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Variational Iteration Method for solving Two-Dimensional Reaction-Diffusion Brusselator System

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Abstract.
This paper proposes the investigation of the variational iteration method to find the solutions for Two-dimensional Reaction-Diffusion Brusselator System . Analytic solutions of the linear partial differential equations with initial data are obtained. It has been shown that the method is quite efficient and is practically well suited for use in these problems. Moreover, a necessary and sufficient condition for convergence is given. Numerical test is investigated to illustrate the pertinent features of the proposed algorithm.

Key Words; Variational iteration method**,** Reaction-Diffusion equation.

1. Introduction

In 1978, Inokuti *et al*. [14] proposed a general Lagrange multiplier method to solve non-linear problems, which was first proposed to solve problems in quantum mechanics. In 1998, the Lagrange multiplier method is modified by He [8-12] into an iteration method that is called variational iteration method (VIM). It is used to solve effectively, easily, and accurately a large class of non-linear problems with approximations converging rapidly to accurate solutions, where the approximate solution of the VIM in the main is readily obtained upon using the obtained Lagrange multiplier and on the selective initial approximate. The variational iteration method changes the differential equation to a recurrence sequence of functions, where the limit of that sequence is considered as the solution of the partial differential equations. The main advantage of the method is that, it can be applied directly to all types of nonlinear differential and integral equations, homogeneous or inhomogeneous, with constant or variable coefficients [1, 15-17]. Moreover, the proposed method is capable of greatly reducing the size of computational work while still maintaining high accuracy of the numerical solution.

One of such important reaction - diffusion equations is known as Brusselator system, where the Reaction-diffusion models frequently arise in the study of chemical and biological systems. A large number of physical problems such as the formation of ozone by atomic oxygen through a triple collision and enzymatic reactions .The brusselator system is occured. The reaction-diffusion Brusselator system contains a pair of variables intermediates with reactant and product chemicals whose concentrations are controlled, and which is used to describe mechanism of chemical reaction-diffusion with non-linear oscillations (see Nicolis and Prigogine [6], Prigogine and Lefever [7] and Tyson [13]).

In recent years, much research has been focused on the numerical solution of systems of brusselator system, Adomian [5] used a decomposition method for the numerical solution of the reaction-diffusion brusselator system. Twizell *et al*. [4] developed a second order finite difference method for diffusion free brusselator system. Wazwaz [2] used modified Adomian decomposition method for which problem whereas Ang [19] applied the dual-reciprocity boundary element method for numerical solution of the reaction-diffusion Brusselator system.

In this paper, we investigate the model of two- dimensional reaction-diffusion brusselator system by using variational iteration method. This study shows that, in this particular context, an averaged description can capture only large scale features of the exact solution, the convergence of which can be made as precise as necessary.

Consider the following 2D reaction-diffusion brusselator system:

$$\frac{\partial u(x,y,t)}{\partial t} = \beta + u^2 v(x,y,t) - (\omega+1)u(x,y,t) + \mu\nabla^2 u(x,y,t) = 0 \quad ; \quad x,y \text{ in } \Omega\times J,\, t\geq 0$$

$$\frac{\partial v(x,y,t)}{\partial t} = \omega u(x,y,t) - u^2 v(x,y,t) + \mu\nabla^2 v(x,y,t) = 0 \quad ,x,y \text{ in } \Omega\times J\,,t\geq 0$$

together with initial conditions

$$\left.\begin{aligned} u(x,y,0) &= h(x,y) \\ v(x,y,0) &= k(x,y) \end{aligned}\right\} \quad ,(x,y)\in\Omega$$

…(1)

where $u(x, y, t)$ and $v(x, y, t)$ represent dimensionless concentrations of two reactants in Banach space, ω and β be constants concentrations of the two reactants, μ (diffusion coefficient) is a constant h and k are known functions, Ω and J are opened and bounded in R^2 , ∇^2 is Laplace operator of second partial differential equation.

The organization of this paper is as follows; section 2 gives brief ideas of VIM. and, the sufficient conditions are presented to guarantee the convergence of the method. In section, 4; present the results of numerical test for Two-dimensional reaction-diffusion Brusselator system are presented to illustrate the effectiveness and the useful of the variational iteration method and discussion the results. Conclusions are presented in the last section.

2. Variational Iteration Method:

2.1 concept of VIM

The idea of variational iteration method depends on the general Lagrange's multiplier method [14]. This method has a main feature, which is the solution of a mathematical problem with linearization assumption used as initial approximation or trial function. This approximation converges rapidly to an accurate solution [11].

