Quantum cryptography has been experimentally proved over different channels including free space [5,6], and optical fiber [7,8], besides them the water channels such as seawater or oceanic water[9,10]. The fulfillment of underwater QKD dealing with photon polarization depends on the polarization preservation of the water channel. Pure water is an ideal optical medium, while seawater and oceanic water are complicated and have different optical characteristics [9]. In this work, pure water was studied at first by measuring the Mueller matrix to consider how the polarization preserves or changes in the water channel. The experiment was performed to measure the Mueller matrix of the water channel as a primary step before QKD with low power levels by a highly attenuated laser source to generate approximately single photons according to the necessity of quantum cryptography. The laser source must be used within the blue-green optical window of seawater (430 to 570 nm) because it has less attenuation in water in contrast to other wavelengths [10].

2. Theory

Polarization properties of electromagnetic radiation passing through a linear medium can be described by either Stokes vector for partially polarized light or, by Jones vector for completely polarized light [11, 12]. A Stokes vector S gives a description of the polarization state of the light beam, it consists of six various measurements of the polarization state in front of the detector [12, 13].

$$s = \begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} I_H + I_V \\ I_H - I_V \\ I_P - I_M \\ I_R - I_L \end{bmatrix} \quad (1)$$

where S_0, S_1, S_2 and S_3 are the entire components of Stokes vector, I_X is the intensity of light measured with a polarizer oriented at different angles in front of the detector, x: H, V, P, M, R and L.so, (I_H) is for horizontal linear polarization, (I_V) for vertical linear polarization, (I_P) for +45° linear polarization, (I_M) for −45° linear polarization, (I_R) for right circular polarization, and (I_L) for left circular polarization [12]. The medium has an impact on the polarization of light passing through it, which modifies the Stokes or Jones vector so that the medium can be represented by a transfer matrix usually known as the Mueller matrix[11]. The Mueller matrix (M) is 4 × 4 matrix widened in 1943 by Swiss–American physicist Hans Mueller[14]. It describes an optical system as shown in Figure 1, which acts as an operator that connects the incoming light with received light.

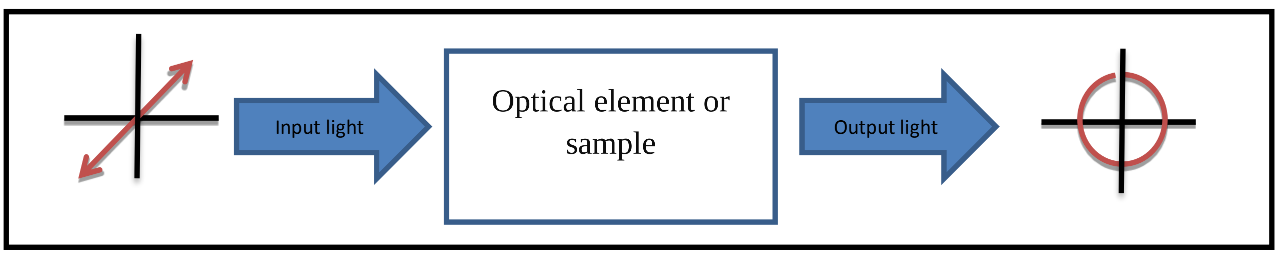

Fig. 1. The transformation of polarization of light by a medium described by the Mueller matrix.

The Mueller–Stokes formalism is given by:

$$s_{out} = M.s_{in} \quad (2)$$

$$\begin{bmatrix} S'_0 \\ S'_1 \\ S'_2 \\ S'_3 \end{bmatrix}_{out} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} . \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix}_{in} \quad (3)$$

where S_{in} and S_{out} are the Stokes vectors of the input and output light, respectively and M is the 4 × 4 Mueller matrix for the system including the optical elements and samples [14,15]. M_{11} describes the transmitted intensity of light [14]. The first row of elements of the Mueller matrix characterizes the diattenuation of materials which describes the difference in transmitted intensity between two incident orthogonal polarizations [14,16]. The first column of elements characterizes the capability of materials to modify the polarization of the light. The enduring elements of the matrix characterize the retardance of the materials [16].

