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The Brauer trees of the symmetric group modulo p =11

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Abstract:

In this paper we find the Brauer trees of the group$\bar{S}_n$ modulo$p$=11 which can give the irreducible modular spin characters of S_{21} modulo $p = 11$, also we give the 11-decompostion matrix of spin characters ofS_{21} .

Key Words : 2000 ; 20C20 ; 20C25 ; 20C30 ; 20C40 .

1. Introduction:

One of the oldest example of permutation representation is furnished by Cayley'stheorem [8]. A group representation of finite group G over a filed K is a group homomorphism $T: G \to GL(n, K)$ [2].Schur showed that the symmetric group S_n has a representation group $\bar{S}_n$ which is of order 2(n!), and it has a central subgroup Z={1,-1} such that $\bar{S}_n/Z \cong S_n$ [12] . The representations of $\bar{S}_n$ fall into two classes [9] , [12]:

1) Those which have Z in their kernel ; these are called the ordinary representation of S_n, the irreducible representations and characters of S_n are indexed by the partitions of n .
2) The representations which do not have Z in their kernel ; these representations are called spin(projective) representations of S_n the irreducible spin representations are indexed by the partitions of n with distinct parts which are called bar partitions of n [10].

For p=11 Yaseen [13] was found the modular irreducible spin character of S_n for$11 \leq n \leq 14$ and for $n = 15,16$ was found by Yaseen[14] and for $n = 17,18$ was found by Jassim[5] and for n=19 by Jassim and Taban[7] and for n=20 by Jassim[6] .

In this paper we found the Brauer trees for the spin characters for S_{21}

2. Preliminaries:

Let p be prime and G be a group of order $p^a.m$ where $(p, m) = 1$

1. Any spin character ofS_n can be written as a linear combination, with

non-negative integer coefficients, of the irreducible spin characters [2].

2. Let H b a subgroup of Gthen [4].
   a) If ψ is principal character of H, then $\psi \uparrow G$ is principal character of G .
   b) If ψ is principal character of G, then $\psi \downarrow H$ is principal character of H .
   c) If ϕ is a modular character of H, then $\phi \uparrow G$ is a modular character of G.
   d) If ϕ is a modular character of G, then $\phi \downarrow H$ is a modular character of H.

3. Let α, β be bar partitions of n which are not p-bar cores. Then$\langle\alpha\rangle(and\ \langle\alpha\rangle'$ if α is odd) and β(and β' if β is odd) are in the same p-block if an only if $\widetilde{\langle\alpha\rangle} = \widetilde{\langle\beta\rangle}$. if α be a bar partition of n and $\langle\alpha\rangle = \widetilde{\langle\alpha\rangle}$, then $\langle\alpha\rangle$(and $\langle\alpha\rangle'$ if α is odd) forms a p-block of defect 0[13].
4. Let $\alpha = (\alpha_1, \dots, \alpha_m)$ be a bar partition of n. The values of characters $\langle\alpha\rangle$ and $\langle\alpha\rangle'$ differ only on the class corresponding to α on which they have values $\mp i^{\frac{n-m+1}{2}}\sqrt{(\alpha_1 \dots \alpha_m)/2}; i = \sqrt{-1}$[9].
5. The degree of the spin characters $\langle\alpha\rangle = \langle\alpha_1, \dots, \alpha_m\rangle$ is: $\deg\langle\alpha\rangle = 2^{[\frac{n-m}{2}]} \frac{n!}{\prod_{i=1}^{m}(\alpha_i!)} \prod_{1\le i\le j\le m}(\alpha_i - \alpha_j)/(\alpha_i + \alpha_j)$ [9],[10].

6. Let B be the block of defect one and let b be the number of p-conjugate characters to the irreducible character χ of G then [11].
   a) There exists a positive integer number N such that the irreducible ordinary characters of G are lying in the block B divided into two disjoint classes : $B_1 = \{\chi \in B | b\ deg\ \chi\ \equiv N\ mod\ p^a\}$, $B_2 = \{\chi \in B | b\ deg\ \chi \equiv -N\ mod\ p^a\}$
   b) Each coefficient of the decomposition matrix of B is one or zero.
   c) If α_1 and α_2 are not p-conjugate characters and belong to the same classesabove, then they have no irreducible modular spin character in common.
   d) For every irreducible ordinary character χ in B_1 , there exists irreducible ordinary character φ in B_2 such that they have one irreducible modular character in common with one multiplicity .

7. If C is principle character of G for an odd prime p and all the entries in C are divisible by a non-negative integer q , then $(1\backslash q)C$ is a principal character of G [4].

8. Let $\alpha = (\alpha_1, \dots, \alpha_m)$ be a bar partition of n not a p bar core , let B be the block containing $\langle\alpha\rangle$ then :
   a) If $n - m - m_0$ is even , then all irreducible modular spin characters in B are double.
   b) If $n - m - m_0$ is odd, then all irreducible modular spin characters in B are associate.

(Here$m_0$the number of parts of α divisible by p)[4]

9. If C is principal character of G for a prime $p$then:$deg\ C \equiv 0\ mode\ p^a$,[3].
10. Let $\beta_{1,}^*, \beta_2, \beta_2', \beta_3, \beta_3'$ be modular spin characters where β_1^* is a double character , $\beta_2 \neq \beta_2'$ are associate modular spin characters (real), and $\beta_3 \neq \beta_3'$ are associate modular spin characters (complex) . Let $\varphi_{1,}^*, \varphi_2, \varphi_2', \varphi_3, \varphi_3'$ be irreducible modular spin character , where φ_1^* is a double character , $\varphi_2 \neq$

φ'_2 *and* $\varphi_3 \neq \varphi'_3$ are associate irreducible modular spin characters (real),(complex) respectively then[13]:

a) $\beta^*_{1,}, \beta_2, \beta'_2$ contains φ_3 *and* φ'_3 with the same multiplicity , β^*_1 which contains φ_2 *and* φ'_2 with the same multiplicity .

b) β_3 *and* β'_3 contains $\varphi^*_{1,}, \varphi_2, \varphi'_2$ with the same multiplicity.

c) $\varphi_3$is constituent of β_3 with the same multiplicity as that of φ'_3 *in* β'_3.

3. Notation:

p.s.	principle spin character.
p.i.s.	Principle indecomposable spin character.
m.s.	Modular spin character.
i.m.s.	Irreducible modular spin character.
≡	Equivalence mod 11 .
d_i	The p.i.s. of S_n.
D_i	The p.i.s. of S_{n-1}.

4. The Brauer trees of the symmetric group$\bar{S}_n$,*p*=11:

The group S_{21} has 114 of irreducible spin characters and $\bar{S}_{21}$ has 99 of $(11,\alpha)$-regular classes , then the decomposition matrix of the spin characters of S_{21} , p=11 has 114 rows and 99 columns[13].