To illustrate the basic concepts of the VIM, we consider the following nonlinear differential equation:

$$L(u)+N(u)+R(u)=g(x,t), \qquad \dots (2a)$$

with specified initial condition: $u_0 = u(x,0)$ …(2b)

where L and R are a linear operator , N is a nonlinear operator, and $g(x)$ is an inhomogeneous term. According to the VIM [3], we can construct a correction functional as follows:

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(\tau)[L(u_n(x,\tau)) + N(\tilde{u}_n(x,\tau) + R(\tilde{u}_n(x,\tau) - \tilde{g}(x,\tau)]d\tau \qquad \text{… (3)}$$

where $\lambda(\tau)$ is called the general Lagrange multiplier [1,15,16], which can be identified optimally via the variational theory and integration by parts. The iterates u_n denote the nth order approximate solutions, where n refers to the number of iterates. $\tilde{u}_n$ is considered as restricted variations so that their variations are zero, $\delta\tilde{u}_n = 0$ [8].

The successive approximation u_{n+1}, $n \geq 0$ of the solution $u(x,t)$ will be obtained by using the determined Lagrange multiplier and any selective function u_0.

To find the optimal value of $\lambda(\tau)$, we applied the restricted variations of correction functional (3) integrating by part to optained $\lambda(\tau)$, in the following form:

$$\begin{aligned}\delta u_{n+1}(x,t) &= \delta u_n(x,t) + \delta\int_0^t \lambda(\tau)[(u_n(x,\tau))_\tau + \\ &\qquad R(\tilde{u}_n(x,\tau)) + N(\tilde{u}_n(x,\tau)) - \tilde{g}(x,t)]d\tau \\ &= \delta u_n(t) + \lambda\delta u_n(\tau)\Big|_{\tau=t} - \int_0^t \lambda'\delta u_n(\tau)d\tau = 0\end{aligned}$$

Consequently, we can write the equation (3) as a successive approximation as follows:

$$\begin{aligned}u_1(x,t) &= u_0(x,t) + \int_0^t (optimal\ \ value\ of\ \ \lambda)[L(u_0(x,\tau)) + \\ &\qquad R(u_0(x,\tau)) + N(u_0(x,\tau)) - g(x,t)]d\tau \\ u_2(x,t) &= u_1(x,t) + \int_0^t (optimal\ \ value\ of\ \ \lambda)[L(u_1(x,\tau)) + \\ &\qquad R(u_1(x,\tau)) + N(u_1(x,\tau)) - g(x,t)]d\tau \\ u_3(x,t) &= u_2(x,t) + \int_0^t (optimal\ \ value\ of\ \ \lambda)[L(u_2(x,\tau)) + \\ &\qquad R(u_2(x,\tau)) + N(u_2(x,\tau)) - g(x,t)]d\tau\end{aligned}$$

So on, where by finding the nth order approximation. Finally summing up iterates to yield,

$$U_M = \sum_{n=0}^{M} u_n, \quad M \geq 1$$

The general solution obtained by the VIM can be written as:

$$u(x,t) = \lim_{M\to\infty} U_M$$

2.2 Convergence of the VIM

Here , we will study the convergence analysis as the same manner in [18] of the variational iteration method to the nonlinear equations. Let us consider the Banach space X , with the set of applications, $u:\Omega \to \Re$,

$$\int_{\Omega} u(x)^2 dx < \infty \ ,$$

and the associated norm: $\left\| u \right\|^2 = \int_{\Omega} u^2(x)\ dx$.

The VIM is convergent if the condition of the following theorem are satisfied .

Theorem 1[18]**:** (Banach's fixed-point theorem)

Assume that X be a Banach space and $A: X \to X$ is a nonlinear mapping, and suppose that $\left\| A[u] - A[\bar{u}] \right\| \le \gamma \left\| u - \bar{u} \right\|$ For some constant $\gamma < 1$. Then A has a unique fixed point.

According to the theorem 2.2, for the nonlinear mapping

$$A[u] = u(x,t) + \int_0^t \lambda [F(u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial \tau}, \frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 u}{\partial \tau^2}, \frac{\partial^2 u}{\partial x \partial \tau})] d\tau$$

.

A sufficient condition for the convergence of the variational iteration method is the strictly the contraction of A , such that for $u, \bar{u} \in X$ we have $\left\| u \right\| \le M$ and $\left\| \bar{u} \right\| \le M$, for M >0.

3 . Numerical Test and discussion

In this section, we present the numerical test for the 2D reaction-diffusion Brusselator system. The variational iteration method will be used for the numerical solution of reaction-diffusion Brusselator system. The accuracy of the scheme is measured in terms of absolute errors.