The theoretical normalization of Mueller matrix elements can be achieved by dividing each of the 16 elements by M_{11}. The normalized values are specified[13,14].

$$\begin{bmatrix} 1 & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} where\ m_{ij} = \frac{M_{ij}}{M_{11}} \qquad (4)$$

The normalized elements (m_{ij}) are constrained between ±1[14].Mueller matrix is normalized to possess polarization-dependent information regardless of the intensity of the light source[14]. Some optical elements can preserve the polarization of incoming light, these polarization preservation properties of that element can be recognized from diagonal elements(m_{22},m_{33},m_{44}) of the Mueller matrix which must be equal to 1 for transmitted light, in this case, these elements refer to the material do not have an impact on the polarization state of incoming light. This means the optical element will not change the polarization state [14]. For example, if m_{33} =1 means that incoming linear 45^0 or -45^0 polarized light will remain with the same polarization state. The normalized elements of the Mueller matrix of an optical sample that will not alter the polarization of light will be expressed as,

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (5)$$

This shape of the normalized Mueller matrix for transmitted light is known as a unit matrix[9]. When the light beam passes through a medium with polarization preservation property, the polarization state of the input beam is equal to the polarization state of the output beam as the diagonal elements are equal to 1 and non-diagonal elements are equal to zero[9,14]. Many optical elements can preserve the polarization state of the light such as a neutral density filter [10], transmissive liquid crystals [13], and oceanic water[17] and in this work, the interested sample is (a water channel) while each individual other linear optical element has its own matrix [18].

3. Experimental Work

The experimental setup is shown in Figure 2.

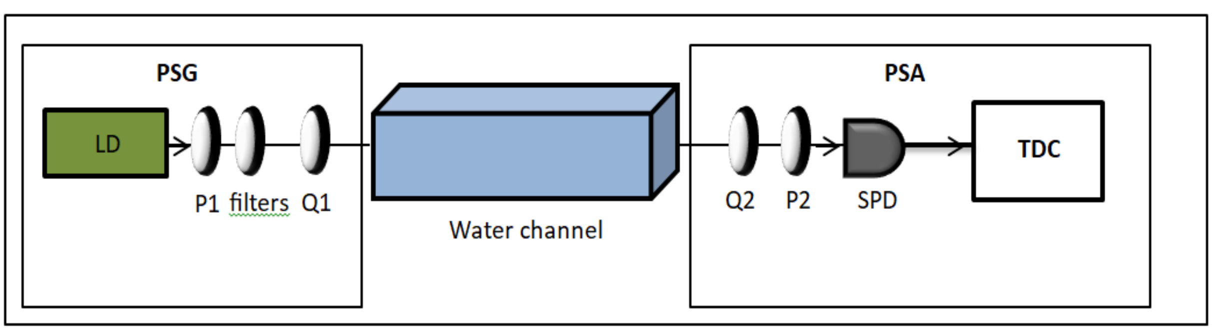

Fig. 2. Experimental setup.

The setup consists of a polarization state generator (PSG) which contains a laser diode **(LD)** (THORLABS, LP520-SF15). The center wavelength of the LD is 520 nm, and the average output optical power is 12 mW with a repetition rate of 1 MHz and a pulse width of 5 ns. Followed by a polarizer (P1), a combination of filters (THORLABS, NE510B-A T=0.1%, NE520B-A T=0.01%, NE13B-A T=0.05% where T is the filter's transmittance) for reaching the desired mean photon number per pulse (μ), and quarter wave plate (Q1) to obtain circular polarization state), followed by water channel; (a glass tank of (120cm × 80cm ×15cm) filled with 90L of pure water. After the water channel, the polarization state analyzer (PSA) is arranged. PSA consists of a quarter wave plate (Q2) and, a polarizer (P2) acting as an analyzer. In addition, the single photon detector module (SPDM) (ID100 single photon detector from id-quantique with a detection efficiency of 35% at 500 nm). The output signal of SPDM was monitored using (ID800) an 8-channel time-to-digital converter (TDC).
To conclude the set of 16 linear equations, the following states were generated as six different polarization states: horizontal (H), vertical(V), 45° linear polarization(D), -45° linear polarization(A), right circular polarization (R) and left circular polarization (L), by changing both of polarizer and analyzer angles and set the quarter wave plates (Q1) at 45^0 , and (Q2) at -45^0 in order to acquire linear polarization state. For circular polarization state, both (Q1) and (Q2) are set at either 45^0 or -45^0. Table 1 lists the elements of Mueller matrix ,when each element represents a combination of P_1 and P_2 .