By using (preliminaries 3) , there are 34 11-blocks of S_{21}, these block are B_1,B_2,B_3, … , B_{34} ,24 of them of defectzero which are B$_{11}$,B$_{12}$, … ,B$_{34}$and the others of defect one .

Lemma (4.1):

The Brauer tree for this blockB_1 is:

$$\langle 21\rangle^* \_ \langle 11,10\rangle = \langle 11,10\rangle' \_ \langle 10,9,2\rangle \_^* \langle 10,8,3\rangle \_^* \langle 10,7,4\rangle \_^* \langle 10,6,5\rangle^*$$

Proof:

deg$\langle 21\rangle^* \equiv$ *deg*$\langle 10,9,2\rangle^* \equiv$*deg*$\langle 10,6,5\rangle^* \equiv$*1 mod 11*

;deg$(\langle 11,10\rangle + \langle 11,10\rangle') \equiv$*deg*$\langle 10,8,3\rangle^* \equiv$*deg*$\langle 10,6,5\rangle^* \equiv$ -1.

By using $(r,\bar{r})$-inducing of p.i.s. of S_{20}[see appendix I] to S_{21} we have :

$D_1 \uparrow^{(2,10)} S_{21} = d_1$,$D_3 \uparrow^{(2,10)} S_{21} = d_2$,$D_7 \uparrow^{(2,10)} S_{21} = d_3$, $D_9 \uparrow^{(2,10)} S_{21} = d_4$, $D_{69} \uparrow^{(3,9)} S_{21} = d_5$; (no sub sum of these p.c.≡0)

So we have theBrauer tree for the block B_1 , and the decomposition matrix for this block in Table (1).

Lemma (4.2):

The Brauer tree for this block $B_2$is:

$$\begin{matrix} \langle 20,1\rangle \_ \langle 12,9\rangle \diagdown & & \diagup \langle 9,8,3,1\rangle \_ \langle 9,7,4,1\rangle \_ \langle 9,6,5,1\rangle \\ & \langle 11,9,1\rangle^* & \\ \langle 20,1\rangle' \_ \langle 12,9\rangle' \diagup & & \diagdown \langle 9,8,3,1\rangle' \_ \langle 9,7,4,1\rangle' \_ \langle 9,6,5,1\rangle' \end{matrix}$$

Proof:

deg$\{\langle 12,9\rangle, \langle 12,9\rangle', \langle 9,8,3,1\rangle, \langle 9,8,3,1\rangle', \langle 9,6,5,1\rangle, \langle 9,6,5,1\rangle'\} \equiv$*7;*

deg$\{\langle 20,1\rangle, \langle 20,1\rangle', \langle 11,9,1\rangle^*, \langle 9,7,4,1\rangle, \langle 9,7,4,1\rangle'\} \equiv$ *-7;*

By using (0,1)-inducing of p.i.s. for S_{20} to S_{21} we have:

$D_1 \uparrow^{(0,1)} S_{21} = k_1$,$D_2 \uparrow^{(0,1)} S_{21} = k_2$,$D_5 \uparrow^{(0,1)} S_{21} = d_5$,$D_6 \uparrow^{(0,1)} S_{21} = d_6$,$D_7 \uparrow^{(0,1)} S_{21} = d_7$,$D_8 \uparrow^{(0,1)} S_{21} = d_8$,$D_9 \uparrow^{(0,1)} S_{21} = d_9$, $D_7 \uparrow^{(0,1)} S_{21} = d_{10}$.

Since $\langle 12,9,1\rangle, \langle 12,9,1\rangle'$ are p.i.s. of S_{22}(of defect zero in S_{22}, p=11) , and

$\langle 12,9,1\rangle \downarrow_{(0,1)} S_{21} = \langle 12,9\rangle + \langle 11,6,1\rangle^* = d_3$are p.s. (Preliminaries 2)

$\langle 12,9,1\rangle' \downarrow_{(0,1)} S_{21} = \langle 12,9\rangle' + \langle 11,6,1\rangle^* = d_4$are p.s. .Either$d_4$ is subtracted from k_1 or not

Suppose d_4 is not subtracted from k_1 , in this case we have $\langle 12,9\rangle' - \langle 20,1\rangle - \langle 20,1\rangle'$ is m.s. for S_{21} , but $(\langle 12,9\rangle' - \langle 20,1\rangle - \langle 20,1\rangle') \downarrow_{(0,1)} S_{20} = \langle 11,9\rangle^* - \langle 20\rangle - \langle 20\rangle'$ is not m.s. forS_{20}, then d_4 is subtracted from $k_1$and $d_3$subtracted from k_2 (d_3, d_4 and $k_1, k_2$are conjugate) .And the decomposition matrix for the block B_2 is given in Table (2).

Lemma (4.3):

The Brauer tree for this blockB_3 is :

$\langle 19,2\rangle\_\langle 13,8\rangle$ \ / $\langle 10,8,2,1\rangle\_\langle 8,7,4,2\rangle\_\langle 8,6,5,2\rangle$

$\langle 11,8,2\rangle^*$

$\langle 19,2\rangle'\_\langle 13,8\rangle'$ / \ $\langle 10,8,2,1\rangle'\_\langle 8,7,4,2\rangle'\_\langle 8,6,5,2\rangle'$

Proof:

$deg\{\langle 19,2\rangle, \langle 19,2\rangle', \langle 11,8,2\rangle^*, \langle 8,7,4,2\rangle, \langle 8,7,4,2\rangle'\} \equiv 8$

$deg\{\langle 13,8\rangle, \langle 13,8\rangle', \langle 10,8,2,1\rangle, \langle 10,8,2,1\rangle', \langle 8,6,5,2\rangle, \langle 8,6,5,2\rangle'\} \equiv -8$

By inducing

$D_{16} \uparrow^{(4,8)} S_{21} = k_1$,$D_{12} \uparrow^{(2,10)} S_{21} = k_2$, $D_{71} \uparrow^{(0,1)} S_{21} = d_5$,$D_{72} \uparrow^{(0,1)} S_{21} = d_6$,$D_{14} \uparrow^{(2,10)} S_{21} = k_3$,$D_{15} \uparrow^{(2,10)} S_{21} = k_4$.

Thus, we have the approximation matrix inTable (3i)

Table (3i)

	ψ_1	ψ_2	φ_5	φ_6	ψ_3	ψ_4	φ_1	φ_2
$\langle 19,2\rangle$	1						a	
$\langle 19,2\rangle'$	1							a
$\langle 13,8\rangle$	1	1					b	
$\langle 13,8\rangle'$	1	1						b
$\langle 11,8,2\rangle^*$		2	1	1			c	c
$\langle 10,8,2,1\rangle$			1		1		d	
$\langle 10,8,2,1\rangle'$				1	1			d
$\langle 8,7,4,2\rangle$					1	1	f	
$\langle 8,7,4,2\rangle'$					1	1		f
$\langle 8,6,5,2\rangle$						1	h	
$\langle 8,6,5,2\rangle'$						1		h
	k_1	k_2	d_5	d_6	k_3	k_4	X	Y

Since $\langle 19,2\rangle \neq \langle 19,2\rangle'$ on $(11, \alpha)$-regular classes then either k_1 is split or there are other two columns. Suppose there are two columns such as X and $Y$to describe columns X and Y :

$\langle 19,2\rangle \downarrow S_{20}$=$(\langle 18,2\rangle^*)^1 + (\langle 19,1\rangle^*)^1$has 2 of i.m.s. in S_{20} (see appendix I) and from Table (3i) we have $a \in \{0,1\}$.