3.1 Test problem

For a particular case, we take $\omega = 1, \beta = 0, \mu = 0.25$ [20] ,thus real rile (1) becomes;

$$\frac{\partial u}{\partial t} = u^2 v - 2u + 0.25\nabla^2 u = 0 \quad ,(x,y)\in(0,1)\times(0,1)$$
$$\frac{\partial v}{\partial t} = u - u^2 v + 0.25\nabla^2 v = 0 \quad ,(x,y)\in(0,1)\times(0,1)$$

subject to the initial condition

$$u(x,y,0) = \exp(-x-y)$$
$$v(x,y,0) = \exp(x+y)$$

$$u(x,y,t) = \exp[-x-y-0.5t]$$
$$v(x,y,t) = \exp[x+y+0.5t]$$

… (4)

To solve the problem (4) by using VIM, we consider a correction functional (3) as:

$$u_{n+1}(x,y,t)=u_n(x,y,t)+\int_0^t \lambda(\tau)[\frac{\partial u_n(x,y,\tau)}{\partial t}-\tilde{u}_n^2(x,y,\tau)\tilde{v}_n(x,y,\tau)+2\tilde{u}_n(x,y,\tau)-0.25(\frac{\partial^2\tilde{u}_n(x,y,\tau)}{\partial x^2}+\frac{\partial^2\tilde{u}_n(x,y,\tau)}{\partial y^2})\,]d\tau$$

$$v_{n+1}(x,y,t)=v_n(x,y,t)+\int_0^t \lambda(\tau)[\frac{\partial v_n(x,y,\tau)}{\partial t}+\tilde{u}_n^2(x,y,\tau)\tilde{v}_n(x,y,\tau)-\tilde{u}_n(x,y,\tau)-0.25(\frac{\partial^2\tilde{v}_n(x,y,\tau)}{\partial x^2}+\frac{\partial^2\tilde{v}_n(x,y,\tau)}{\partial y^2})\,]d\tau$$

… (5)

where, λ is a general Lagrange multiplier. The value of λ can be found by considering $\tilde{u}_n(x,y,\tau),\tilde{v}_n(x,y,\tau),\frac{\partial^2\tilde{u}_n(x,y,\tau)}{\partial x^2},\frac{\partial^2\tilde{u}_n(x,y,\tau)}{\partial y^2},\frac{\partial^2\tilde{v}_n(x,y,\tau)}{\partial x^2}$ *and* $\frac{\partial^2\tilde{v}_n(x,y,\tau)}{\partial y^2}$ as restricted variations(i.e

$$\delta(\tilde{u}_n(x,y,\tau))=\delta(\tilde{v}_n(x,y,\tau))=\delta(\frac{\partial^2\tilde{u}_n(x,y,\tau)}{\partial x^2})=\delta(\frac{\partial^2\tilde{u}_n(x,y,\tau)}{\partial y^2})=\delta(\frac{\partial^2\tilde{v}_n(x,y,\tau)}{\partial x^2})=\delta(\frac{\partial^2\tilde{v}_n(x,y,\tau)}{\partial y^2})=0.$$

in Equation(5), then integrating the result by part to obtain λ yields the following stationary conditions:

$$\delta(u_n):\lambda'=0$$
$$\delta(u_n):1+\lambda\big|_{\tau=t}=0$$

So, the Lagrange multiplier in this case can be identified as follows: $\lambda=-1$

Then the correction functional (5) becomes in the following formula:

$$u_{n+1}(x,y,t)=u_n(x,y,t)-\int_0^t [\frac{\partial u_n(x,y,\tau)}{\partial t}-u_n^2(x,y,\tau)v_n(x,y,\tau)+2u_n(x,y,\tau)-0.25(\frac{\partial^2 u_n(x,y,\tau)}{\partial x^2}+\frac{\partial^2 u_n(x,y,\tau)}{\partial y^2})\,]d\tau$$

$$v_{n+1}(x,y,t)=v_n(x,y,t)-\int_0^t [\frac{\partial v_n(x,y,\tau)}{\partial t}+u_n^2(x,y,\tau)v_n(x,y,\tau)-u_n(x,y,\tau)-0.25(\frac{\partial^2 v_n(x,y,\tau)}{\partial x^2}+\frac{\partial^2 v_n(x,y,\tau)}{\partial y^2})\,]d\tau$$

… (6)