Table 1. The elements of Mueller Matrix

M_{11}	**M_{12}**	**M_{13}**	**M_{14}**
(HH+HV+VH+VV)	(HH+HV+VH-VV)	(DH+DV-AH-AV)	(RH+RV-LH-LV)
M_{21}	**M_{22}**	**M_{23}**	**M_{24}**
(HH-HV+VH-VV)	(HH-HV-VH+VV)	(DH-DV-AH+AV)	(RH-RV-LH+LV)
M_{31}	**M_{32}**	**M_{33}**	**M_{34}**
(HD-HA+VD-VA)	(HD-HA-VD+VA)	(DD-DA-AD+AA)	(RD-RA-LD+LA)
M_{41}	**M_{42}**	**M_{43}**	**M_{44}**
(HR-HL+VR-VL)	(HR-HL-VR+VL)	(DR-DL-AR+AL)	(LL-RL-LR+RR)

4. Steps of measurements

1- At the beginning, the appropriate power of the LD must be set, by the aid of using optical filters (attenuators) the desired power that produces (μ) equal to 0.1 can be reached , this value of μ is the suitable value used in QKD systems.
2-The power of LD was measured before and after the water channel using a power meter [PM100D] in order to calculate the transmittance.
3-To calculate the elements of mueller matrix, for example (M_{21}), four measurements are necessary: (HH), (VV), (HV) and (VH). These measurements can be achieved as follows: the measurement (HH) is obtained by setting P1 at 90^0, P2 is set at 90^0, (Q1) and (Q2) were adjusted at $+45^0$, -45^0 respectively to preserve linear polarization state, after setting the polarizers and quarter wave plates at the appropriate orientations, the output signal from the SPDM is recorded as counts by using TDC. This step was repeated for (VV), (HV) and (VH) by changing the polarization angles of P1 and P2 once at 0^0 to get vertical polarization state and 90^0 to get horizontal polarization state.
4- In order to conclude element M23, the required measurements are (DH, DV, AH, and AV), which can be achieved by changing the angle of P1 at D= 450 and A= -450 linear polarization state, P2 is set at 90 for

horizontal polarization state and 0 for vertical polarization state, and taking into consideration both of Q1 and Q2 were adjusted at 45,-45 to keep linear polarization state.
5-If the measurement requires a circular polarization state, such as M14 (RH,RV,LH,LV). The measurement was achieved by setting P1 at 45 0 and P2 was changed between 00 and 900, Q1 was rotated at 450 to obtain right circular polarization R and 1350 to obtain left circular polarization L, and Q2 was adjusted at 0 0 to keep the linear polarization at PSA.
6- In order to calculate the entire set of Mueller matrix elements, the previous procedure was repeated considering different polarization states for each element.

5. Results and discussion

The experimental results can be summarized as follows:
The measured average optical power (Pavg) of LD before and after the water channel is 8.75µW and 6.94 µW respectively, and the transmittance of the water channel was 0.793%.
The counts measured by TDC resulting from the procedure of finding Mueller matrix elements are recorded for all the desired polarization states, as listed in Table 2.
Using the results from step 2, (Pavg) is calculated using the following equation (Table 2):

$$p_{avg} = N * P_{single} \tag{6}$$

Where:
N: the number of counts
Psingle: the average power of a single photon and it is calculated as:

$$P_{single} = h\nu f = \frac{hc}{\lambda} \tag{7}$$

Where:
h: planks constant 6.63*10-34 j.s.
ν: the frequency in Hz.
c: speed of light in free space 3*108 m/s.
λ: the wavelength of laser beam.
f: repetition rate in MHz.
$P_{single} = \frac{6.63*10^{-34}*3*10^{8}}{520*10^{-9}} 10^{6} = 3.825 * 10^{-13}\, w.$

The normalized value for each calculated value of the power is found by dividing all 16 elements by M11 as listed in Table 2.