If $a = 1$, $k_1$musthave a conjugate p.c. so $\langle 19,2\rangle$has 3 m.c. contradiction since $\langle 19,2\rangle$ have at most two m.c. so $a = 0$ and k_1 split to give $\langle 19,2\rangle + \langle 18,3\rangle$ and $\langle 19,2\rangle' + \langle 18,3\rangle'$.

Either k_2 split or there are other columns$X$ and Y (as above with a=0)

1) Since $\langle 13,8\rangle \downarrow S_{20} = (\langle 12,8\rangle^*)^2 + (\langle 13,7\rangle^*)^2$ has 4 of i.m.s. we have $b \in \{0,1\}$, otherwise we have contradiction .
2) Since $\langle 11,8,2\rangle^* \downarrow S_{20} = (\langle 10,8,2\rangle)^1 + (\langle 10,8,2\rangle')^1 + \langle 11,7,2\rangle^2 + (\langle 11,7,2\rangle')^2 + (\langle 11,8,1\rangle)^2 + (\langle 11,8,1\rangle')^2$ has 10 of i.m.s. we have $c \in \{0,1,2,3\}$.
3) $\langle 10,8,2,1\rangle \downarrow S_{20}$= $(\langle 9,8,2,1\rangle^*)^2 + (\langle 10,7,2,1\rangle^*)^2 + (\langle 10,8,2\rangle)^1$ has 5 of i.m.s. we have $d \in \{0,1,2\}$.
4) $\langle 8,7,4,2\rangle \downarrow S_{20}$=$(\langle 8,6,4,2\rangle^*)^1 + (\langle 8,7,3,2\rangle^*)^2 + (\langle 10,8,2\rangle^*)^2$ has 5 of i.m.s. we have $f \in \{0,1,2\}$.
5) $\langle 8,6,5,2\rangle \downarrow S_{20}$= $(\langle 7,6,5,2\rangle^*)^1 + (\langle 8,6,4,2\rangle^*)^1 + (\langle 8,6,5,1\rangle^*)^1$ has 3 of i.m.s. we have $h \in \{0,1\}$.

Now if $b = 1$

1) There is no i.m.s. in $\langle 13,8\rangle \downarrow S_{20} \cap \langle 10,8,2,1\rangle \downarrow S_{20}$, so d = 0 ;
2) There is no i.m.s. in $\langle 13,8\rangle \downarrow S_{20} \cap \langle 8,7,4,2\rangle \downarrow S_{20}$, so f = 0 ;
3) There is no i.m.s. in $\langle 13,8\rangle \downarrow S_{20} \cap \langle 8,6,5,2\rangle \downarrow S_{20}$, so h = 0 .

We,get the possible columns :
$X = \langle 13,8\rangle + c\langle 11,8,2\rangle^*$,
$Y = \langle 13,8\rangle' + c\langle 11,8,2\rangle^*, c \in \{0,1,2,3\}$
$deg\, X \equiv 0$ and $deg\, Y \equiv 0$ only when $b = c = 1$.
So k_2 splits to give $\langle 13,8\rangle + \langle 11,8,2\rangle^*$ and $\langle 13,8\rangle' + \langle 11,8,2\rangle^*$ which is the same when b=0 .
Since $\langle 8,6,5,2\rangle \neq \langle 8,6,5,2,\rangle'$ on $(11,\alpha)$-regular classes then either k_4 is split or there are other two columns. If we Suppose there are two columns such as $X\ an\ Y$as in Table (3i) with a=b=0 . To describe $X\ an\ Y$:
If $h = 1$:

1) There is no i.m.s. in $\langle 8,6,5,2\rangle \downarrow S_{20} \cap \langle 11,8,2\rangle^* \downarrow S_{20}$, so c = 0;
2) There is no i.m.s. in $\langle 8,6,5,2\rangle \downarrow S_{20} \cap \langle 10,8,2,1\rangle \downarrow S_{20}$, so d = 0 .

We, get the possible columns :
$X = f\langle 8,7,4,2\rangle + \langle 8,6,5,2\rangle$,
$Y = f\langle 8,7,4,2\rangle' + \langle 8,6,5,2\rangle', f \in \{0,1,2,3\}$.
, $\deg X \equiv 0\ and\ \deg Y \equiv 0\ only\ when\ f = 1$.
So k_4 splits to
$\langle 8,7,4,2\rangle + \langle 8,6,5,2\rangle\ and\ \langle 8,7,4,2\rangle' + \langle 8,6,5,2\rangle'$ which is the same when $h = 0$.
Since $\langle 10,8,2,1\rangle \neq \langle 10,8,2,1\rangle'$ on $(11,\alpha)$-regular classes then the last column k_3 must split to
$\langle 10,8,2,1\rangle + \langle 8,7,4,2\rangle\ and\ \langle 10,8,2,1\rangle' + \langle 8,7,4,2\rangle'$[Preliminaries 10],so we get the Brauer tree for the block B_3, and the decomposition matrix for this block in Table (3).

Lemma (4.4):
The Brauer tree for this block $B_4$is:

⟨18,3⟩_⟨14,7⟩ \\ / ⟨10,7,3,1⟩_⟨9,7,3,2⟩_⟨7,6,5,3⟩
⟨11,7,3⟩*
⟨18,3⟩′_⟨14,7⟩′ / \\ ⟨10.7.3.1⟩′_⟨9.7.3.2⟩′_⟨7.6.5.3⟩′

Proof:
$deg\{\langle 18,3\rangle, \langle 18,3\rangle', \langle 11,8,2\rangle^*, \langle 9,7,3,2\rangle, \langle 9,7,3,2\rangle'\} \equiv 9$
$deg\{\langle 14,7\rangle, \langle 14,7\rangle', \langle 10,7,3,1\rangle, \langle 10,7,3,1\rangle', \langle 7,6,5,3\rangle, \langle 7,6,5,3\rangle'\} \equiv -9$

By using $(r,\bar{r})$-inducing of p.i.s. of $S_{20}$to S_{21} we have:
$D_{16} \uparrow^{(3,9)} S_{21} = k_1$,$D_{17} \uparrow^{(3,9)} S_{21} = k_2$,$D_{73} \uparrow^{(0,1)} S_{21} = d_5$,$D_{74} \uparrow^{(0,1)} S_{21} = d_6$,$D_{19} \uparrow^{(3,9)} S_{21} = k_3$,$D_{20} \uparrow^{(3,9)} S_{21} = k_4$.
We have the approximation matrix (Table (4i))