Using the above iteration formulas (6) and the initial approximations, we can obtain the following approximations:

$$u_1(x,y,t) = \exp(-x-y) - 1.5\exp(-x-y)t + \exp(-x-y)^2\exp(x+y)t$$
$$v_1(x,y,t) = \exp(x+y) + \exp(-x-y)\,t - \exp(-x-y)^2\exp(x+y)t + 0.5\exp(x+y)t$$

$$u_2(x,y,t) = \exp(-x-y) - 1.5\exp(-x-y)t + \exp(-x-y)^2\exp(x+y)t + 0.25(-1.5\exp(-x-y) + \exp(-x-y)^2\exp(x+y))^2(\exp(-x-y) - \exp(-x-y)^2\exp(x+y) + 0.5\exp(x+y))t^4 + \dots$$

$$v_2(x,y,t) = \exp(x+y) + \exp(-x-y)t - \exp(-x-y)^2\exp(x+y)t + 0.5\exp(x+y)t - 0.25(-1.5\exp(-x-y) + \exp(-x-y)^2\exp(x+y))^2(\exp(-x-y) - \exp(-x-y)^2\exp(x+y) + 0.5\exp(x+y))t^4 + \dots$$

.
.
.
and …...
Now , from theorem 1 we have the proof of the convergence as the following nonlinear mapping form:

$$A[u] = u - \int_0^t [\frac{\partial u}{\partial \tau} - u^2 v + 2u - 0.25(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2})]d\tau$$
$$A[\overline{u}] = \overline{u} - \int_0^t [\frac{\partial \overline{u}}{\partial \tau} - \overline{u}^2 v + 2\overline{u} - 0.25(\frac{\partial^2 \overline{u}}{\partial x^2} + \frac{\partial^2 \overline{u}}{\partial y^2})]d\tau$$

where $u = u(x,y,t)$. According to the above theorem, we can get

$$A[u] - A[\overline{u}] = (u-\overline{u}) - \int_0^t \lambda[\frac{\partial(u-\overline{u})}{\partial \tau} - (u^2 - \overline{u}^2)v + 2(u-\overline{u}) - 0.25(\frac{\partial^2(u-\overline{u})}{\partial x^2} + \frac{\partial^2(u-\overline{u})}{\partial y^2})]d\tau$$

and then

$$\left\|A[u]-A[\bar{u}]\right\|=\left\|(u-\bar{u})-\int_0^t[\frac{\partial(u-\bar{u})}{\partial\tau}-(u^2-\bar{u}^2)v+2(u-\bar{u})-0.25(\frac{\partial^2(u-\bar{u})}{\partial x^2}+\frac{\partial^2(u-\bar{u})}{\partial y^2})]d\tau\right\|$$

$$\geq\left\|u-\bar{u}\right\|-\int_0^t\left\|\frac{\partial(u-\bar{u})}{\partial\tau}-(u^2-\bar{u}^2)v+2(u-\bar{u})-0.25(\frac{\partial^2(u-\bar{u})}{\partial x^2}+\frac{\partial^2(u-\bar{u})}{\partial y^2})\right\|d\tau$$

Since $\left\|\frac{\partial(u-\bar{u})}{\partial\tau}\right\|\leq\delta_1\left\|(u-\bar{u})\right\|$, $\left\|\frac{\partial^2(u-\bar{u})}{\partial x^2}\right\|\leq\delta_2\left\|(u-\bar{u})\right\|$, $\left\|\frac{\partial^2(u-\bar{u})}{\partial y^2}\right\|\leq\delta_3\left\|(u-\bar{u})\right\|$

and $\left\|u^2-\bar{u}^2\right\|=\left\|(u-\bar{u})(u+\bar{u})\right\|\leq 2M\left\|(u-\bar{u})\right\|$ *,where* $\left\|u\right\|\leq M$ *and* $\left\|v\right\|\leq M$

then the inequality becomes

$$\geq\left\|u-\bar{u}\right\|-\int_0^t\alpha\left\|u-\bar{u}\right\|d\tau \quad , \qquad \text{where}\ \alpha=\delta_1-2M^2-0.25(\delta_2+\delta_3)+2$$

$$\geq(1-\int_0^t\alpha\, d\tau)\left\|u-\bar{u}\right\|$$
$$\leq(\int_0^t\alpha\, d\tau-1)\left\|u-\bar{u}\right\|$$
$$\leq\gamma_1\left\|u-\bar{u}\right\| \qquad \text{where}\ \gamma_1=(\int_0^t\alpha\, d\tau-1)$$

In the same way we found the proof of convergence of the component $v(x, y, t)$ as the following form

$$A[v]=v-\int_0^t[\frac{\partial v}{\partial\tau}+u^2v-u-0.25(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})]d\tau$$
$$A[\bar{v}]=\bar{v}-\int_0^t[\frac{\partial\bar{v}}{\partial\tau}+u^2\bar{v}-u-0.25(\frac{\partial^2\bar{v}}{\partial x^2}+\frac{\partial^2\bar{v}}{\partial y^2})]d\tau$$