The resulting Mueller matrix for the water channel used (pure water) is shown below:

$$M_{pure\ water} = \begin{vmatrix} 1.000 & -0.908 & 0.128 & 0.014 \\ -0.953 & 0.972 & 0.007 & -0.008 \\ -0.152 & -0.257 & 0.989 & 0.161 \\ -0.058 & 0.035 & 0.080 & 0.992 \end{vmatrix}$$

Table. 2. Calculated Mueller matrix elements

M_{ij}	**Calculation of Mueller matrix elements based on TDC counts**	P_{avg} **(nw)**	m_{ij}
M11	HH+HV+VH+VV 64+39+221+25383= 25707	9.8329275	1.000
M12	HH+HV+VH-VV 52+32+558-24829= -24187	-9.2515275	-0.908
M13	PH+PV-MH-MV 8585+6206-5177-6300=3314	1.267605	0.128
M14	RH+RV-LH-LV 14378+10826-13802-11038= 364	0.13923	0.014
M21	HH-HV+VH-VV 157-31+393-24736= -24499	-9.3708675	-0.953
M22	HH-HV-VH+VV 98-24-160+25144= 25058	9.564685	0.972
M23	PH-PV-MH+MV 8780-7967-5052+6289= 2050	0.784125	0.007
M24	RH-RV-LH+LV 10729-14465-11694+13302= -2128	-0.81396	-0.008
M31	HP-HM+VP-VM 78-107+9977-13868= -3920	-1.4994	-0.152
M32	HP-HM-VP+VM 52-37-15393+8747= -6631	-2.5363575	-0.257
M33	PP-PM-MP+MM 14393-294-87+11415= 25427	9.7258275	0.989
M34	RP-RM-LP+LM 12464-12318-10278+14287= 4155	1.5892875	0.161
M41	HR-HL+VR-VL 58-35+9989-11581= -1506	-0.576045	-0.058
M42	HR-HL-VR+VL 44-34-10905+11796= 901	0.3446325	0.035
M43	PR-PL-MR+ML 9056-7455-3923+4373=2057	0.7868025	0.080
M44	LL-RL-LR+RR 12088-248-222+13898= 25516	9.75987	0.992

In this work, the water channel was prepared in order to simulate seawater, and its characteristics were studied by the 4 × 4 Mueller matrix to investigate its polarization preservation properties, the 16 elements of the Mueller matrix were experimentally measured using an attenuated coherent optical source [LD and a combination of filters] to get an approximate mean photon number equals to 0.1. Polarizers and quarter-wave plates at PSG and PSA are changed, and the low-level optical pulses are detected by SPD and recorded in terms of counts by TDC. At each step, the orientation of the optical elements was changed so as to obtain new readings for each element of the matrix. The resultant calculated 16 elements were normalized to total transmitted intensity and the achieved Mueller matrix is nearly to the unit matrix which shows the ability of the water channel to preserve the polarization of the input beam.

6. Conclusions

According to the resulted normalized Mueller matrix of the water channel, and in comparison with an ideal normalized matrix it was found that the Mueller matrix of transmitted light passing through the water channel is nearly a unit matrix. The minimum value of the diagonal element is equal to 0.972, and the maximum value of the non-diagonal element is equal to 0.161, which means that the water channel used did not alter the polarization state of the light passing through it. In consequence, the Mueller matrix can be used as a method to study the effect of a medium on the polarization of the light passing through it.

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التحقق من المحافظة على الاستقطاب في قناة الماء باستخدام مصفوفة مولر بالاعتماد على كشف الفوتون المنفرد

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الخلاصة: ان بروتوكول توزيع المفتاح الكمي المعتمد على ترميز الاستقطاب هو بروتوكول يتم استخدامه بشكل واسع من اجل مشاركة مفتاح سري ما بين المرسل والمستقبل. بينما تنتقل النبضات البصرية التي يتم ترميز استقطابها من خلال القناة الكمية, فان حالات الاستقطاب لتلك النبضات من الممكن ان تتغير. لذلك من المطلوب التحقق المسبق من صفات القناة من اجل التحقق من تأثير القناة على استقطاب النبضات التي تمر من خلالها. في هذا العمل, التحقق عمليا من المحافظة على الاستقطاب للماء النقي يتم تقديمه. التحقق يستند على تحقيق مصفوفة مولرمن اجل التأكد من محافظة القناة على الاستقطاب. تم اجراء القياسات باستخدام كواشف الفوتون المنفرد. اظهرت النتائج انه يتم المحافظة على استقطاب النبضات البصرية عند مرورها من خلال قناة الماء النقي ويمكن استخدامها كقناة كمية في توزيع انظمة المفتاح الكمي.