Table (4i))

	ψ_1	ψ_2	φ_5	φ_6	φ_3	φ_4	φ_1	φ_2
⟨18,3⟩	1						a	
⟨18,3⟩′	1							a
⟨14,7⟩	1	1					b	
⟨14,7⟩′	1	1						b
⟨11,7,3⟩*		2	1	1			c	c
⟨10,7,3,1⟩			1		1		d	
⟨10,7,3,1⟩′				1	1			d
⟨9,7,3,2⟩					1	1	f	
⟨9,7,3,2⟩′					1	1		f
⟨7,6,5,3⟩						1	h	
⟨7,6,5,3⟩′						1		h
	k_1	k_2	d_5	d_6	k_3	k_4	X	Y

Since $\langle 18,3\rangle \neq \langle 18,3\rangle'$ on $(11,\alpha)$-regular classes then either k_1 is split or there are other two columns, suppose there are two columns such as $X\ and\ Y$

To describe columns $X\ and\ Y$
$\langle 18,3\rangle \downarrow S_{20}$= $((\langle 18,2\rangle^{*})^{1} + (\langle 17,3\rangle^{*})^{1}$has 2 of i.m.s. and from (Table (4i)) we have a = 0, otherwise we have contradiction so k_1 must split to give$\langle 18,3\rangle + \langle 14,7\rangle\ and\ \langle 18,3\rangle' + \langle 14,7\rangle'$. Either k_2 split or there are other column X and Y (as above with a=0)

1) Since $\langle 14,7\rangle \downarrow S_{20}$ = $((\langle 14,6\rangle^{*})^{2} + (\langle 13,7\rangle^{*})^{2}$ has 4 of i.m.s,we have $b \in \{0,1\}$.
2) Since $\langle 11,7,3\rangle^{*} \downarrow S_{20}$ = $((\langle 10,7,3\rangle)^{1} + (\langle 10,7,3\rangle')^{1} + \langle 11,6,3\rangle^{2} + (\langle 11,6,3\rangle')^{2} + (\langle 11,7,2\rangle)^{2} + (\langle 11,7,2\rangle')^{2}$ has 10 of i.m.s. we have $c \in \{0,1,2,3\}$.
3) Since $\langle 10,7,3,1\rangle \downarrow S_{20} =$ $((\langle 9,7,3,1\rangle^{*})^{1} + (\langle 10,6,3,1\rangle^{*})^{2} + (\langle 10,7,2,1\rangle^{*})^{1} + (\langle 10,7,3\rangle)^{1}$has 5 of i.m.s, we have $d \in \{0,1,2\}$.
4) $\langle 9,7,3,2\rangle \downarrow S_{20}$=$(\langle 8,7,3,2\rangle^{*})^{2} + (\langle 8,6,3,2\rangle^{*})^{2} + (\langle 9,7,3,1\rangle^{*})^{1}$ has 5 of i.m.s. we have $f \in \{0,1,2\}$.
5) $\langle 7,6,5,3\rangle \downarrow S_{20}$= $(\langle 7,6,4,3\rangle^{*})^{1} + (\langle 7,6,5,2\rangle^{*})^{1}$has 3 of i.m.s. we have $h \in \{0,1\}$.

Now if $b = 1$:

1) There is no i.m.s. in $\langle 14,7\rangle \downarrow S_{20} \cap \langle 10,7,3,1\rangle \downarrow S_{20}$, so d = 0 ;
2) There is no i.m.s. in $\langle 14,7\rangle \downarrow S_{20} \cap \langle 9,7,3,2\rangle \downarrow S_{20}$, so f = 0 ;
3) There is no i.m.s. in $\langle 13,8\rangle \downarrow S_{20} \cap \langle 7,6,5,3\rangle \downarrow S_{20}$, so h = 0 .

We, get the possible columns :
$X = \langle 14,7\rangle + c\langle 11,7,3\rangle^{*}$, $Y = \langle 14,7\rangle' + c\langle 11,7,2\rangle^{*}$, $b = 1, c \in \{0,1,2,3,4,5,6\}$
$\deg X \equiv 0\ and\ \deg Y \equiv 0\ only\ when\ b = c = 1$.
So k_2 splits to give $\langle 14,7\rangle + \langle 11,7,3\rangle^{*}$ and $\langle 14,7\rangle' + \langle 11,7,2\rangle^{*}$ which is the same when b=0 .
Since $\langle 7,6,5,3\rangle \neq \langle 7,6,5,3,\rangle'$ on $(11, \alpha)$-regular classes then either $k_4$is split or there are other two columns. If we Suppose there are two columns such as $X\ and\ Y$ (as in Table (4i)) with a=b=0 . To describe $X\ and\ Y$:
Now if h=1

1) There is no i.m.s. in $\langle 7,6,5,3\rangle \downarrow S_{20} \cap \langle 11,7,2\rangle^{*} \downarrow S_{20}$, so c = 0;
2) There is no i.m.s. in $\langle 7,6,5,3\rangle \downarrow\cap \langle 10,7,3,1\rangle \downarrow S_{20}$, so d = 0 .

We, get the possible columns
$X = f\langle 9,7,3,2\rangle + \langle 7,6,5,3\rangle$, $Y = f\langle 9,7,3,2\rangle' + \langle 7,6,5,3\rangle'$, $f \in \{0,1,2\}$
$\deg X \equiv 0\ and\ \deg Y \equiv 0\ only\ when\ f = 1$ .So k_4 splits to
$\langle 9,7,3,2\rangle + \langle 7,6,5,3\rangle\ and\ \langle 9,7,3,2\rangle' + \langle 7,6,5,3\rangle'$, which is the same when b=0 .
Since $\langle 10,7,3,1\rangle \neq \langle 10,7,3,1\rangle'$ on $(11, \alpha)$-regular classes then the last columns k_3 must split to $\langle 10,7,3,1\rangle + \langle 9,7,3,2\rangle\ and\ \langle 10,7,3,1\rangle' + \langle 9,7,3,2\rangle'$.So we get the Brauer tree for the block B_4, and the decomposition matrix for this block in Table (4).