Where $v=v(x,y,t)$. According to the above theorem, we get

$$A[\bar{v}]-A[v]=(v-\bar{v})-\int_0^t\lambda[\frac{\partial(v-\bar{v})}{\partial\tau}-(v-\bar{v})u^2-0.25(\frac{\partial^2(v-\bar{v})}{\partial x^2}+\frac{\partial^2(v-\bar{v})}{\partial y^2})]d\tau$$

and then

$$\left\|A[v]-A[\bar{v}]\right\| = \left\|(v-\bar{v})+\int_0^t \left[\frac{\partial(v-\bar{v})}{\partial\tau}+(v-\bar{v})u^2-0.25\left(\frac{\partial^2(v-\bar{v})}{\partial x^2}+\frac{\partial^2(v-\bar{v})}{\partial y^2}\right)\right]d\tau\right\|$$

$$\geq \left\|v-\bar{v}\right\| + \int_0^t \left\|\frac{\partial(v-\bar{v})}{\partial\tau}+(v-\bar{v})u^2-0.25\left(\frac{\partial^2(v-\bar{v})}{\partial x^2}+\frac{\partial^2(v-\bar{v})}{\partial y^2}\right)\right] d\tau\right\|$$

Since $\left\|\frac{\partial(v-\bar{v})}{\partial\tau}\right\| \leq \delta_4\left\|(u-\bar{u})\right\|$, $\left\|\frac{\partial^2(v-\bar{v})}{\partial x^2}\right\| \leq \delta_5\left\|(u-\bar{u})\right\|$, $\left\|\frac{\partial^2(v-\bar{v})}{\partial y^2}\right\| \leq \delta_6\left\|(u-\bar{u})\right\|$

and $\left\|v-\bar{v}\right\|u^2 \leq M^2\left\|v-\bar{v}\right\|$,*where* $\left\|u\right\| \leq M$

then the inequality becomes

$$\geq \left\|v-\bar{v}\right\| + \int_0^t \alpha_1 \left\|v-\bar{v}\right\| d\tau \quad , \qquad \text{where } \alpha_1 = \delta_4 + M^2 - 0.25(\delta_5+\delta_6)$$

$$\geq \left(1-\int_0^t \alpha_1\, d\tau\right)\left\|u-\bar{u}\right\|$$

$$\leq \left(\int_0^t \alpha_1\, d\tau - 1\right)\left\|u-\bar{u}\right\|$$

$$\leq \gamma_1\left\|u-\bar{u}\right\| \qquad \text{where } \gamma_1 = \left(\int_0^t \alpha_1\, d\tau - 1\right)$$

(where, $\delta's$ are the absolute values of differential operators which appear in partial differential equations).Therefore, the proof is completed.

Table 1a: Comparison between the VIM and the exact solution u(x,y) ,and the absolute errors for several iterations of the VIM solution at *t=0.1*

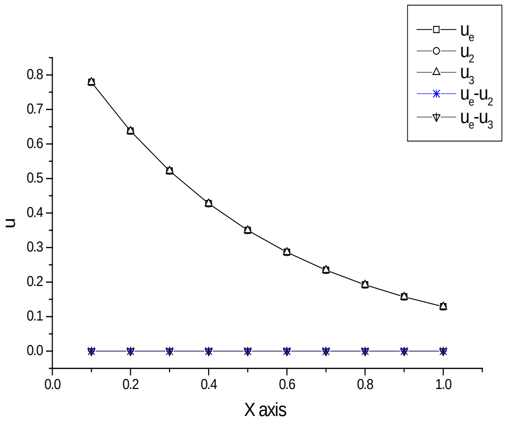
$x=y$	u_e	$u_{2(VIM)}$	$u_{3(VIM)}$	$\left\|u_e-u_{2(VIM)}\right\|$	$\left\|u_e-u_{3(VIM)}\right\|$
0.1	0.7788007831	0.7787519597	0.7788007932	4.88232940E-5	
0.2	0.6376281516	0.6375881786	0.6376278837	3.99730727E-5	1.0163830E-8
0.3	0.5220457768	0.5220130495	0.5220454056	3.27272426E-5	2.6800532E-7
0.4	0.4274149319	0.4273881372	0.4274145449	2.67947444E-5	3.7116680E-7
0.5	0.3499377491	0.3499158114	0.3499373865	2.19376453E-5	3.8707608E-7
0.6	0.2865047969	0.2864868358	0.2865044748	1.79611098E-5	3.6256141E-7
0.7	0.2345702881	0.2345555828	0.2345700108	1.47053260E-5	3.2199835E-7
0.8	0.1920499086	0.1920378689	0.1920496738	1.20396357E-5	2.7740344E-7
0.9	0.1572371663	0.1572273091	0.1572369701	9.85724840E-6	2.3460368E-7
1.0	0.1287349036	0.1287268331	0.1287347407	8.07047350E-6	1.9625035E-7
					1.6299179E-7