Lemma (4.5):
The Brauer tree for the block B_5 is:

$$\langle 18,2,1\rangle^{*}\_\langle 13,7,1\rangle^{*}\_\langle 12,7,2\rangle^{*}\_\langle 11,7,2,1\rangle = \langle 11,7,2,1\rangle'\_\langle 8,7,3,2,1\rangle^{*}\_\langle 7,6,5,2,1\rangle^{*}$$

Proof:

$$deg\langle 13,7,1\rangle^{*} \equiv deg(\langle 11,7,2,1\rangle + \langle 11,7,2,1\rangle') \equiv deg\langle 7,6,5,2,1\rangle^{*} \equiv 8deg\langle 18,2,1\rangle^{*}$$
$$\equiv deg\langle 12,7,2\rangle^{*} \equiv deg\langle 8,7,3,2,1\rangle^{*} \equiv -8$$

By inducing $D_{16} \uparrow^{(0,1)} S_{20} = d_1$,$D_{28} \uparrow^{(7,5)} S_{20} = d_2$, $D_{18} \uparrow^{(0,1)} S_{20} = 2d_3$,$D_{19} \uparrow^{(0,1)} S_{20} = d_4$,$D_{20} \uparrow^{(0,1)} S_{20} = d_5$.
Using [Preliminaries 7] we get d_3 is p.c. .So, we get the Brauer tree for the block B_5, and the decomposition matrix for this block in (Table (5)).

Lemma (4.6):
The Brauer tree for the block B_6 is:

$$\begin{array}{ccc} \langle 17,4\rangle\_\langle 15,6\rangle \diagdown & & \diagup \langle 10,6,4,1\rangle\_\langle 9,6,4,2\rangle\_\langle 8,6,4,3\rangle \\ & \langle 11,6,4\rangle^* & \\ \langle 17,4\rangle'\_\langle 15,6\rangle' \diagup & & \diagdown \langle 10,6,4,1\rangle^*\_\langle 9,6,4,2\rangle^*\_\langle 8,6,4,3\rangle^* \end{array}$$

Proof:
$deg\{\langle 17,4\rangle, \langle 17,4\rangle', \langle 11,6,4\rangle^*, \langle 9,6,4,2\rangle, \langle 9,6,4,2\rangle'\} \equiv 10$
$deg\{\langle 15,6\rangle, \langle 15,6\rangle', \langle 10,6,4,1\rangle, \langle 10,6,4,1\rangle', \langle 8,6,4,3\rangle, \langle 8,6,4,3\rangle'\} \equiv -10$
By inducing $D_{21} \uparrow^{(4,8)} S_{21} = k_1$, $D_{22} \uparrow^{(4,8)} S_{21} = k_2$, $D_{75} \uparrow^{(0,1)} S_{21} = d_5$, $D_{76} \uparrow^{(0,1)} S_{21} = d_6$, $D_{24} \uparrow^{(4,8)} S_{21} = k_3$, $D_{25} \uparrow^{(4,8)} S_{21} = k_4$.
Thus, we have the approximation matrix Table ($6i$)

Table ($6i$)

	ψ_1	ψ_2	φ_5	φ_6	ψ_3	ψ_4	φ_1	φ_2
$\langle 17,4\rangle$	1						a	
$\langle 17,4\rangle'$	1							a
$\langle 15,6\rangle$	1	1					b	
$\langle 15,6\rangle'$	1	1						b
$\langle 11,6,4\rangle^*$		2	1	1			c	c
$\langle 10,6,4,1\rangle$			1		1		d	
$\langle 10,6,4,1\rangle'$				1	1			d
$\langle 9,6,4,2\rangle$					1	1	f	
$\langle 9,6,4,2\rangle'$					1	1		f
$\langle 8,6,4,3\rangle$						1	h	
$\langle 8,6,4,3\rangle'$						1		h
	k_1	k_2	d_5	d_6	k_3	k_4	X	Y

Since $\langle 17,4\rangle \neq \langle 17,4\rangle'$ on $(11,\alpha)$-regular classes then either k_1 is split or there are other two columns, suppose there are two columns such as X an Y .To describe columns X and Y :

1) Since $\langle 17,4\rangle \downarrow S_{20}$= $(\langle 17,3\rangle^*)^1 + (\langle 16,4\rangle^*)^1$has 2 of i.m.s. and from Table (6i)) we have a = 0, so k_1 must split to give $\langle 17,4\rangle + \langle 15,6\rangle$ and $\langle 17,4\rangle' + \langle 15,6\rangle'$.

Either k_2 split or there are another column X and Y (as above with a =0)

1) Since $\langle 15,6\rangle \downarrow S_{20}$ = $(\langle 15,5\rangle^*)^2 + (\langle 14,6\rangle^*)^2$ has 4 of i.m.s.we have $b \in \{0,1\}$.
2) Since $\langle 11,6,4\rangle^* \downarrow S_{20}$ = $(\langle 11,6,3\rangle)^2 + (\langle 11,6,3\rangle')^2 + \langle 11,5,4\rangle^2 + (\langle 11,5,4\rangle')^2 + (\langle 10,6,4\rangle)^1 + (\langle 10,6,4\rangle')^1$ has 10 of i.m.s. we have $c \in \{0,1,2,3\}$.
3) Since$\langle 10,6,4,1\rangle \downarrow S_{20}$=$(\langle 10,6,4\rangle)^1 + (\langle 10,6,3,1\rangle^*)^2 + (\langle 10,5,4,1\rangle^*)^2 + (\langle 9,6,4,1\rangle^*)^1$ has 6 of i.m.s. we have $d \in \{0,1,2,3\}$.
4) $\langle 9,6,4,2\rangle \downarrow S_{20}$=$(\langle 9,6,4,1\rangle^*)^1 + (\langle 9,6,3,2\rangle^*)^2 + (\langle 9,5,4,2\rangle^*)^2 + (\langle 8,6,4,2\rangle^*)^1$ has 6 of i.m.s. we have $f \in \{0,1,2\}$.
5) $\langle 8,6,4,3\rangle \downarrow S_{20}$= $(\langle 8,6,4,2\rangle^*)^1 + (\langle 8,5,4,3\rangle^*)^1 + (\langle 7,6,4,3\rangle^*)^1$ has 3 of i.m.s. we have $h \in \{0,1\}$.

Now if $b = 1$:

1) There is no i.m.s. in $\langle 15,6\rangle \downarrow S_{20} \cap \langle 10,6,4,1\rangle \downarrow S_{20}$, so d = 0 ;
2) There is no i.m.s. in $\langle 15,6\rangle \downarrow S_{20} \cap \langle 9,6,4,2\rangle \downarrow S_{20}$, so f = 0 ;
3) There is no i.m.s. in $\langle 15,6\rangle \downarrow S_{20} \cap \langle 8,6,4,3\rangle \downarrow S_{20}$, so h = 0 .

We, get the possible columns :
$X = \langle 15,6\rangle + c\langle 11,6,4\rangle^*$, $Y = \langle 15,5\rangle' + c\langle 11,6,4\rangle^*$, $b = 1, c \in \{0,1,2,3\}$
$\deg X \equiv 0$ and $\deg Y \equiv 0$ $only$ $when$ $b = c$.
So k_2 splits to give $\langle 15,6\rangle + \langle 11,6,4\rangle^*$ and $\langle 15,6\rangle' + \langle 11,6,4\rangle^*$ which is the same when b=0 .

Since $\langle 8,6,4,3\rangle \neq \langle 8,6,5,3,\rangle'$ on $(11,\alpha)$-regular classes then either k_4 is split or there are other two columns. If we Suppose there are two columns such as $X\ an\ Y$ (as in Table (6i)) with a=b=0 . To describe $X\ an\ Y$:

If $h = 1$

1) There is no i.m.s. in $\langle 8,6,4,3\rangle \downarrow S_{20} \cap \langle 11,6,4\rangle^{*} \downarrow S_{20}$, so c = 0;
2) There is no i.m.s. in $\langle 8,6,4,3\rangle \downarrow S_{20} \cap \langle 10,6,41\rangle \downarrow S_{20}$, so d = 0 .

We, get the possible columns

$X = f\langle 9,6,4,2\rangle + \langle 8,6,4,3\rangle$, $Y = f\langle 9,6,4,2\rangle' + \langle 8,6,4,3\rangle'$, $f \in \{0,1,2\}, h = 1$

$\deg X \equiv 0\ and\ \deg Y \equiv 0\ only\ when\ f = h$.

So k_4 splits to $\langle 9,6,4,2\rangle + \langle 8,6,4,3\rangle\ and\ \langle 9,6,4,2\rangle' + \langle 8,6,4,3\rangle'$ which is the same when $h = 0$.

Since $\langle 10,6,4,1\rangle \neq \langle 10,6,4,1\rangle'$ on $(11,\alpha)$-regular classes then the last columns k_3 must split to $\langle 10,6,4,1\rangle + \langle 9,6,4,2\rangle\ and\ \langle 10,6,4,1\rangle' + \langle 9,6,4,2\rangle'$ [Preliminaries 10].

So we get the Brauer tree for the block B_6, and the decomposition matrix for this block in Table (6).

Lemma (4.7):

The Brauer tree for this block $B_7$is:

$$\langle 17,3,1\rangle^{*}\_\langle 14,6,1\rangle^{*}\_\langle 12,6,3\rangle^{*}\_\langle 11,6,3,1\rangle = \langle 11,6,3,1\rangle'\_\langle 9,6,3,2,1\rangle^{*}\_\langle 7,6,4,3,1\rangle^{*}$$

Proof:

$$deg\langle 14,6,1\rangle^{*} \equiv deg(\langle 11,6,3,1\rangle + \langle 11,6,3,1\rangle') \equiv deg\langle 7,6,4,3,1\rangle^{*} \equiv 6$$
$$deg\langle 17,3,1\rangle^{*} \equiv deg\langle 12,6,3\rangle^{*} \equiv deg\langle 9,6,3,2,1\rangle^{*} \equiv -6$$

By inducing$D_{21} \uparrow^{(0,1)} S_{20} = d_1$,$D_{28} \uparrow^{(0,1)} S_{20} = d_2$,$D_{30} \uparrow^{(3,9)} S_{20} = d_3$,$D_{24} \uparrow^{(0,1)} S_{20} = d_4$,$D_{25} \uparrow^{(0,1)} S_{20} = d_5$.

So, we get the Brauer tree for the block B_7, and the decomposition matrix for this block in (Table (7)).

Lemma (4.8):

The Brauer tree for this block$B_8$is:

$$\langle 16,4,1\rangle^{*}\_\langle 15,5,1\rangle^{*}\_\langle 12,5,4\rangle^{*}\_\langle 11,5,4,1\rangle = \langle 11,5,4,1\rangle'\_\langle 9,5,4,2,1\rangle^{*}\_\langle 8,5,4,3,1\rangle^{*}$$

Proof:

$deg\langle 15,5,1\rangle^{*} \equiv deg(\langle 11,5,4,1\rangle + \langle 11,5,4,1\rangle') \equiv deg\langle 8,5,4,3,1\rangle^{*} \equiv 6$

$deg\langle 16,4,1\rangle^{*} \equiv deg\langle 12,5,4\rangle^{*} \equiv deg\langle 9,5,4,2,1\rangle^{*} \equiv -6$

By inducing:$D_{36} \uparrow^{(0,1)} S_{20} = d_1$,$D_{43} \uparrow^{(0,1)} S_{20} = d_2$,$D_{30} \uparrow^{(3,9)} S_{20} = d_3$,$D_{39} \uparrow^{(0,1)} S_{20} = d_4$,$D_{40} \uparrow^{(0,1)} S_{20} = d_5$.

So, we get the Brauer tree for the block B_8, and the decomposition matrix for this block in (Table (8)).

Lemma (4.9):

The Brauer tree for this block $B_9$is:

$$\langle 16,3,2\rangle^{*}\_\langle 14,5,2\rangle^{*}\_\langle 13,5,2\rangle^{*}\_\langle 11,5,3,2\rangle = \langle 11,5,3,2\rangle'\_\langle 10,5,3,2,1\rangle^{*}\_\langle 7,5,4,3,2\rangle^{*}$$

Proof :

$deg\langle 14,5,2\rangle^{*} \equiv \deg(\langle 11,5,3,2\rangle + \langle 11,5,3,2\rangle') \equiv deg\langle 7,5,4,3,2\rangle^{*} \equiv 6$

$deg\langle 16,3,2\rangle^{*} \equiv deg\langle 13,5,2\rangle^{*} \equiv deg\langle 10,5,3,2,1\rangle^{*} \equiv -6$

By using (2,10)-inducing we get :

$D_{41} \uparrow^{(2,10)} S_{20} = d_1$,$D_{43} \uparrow^{(2,10)} S_{20} = d_2$,$D_{45} \uparrow^{(2,10)} S_{20} = d_3$,$D_{47} \uparrow^{(2,10)} S_{20} = d_4$,$D_{49} \uparrow^{(3,9)} S_{20} = d_5$.

So, we get the Brauer tree for the block B_9, and the decomposition matrix for this block in (Table (9)).

Lemma (4.10):

The Brauer tree for the block B_{10} is:

$$\begin{array}{ccccc} \langle 15,3,2,1\rangle\_\langle 14,4,2,1\rangle\_\langle 13,4,3,1\rangle\_\langle 12,4,3,2\rangle & \backslash & & / & \langle 6,5,4,3,2,1\rangle \\ & & \langle 11,4,3,2,1\rangle^{*} & & \\ \langle 15,3,2,1\rangle'\_\langle 14,4,2,1\rangle'\_\langle 13,4,3,1\rangle'\_\langle 12,4,3,2\rangle' & / & & \backslash & \langle 6,5,4,3,2,1\rangle' \end{array}$$

Proof:

$$deg\{\langle 14,4,2,1\rangle,\langle 12,4,3,2\rangle,\langle 6,5,4,3,2,1\rangle\} \equiv 8\, deg\{\langle 15,3,2,1\rangle,\langle 13,4,3,1\rangle,\langle 11,4,3,2,1\rangle^{*}\} \equiv -8$$

By using $(r,\bar{r})$-inducing we get:

$D_{51}\uparrow^{(0,1)} S_{20}=d_1$,$D_{52}\uparrow^{(0,1)} S_{20}=d_2$,$D_{53}\uparrow^{(0,1)} S_{20}=d_3$,$D_{54}\uparrow^{(0,1)} S_{20}=d_4$, $D_{70}\uparrow^{(2,10)} S_{20}=k_1$, $D_{55}\uparrow^{(0,1)} S_{20}=k_2$, $D_{56}\uparrow^{(0,1)} S_{20}=k_3$, $D_{57}\uparrow^{(0,1)} S_{20}=k_4$, $D_{59}\uparrow^{(0,1)} S_{20}=d_9$,$D_{60}\uparrow^{(0,1)} S_{20}=d_{10}$.