Table 2a: Comparison between the VIM and the exact solution ,and the absolute errors for several iterations of the VIM solution at *t=0.5*

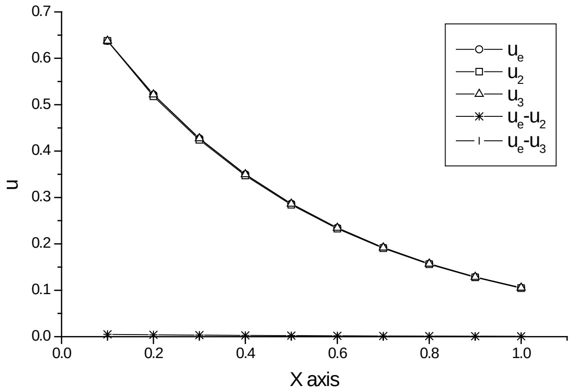
$x = y$	u_e	$u_{2(VIM)}$	$u_{3(VIM)}$	$\lvert u_e - u_{2(VIM)} \rvert$	$\lvert u_e - u_{3(VIM)} \rvert$
0.1	0.6376281516	0.6375881786	0.6375243254	4.92411286E-3	
0.2	0.5220457768	0.5180142544	0.5218510795	4.03152231E-3	1.03826154E-4
0.3	0.4274149319	0.4241142005	0.4271953272	3.30073134E-3	1.94697146E-4
0.4	0.3499377491	0.3472353388	0.3497249135	2.70241034E-3	2.19604714E-4
0.5	0.2865047969	0.2842922504	0.2863124096	2.21254644E-3	2.12835758E-4
0.6	0.2345702881	0.2327588082	0.2344028236	1.81147981E-3	1.92387095E-4
0.7	0.1920499086	0.1905667942	0.1919073390	1.48311433E-3	1.67464249E-4
0.8	0.1572371663	0.1560228951	0.1571174431	1.21427120E-3	1.42569471E-4
0.9	0.1287349036	0.1277407424	0.1286352377	9.94161277E-4	1.19723079E-4
1.0	0.1053992246	0.1045852742	0.1053167222	8.13950408E-4	9.96659750E-5
					8.25023220E-5

Table 3a: Comparison between the VIM and the exact solution ,and the absolute errors for several iterations of the VIM solution at *t=1* .

$x = y$	u_e	$u_{2(VIM)}$	$u_{3(VIM)}$	$\lvert u_e - u_{2(VIM)} \rvert$	$\lvert u_e - u_{3(VIM)} \rvert$
0.1	0.6376281516	0.4690644929	0.4942779725	2.75208109E-2	
0.2	0.5220457768	0.3840375269	0.4037375909	2.25321328E-2	2.30733147E-3
0.3	0.4274149319	0.3144233329	0.3300348587	1.84477507E-2	2.83206892E-3
0.4	0.3499377491	0.2574280519	0.2699256659	1.51037410E-2	2.83622496E-3
0.5	0.2865047969	0.2107642632	0.2208405694	1.23658968E-2	2.60612710E-3
0.6	0.2345702881	0.1725591840	0.1807234199	1.01243400E-2	2.28959066E-3
0.7	0.1920499086	0.1412795106	0.1479168729	8.28910862E-3	1.96010425E-3
0.8	0.1572371663	0.1156698801	0.1210783270	6.78654818 E-3	1.65174624E-3
0.9	0.1287349036	0.09470248801	0.09911640897	5.55635569E-3	1.37810139E-3
1.0	0.1053992246	0.07753583934	0.08114189171	4.54915926E-3	1.14243472E-3
					9.43106900E-4



(a)



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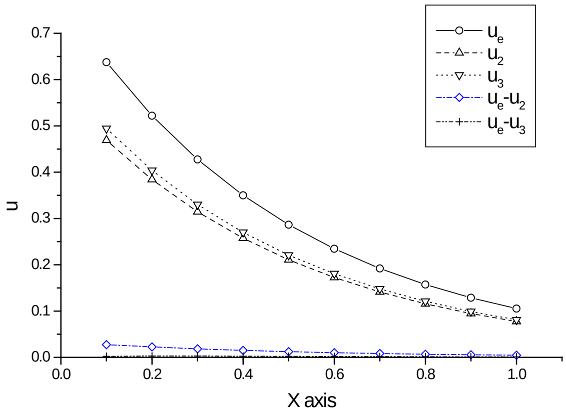