Now on $(11,\alpha)$ regular classes we have $k_2+k_3-k_4=k_1$

Since $\langle 12,4,3,2,1\rangle,\langle 12,4,3,2,1\rangle'$ are p.i.s. of S_{22} of defect zero in S_{22}

$\langle 12,4,3,2,1\rangle\downarrow(1,0)=\langle 12,4,3,2\rangle+\langle 11,4,3,2,1\rangle^{*}$ are p.i.s ;

$\langle 12,4,3,2,1\rangle'\downarrow(1,0)=\langle 12,4,3,2\rangle'+\langle 11,4,3,2,1\rangle^{*}$ are p.i.s. .So $\langle 12,4,3,2\rangle+\langle 12,4,3,2\rangle'+\langle 11,4,3,2,1\rangle^{*}$ must be split to give $\langle 12,4,3,2\rangle+\langle 11,4,3,2,1\rangle^{*}=d_7$ and $\langle 12,4,3,2\rangle'+\langle 11,4,3,2,1\rangle^{*}=d_8$.

$k_1=k_2+k_3-d_7-d_8$.

The only possibility is k_2-d_8 , k_3-d_7 (otherwise negative entries) so k_1 split to give k_2-d_8= d_5 and k_3-d_7=d_6 .

So, we get the Brauer tree for the block B_{10}, and the decomposition matrix for this block in (Table (10)).

Appendix I

(The decomposition matrix for the spin characters of S_{20} , p=11) [A.H.Jassim]

The spin characters	The decomposition matrix for the block B_1									
⟨20⟩	1									
⟨20⟩′		1								
⟨11,9⟩*	1	1	1	1						
⟨10,9,1⟩			1		1					
⟨10,91⟩′				1		1				
⟨9,8,3⟩					1		1			
⟨9,8,3⟩′						1		1		
⟨9,7,4⟩							1		1	
⟨9,7,4⟩′								1		1
⟨9,6,5⟩									1	
⟨9,6,5⟩′										1
	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}

The spin characters	The decomposition matrix for the block B_2				
⟨19,1⟩*	1				
⟨12,8⟩*	1	1			
⟨11,8,1⟩		1	1		
⟨11,8,1⟩′		1	1		
⟨9,8,2,1⟩*			1	1	
⟨8,7,4,1⟩*				1	1
⟨8,6,5,1⟩*					1
	D_{11}	D_{12}	D_{13}	D_{14}	D_{15}

The spin characters	The decomposition matrix for the block B_3				
$\langle 18,2\rangle^*$	1				
$\langle 13,7\rangle^*$	1	1			
$\langle 11,7,2\rangle$		1	1		
$\langle 11,7,2\rangle'$		1	1		
$\langle 10,7,2,1\rangle^*$			1	1	
$\langle 8,7,3,2\rangle^*$				1	1
$\langle 7,6,5,2\rangle^*$					1
	D_{16}	D_{17}	D_{18}	D_{19}	D_{20}

The spin characters	The decomposition matrix for the block B_4				
$\langle 17,3\rangle^*$	1				
$\langle 14,6\rangle^*$	1	1			
$\langle 11,6,3\rangle$		1	1		
$\langle 11,6,3\rangle'$		1	1		
$\langle 10,6,3,1\rangle^*$			1	1	
$\langle 9,6,3,2\rangle^*$				1	1
$\langle 7,6,4,3\rangle^*$					1
	D_{21}	D_{22}	D_{23}	D_{24}	D_{25}

The spin characters	The decomposition matrix for the block B_5									
$\langle 17,2,1\rangle$	1									
$\langle 17,2,1\rangle'$		1								
$\langle 13,6,1\rangle$	1		1							
$\langle 13,6,1\rangle'$		1		1						
$\langle 12,6,2\rangle$			1		1					
$\langle 12,6,2\rangle'$				1		1				
$\langle 11,6,2,1\rangle^*$					1	1	1	1		
$\langle 8,6,3,2,1\rangle$							1		1	
$\langle 8,6,3,2,1\rangle'$								1		1
$\langle 7,6,4,2,1\rangle$									1	
$\langle 7,6,4,2,1\rangle'$										1
	D_{26}	D_{27}	D_{28}	D_{29}	D_{30}	D_{31}	D_{32}	D_{33}	D_{34}	D_{35}

The spin characters	The decomposition matrix for the block B_6				
$\langle 16,4\rangle^*$	1				
$\langle 15,5\rangle^*$	1	1			
$\langle 11,5,4\rangle$		1	1		
$\langle 11,5,4\rangle'$		1	1		
$\langle 10,5,4,1\rangle^*$			1	1	
$\langle 9,5,4,2\rangle^*$				1	1
$\langle 8,5,4,3\rangle^*$					1
	D_{36}	D_{37}	D_{38}	D_{39}	D_{40}

The spin characters	The decomposition matrix for the block B_7									
$\langle 16,3,1\rangle$	1									
$\langle 16,3,1\rangle'$		1								
$\langle 14,5,1\rangle$	1		1							
$\langle 14,5,1\rangle'$		1		1						
$\langle 12,5,3\rangle$			1		1					
$\langle 12,5,3\rangle'$				1		1				
$\langle 11,5,3,1\rangle^*$					1	1	1	1		
$\langle 9,5,3,2,1\rangle$							1		1	
$\langle 9,5,3,2,1\rangle'$								1		1
$\langle 7,5,4,3,1\rangle$									1	
$\langle 7,5,4,3,1\rangle'$										1
	D_{41}	D_{42}	D_{43}	D_{44}	D_{45}	D_{46}	D_{47}	D_{48}	D_{49}	D_{50}

The spin characters	The decomposition matrix for the block B_8									
⟨15,3,2⟩	1									
⟨15,3,2⟩′		1								
⟨14,4,2⟩	1		1							
⟨14,4,2⟩′		1		1						
⟨13,4,3⟩			1		1					
⟨13,4,3⟩′				1		1				
⟨11,4,3,2⟩*					1	1	1	1		
⟨10,4,3,2,1⟩							1		1	
⟨10,4,3,2,1⟩′								1		1
⟨6,5,4,3,2⟩									1	
⟨6,5,4,2⟩′										1
	D_{51}	D_{52}	D_{53}	D_{54}	D_{55}	D_{56}	D_{57}	D_{58}	D_{59}	D_{60}