(c)

Figure (1a,b,c) Comparison between exact solution and VIM solutions for the component *u*(*x, t*) at $t = 0.1, 0.5, 1$ **respectively.**

Table 1b: Comparison between the VIM and the exact solution v(x,y) ,and the absolute errors for several iterations of the VIM solution at *t=0.1*

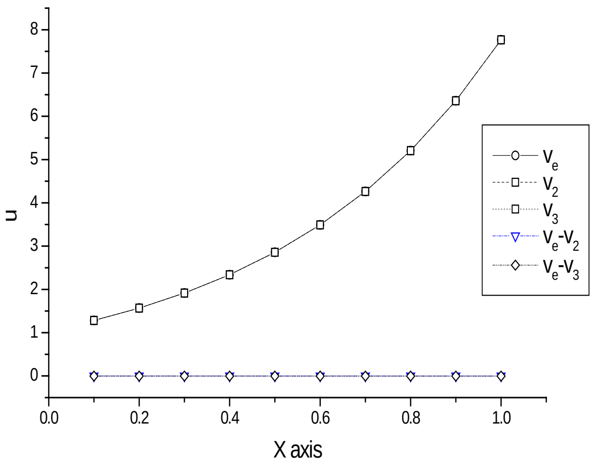
$x = y$	v_e	$v_{2(VIM)}$	$v_{3(VIM)}$	$\left\lvert v_e - v_{2(VIM)} \right\rvert$	$\left\lvert v_e - v_{3(VIM)} \right\rvert$
0.1	1.284025417	1.284065318	1.284026527	3.9901391E-5	
0.2	1.568312185	1.568334479	1.568313243	2.2294006E-5	1.11145180E-6
0.3	1.915540829	1.915546408	1.915541689	5.5787770E-6	1.05822393E-6
0.4	2.339646852	2.339635940	2.339647444	1.0911511E-5	8.58990681E-7
0.5	2.857651118	2.857623278	2.857651413	2.7839319E-5	5.93255589E-7
0.6	3.490342957	3.490297074	3.490342939	4.5883360E-5	2.95736570E-7
0.7	4.263114515	4.263048744	4.263114161	6.5770509E-5	1.95850300E-8
0.8	5.206979827	5.206891530	5.206979115	8.8297487E-5	3.54204350E-7
0.9	6.359819523	6.359705154	6.359818417	1.1436812E-4	7.12521800E-7
1.0	7.767901106	7.767756079	7.767899565	1.4502685E-4	1.10448320E-6
					1.54166590E-6

Table 2b: Comparison between the VIM and the exact solution ,and the absolute errors for several iterations of the VIM solution at *t=0.5*

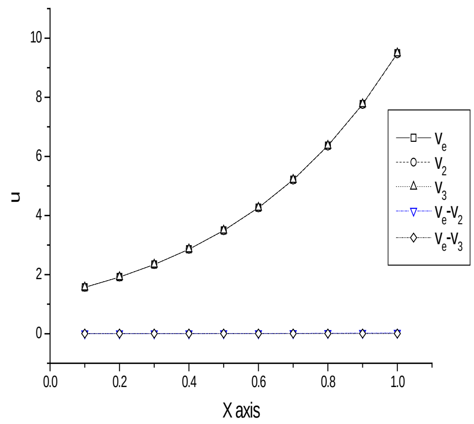
$x = y$	v_e	$v_{2(VIM)}$	$v_{3(VIM)}$	$\lvert v_e - v_{2(VIM)} \rvert$	$\lvert v_e - v_{3(VIM)} \rvert$
0.1	1.568312185	1.571851645	1.568986130	353946073E-3	
0.2	1.915540829	1.917073675	1.916118076	153284683E-3	6.73946371E-4
0.3	2.339646852	2.339234602	2.340076789	41224936E-4	5.77247366E-4
0.4	2.857651118	2.855277228	2.857910510	237388953E-3	4.29936805E-4
0.5	3.490342957	3.485912155	3.490419994	443080192E-3	2.59392075E-4
0.6	4.263114515	4.256448978	4.263000092	66655374E-3	7.70365220E-5
0.7	5.206979827	5.197812041	5.206662659	91677853E-3	1.14422105E-4
0.8	6.359819523	6.347781553	6.359283207	120379702E-2	3.17166859E-4
0.9	7.767901106	7.752509827	7.767122104	153912788E-2	5.36315410E-4
1.0	9.487735836	9.468373543	9.486681778	193622931E-2	7.79001710E-4
					1.05405807E-3