Appendix II

The decomposition matrix for the spin characters of S_{21} ,p=11

Table (1)

degree of characters module p=11	The spin characters	The decomposition matrix for the blockB_1				
1	⟨21⟩*	1				
5	⟨11,10⟩	1	1			
5	⟨11,10⟩′	1	1			
1	⟨10,9,2⟩*		1	1		
10	⟨10,8,3⟩*			1	1	
1	⟨10,7,4⟩*				1	1
10	⟨10,6,5⟩*					1
		d_1	d_2	d_3	d_4	d_5

Table (2)

degree of characters module p=11	The spin characters	The decomposition matrix for the blockB_2									
4	⟨20,1⟩	1									
4	⟨20,1⟩′		1								
7	⟨12,9⟩	1		1							
7	⟨12,9⟩′		1		1						
4	⟨11,9,1⟩*			1	1	1	1				
7	⟨9,8,3,1⟩					1		1			
7	⟨9,8,3,1⟩′						1		1		
4	⟨9,7,4,1⟩							1		1	
4	⟨9,7,4,1⟩′								1		1
7	⟨9,6,5,1⟩									1	
7	⟨9,6,5,1⟩′										1
		d_6	d_7	d_8	d_9	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}

Table (3)

degree of characters module p=11	The spin characters	The decomposition matrix for the blockB_3									
8	⟨19,2⟩	1									
8	⟨19,2⟩′		1								
3	⟨13,8⟩	1		1							
3	⟨13,8⟩′		1		1						
8	⟨11,8,2⟩*			1	1	1	1				
3	⟨10,8,2,1⟩					1		1			
3	⟨108,2,1⟩′						1		1		
8	⟨8,7,4,2⟩							1		1	
8	⟨8,7,4,2⟩′								1		1
3	⟨8,6,5,2⟩									1	
3	⟨8,6,5,2⟩′										1
		d_{16}	d_{17}	d_{18}	d_{19}	d_{20}	d_{21}	d_{22}	d_{23}	d_{24}	d_{25}

Table (4)

degree of characters module p=11	The spin characters	The decomposition matrix for the blockB_4									
2	⟨18,3⟩	1									
2	⟨18,3⟩′		1								
9	⟨14,7⟩	1		1							
9	⟨14,7⟩′		1		1						
2	⟨11,7,3⟩*			1	1	1	1				
9	⟨10,7,3,1⟩					1		1			
9	⟨10,7,3,1⟩′						1		1		
2	⟨9,7,3,2⟩							1		1	
2	⟨9,7,3,2⟩′								1		1
9	⟨7,6,5,3⟩									1	
9	⟨7,6,5,3⟩′										1
		d_{26}	d_{27}	d_{28}	d_{29}	d_{30}	d_{31}	d_{32}	d_{33}	d_{34}	d_{35}

Table (5)

degree of characters module p=11	The spin characters	The decomposition matrix for the blockB_5				
3	⟨18,2,1⟩*	1				
8	⟨13,7,1⟩*	1	1			
3	⟨12,7,2⟩*		1	1		
8	⟨11,7,2,1⟩			1	1	
8	⟨11,7,2,1⟩′			1	1	
3	⟨8,7,3,2,1⟩*				1	1
8	⟨7,6,5,2,1⟩*					1
		d_{36}	d_{37}	d_{38}	d_{39}	d_{40}

Table (6)

degree of characters module p=11	The spin characters	The decomposition matrix for the blockB_6									
10	⟨17,7⟩	1									
10	⟨17,7⟩′		1								
1	⟨15,6⟩	1		1							
1	⟨15,6⟩′		1		1						
10	⟨11,6,4⟩*			1	1	1	1				
1	⟨10,6,4,1⟩					1		1			
1	⟨10,6,4,1⟩′						1		1		
10	⟨9,6,4,2⟩							1		1	
10	⟨9,6,4,2⟩′								1		1
1	⟨8,6,4,3⟩									1	
1	⟨8,6,4,3⟩′										1
		d_{41}	d_{42}	d_{43}	d_{44}	d_{45}	d_{46}	d_{47}	d_{48}	d_{49}	d_{50}

Table (7)

degree of characters module p=11	The spin characters	The decomposition matrix for the blockB_7				
6	⟨17,3,1⟩*	1				
5	⟨14,6,1⟩*	1	1			
6	⟨12,6,3⟩*		1	1		
5	⟨11,6,3,1⟩			1	1	
5	⟨11,6,3,1⟩′			1	1	
6					1	1
5						1
		d_{51}	d_{52}	d_{53}	d_{54}	d_{55}

Table (8)

degree of characters module p=11	The spin characters	The decomposition matrix for the block B_8				
5	⟨16,4,1⟩*	1				
6	⟨15,5,1⟩*	1	1			
5	⟨12,5,4⟩*		1	1		
6	⟨11,5,4,1⟩			1	1	
6	⟨11,5,4,1⟩′			1	1	
5					1	1
6						1
		d_{56}	d_{57}	d_{58}	d_{59}	d_{60}

Table (9)

degree of characters module p=11	The spin characters	The decomposition matrix for the blockB_9				
5	⟨16,3,2⟩*	1				
6	⟨14,5,2⟩*	1	1			
5	⟨13,5,2⟩*		1	1		
6	⟨11,5,3,2⟩			1	1	
6	⟨11,5,3,2⟩′			1	1	
5	⟨10,5,3,2,1⟩*				1	1
6	⟨7,5,4,3,2⟩*					1
		d_{61}	d_{62}	d_{63}	d_{64}	d_{65}

Table (10)

degree of characters module p=11	The spin characters	The decomposition matrix for the block B_{10}									
3	⟨15,3,2,1⟩	1									
3	⟨15,3,2,1⟩′		1								
8	⟨14,4,2,1⟩	1		1							
8	⟨14,4,2,1⟩′		1		1						
3	⟨13,4,3,1⟩			1		1					
3	⟨13,4,3,1⟩′				1		1				
8	⟨12,4,3,2⟩					1		1			
8	⟨12,4,3,2⟩′						1		1		
3	⟨11,4,3,2,1⟩*							1	1	1	1
8	⟨6,5,4,3,2,1⟩									1	
8	⟨6,5,4,3,2,⟩′										1
		d_{66}	d_{67}	d_{68}	d_{69}	d_{70}	d_{71}	d_{72}	d_{73}	d_{74}	d_{75}

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شجرات براور للزمرة التناظرية $\overline{S}_{21}$ عندما $p = 11$

المستخلص

في هذا البحث وجدنا شجرات براور للزمرة $\overline{S}_{21}$ عندما $p = 11$ والتي تمكننا من حساب المشخصات الإسقاطيةالمعيارية للزمرة التناظرية S_{21} عندما $p = 11$ ، كذلك حصلنا على مصفوفة التجزئة للمشخصات الإسقاطية للزمرة S_{21} .