Table 3b: Comparison between the VIM and the exact solution ,and the absolute errors for several iterations of the VIM solution at *t=1*

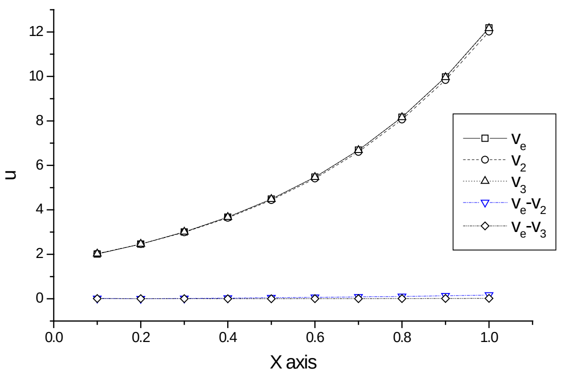
$x = y$	v_e	$v_{2(VIM)}$	$v_{3(VIM)}$	$\lvert v_e - v_{2(VIM)} \rvert$	$\lvert v_e - v_{3(VIM)} \rvert$
0.1	2.013752707	2.027421709	2.022537054	1.36690020E-2	
0.2	2.459603111	2.459127637	2.466317753	4.75474600E-4	8.78434627E-3
0.3	3.004166024	2.989526989	3.008446198	1.46390346E-2	6.71464238E-3
0.4	3.669296668	3.639906558	3.670966075	2.93901095E-2	4.28017530E-3
0.5	4.481689070	4.436368357	4.480623676	4.53207120E-2	1.66940660E-3
0.6	5.473947392	5.410877199	5.469999602	6.30701930E-2	1.06539323E-3
0.7	6.685894442	6.602543539	6.678848278	8.33509031E-2	3.94779010E-3
0.8	8.166169913	8.059193133	8.155711009	1.06976780E-1	7.04616392E-3
0.9	9.974182455	9.839286446	9.959873729	1.34896009E-1	1.04589050E-2
1.0	12.18249396	12.01426487	12.16375163	1.68229086E-1	1.43087258E-2
					1.87423155E-2



(a)



(b)



(c)

Figure (2a,b,c) Comparison between exact solution and VIM solutions for the component *v*(*x*, *t*) at $t = 0.1, 0.5, 1$ **respectively.**

3.2 Discussion

In this paper, we consider VIM to obtain the approximate solution to a nonlinear system of second-order partial differential equations with initial conditions as a function of x and y on the open and bound region $\Omega = \{(x, y) : 0 < x < 1\}$ and J = $\{(x, y) : 0 < y < 1\}$ respectively, $t \geq 0$. The obtained results from the variational iteration solution are found at $t = 0.1, 0.5, 1$. We found the numerical solutions in second-third order approximation. The comparison of the obtained results from the VIM with the exact solution for both components $u(x,y,t)$ or $v(x,y,t)$is made and clarified in Tables (1,2,3 a,b,c) and represented graphically in Figures (1,2a,b,c) at $t = 0.1, 0.5, 1$ respectively. From The tables, one can see that the accuracy of this method increases with increasing the iterations. More precisely, the errors are decreasing with increasing the number of iterations and, also we noted that the accuracy of this method increases with decreasing the value of time. The results obtained show that VIM solutions with less iterations converge rapidly to the exact solution $u(x, y, t)$ or $v(x, y, t)$.

4. Conclusions

In this paper, the variation iteration method has been successfully employed to obtain the approximate analytical solutions of Two-Dimensional Reaction-Diffusion Brusselator System. The method has been applied directly without using linearization or any restrictive assumptions. The comparison of the numerical results of VIM with exact solution shows that the variational iteration method is a powerful mathematical tool to solving nonlinear partial differential equations.

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طريقة التغايرالتكرارية لحل *Two-Dimensional Reaction-Diffusion Brusselator System* ذات البعدين

المستخلص:

في هذا البحث اقترحنا استعمال طريقة التغايرالتكرارية *Two-Dimensional Reaction-Diffusion Brusselator System* ذات البعدين . تم الحصول على الحل التحليلي باستعمال هذه الطريقة لهذا النوع من المعادلات التفاضلية الجزئية مع شروطها الابتدائية، حيث بينت الحلول التي حصلنا عليها دقة الطريقة وملاءمتها لحل هذه المسالة. كما تم اعطاء الشرط الضروري للتقارب. ايضا تم تقديم مثال توضيحي لبيان عمل الطريقة.