The ARMA(p,q) model is estimated when the error is random and has a Laplace distribution by practical rule

A.S. Azhar Kadhim Jbarah / azkdfr_2017@uomustansiriyah.edu.iq .
luma Tariq Abbas / Research / Lumaamer67@gmail.com .

P: ISSN : 1813-6729
E : ISSN : 2707-1359
https://doi.org/10.31272/jae.i142.1041

تأريخ أستلام البحث : 2023/10/11
مقبول للنشر بتأريخ :2023/12/20

Abstract

This research dealt with one of the types of studies proposed by Box-Jenkins, which is the ARMA (1,1) mixed model. Which affects the handling of timelines, whether they exist or not. It was identified with the indirect model with the non-normal distribution, and the Laplace distribution was one of the members of the masses. It touched on the most important topics targeted by the model, as the parameters of the ARMA model (1,1) were estimated using the MLE method. In the side application, a set of real data was analyzed, which represents a number of new data different according to the months from 2015-2022, and they determine the distribution of the main data and verify the interception of the tree, and in the diagnostic model it was found that the appropriate model is ARMA (1,1) .

Keywords : mixed model ARMA **(p,q)** , Laplace Maximum Likelihood Method , Time series .

مجلة الادارة والاقتصاد
مجلد 49 العدد 142 / أذار / 2024
الصفحات : 200 - 212

1- Introduction

Analysis using time series is considered one of the important statistical methods that has wide applications in the economic, health, social, demographic and other fields. Research and studies have followed one another, and the researchers' interest has focused on studying time series when the random error follows an abnormal distribution, as the autoregressive model of degree has been studied. The first is natural and abnormal AR (1), as well as for other models. In previous studies, we relied on a normal distribution of errors without verifying their original distribution. These assumptions cause problems in estimation, construction, and prediction. Accordingly, in this research, attention was focused on the ARMA (1,1) model. And its characteristics when the random error follows a continuous distribution (Lablas).

When the characteristics of the time series change with time, the series is unstable. The lack of stability in the time series means that systematic changes occur about the mean and variance, and the presence of periodic changes in the series may sometimes lead to its instability. Before analyzing the time series, its instability must be removed, because its presence will give shaded results.

2- The theoretical aspect

2-1 First-order mixed model ARMA(1,1)

The mixed model is defined when from the following equation

$yt = \varphi 1yt + at - \theta 1a_{t-1}$ **(1)**

At ((White noise) represents a series of independent random errors with a distribution that may be normal or abnormal.

From the displacement factor B, the model can be written in the form:

$\varphi(B)yt = \theta(B)at$ **(2)**

The ARMA model can be written in AR and autoregressive form

$\pi(B)yt = at$ **(3)**

whereas

$(B) = (1 - \pi 1B - \pi 2B2 \cdots)$ **(4)**

That is

$[1-(\pi 1+\theta 1)B-(\pi 2-\pi 1\theta)B2-(\pi 3-\pi 2\theta 1)B2\cdots]=(1-\phi 1B)$ **(5)**

With equal transactions **($j=1,2\ldots\ldots B^j$)** At both ends we get

$\pi j=\theta j-1(\phi 1-\theta 1)\ for\ j\geq 1$ (6)

That is, the ARMA (1,1) model can be used as a suitable approximation for autoregressive models.[8,1]

In addition , the mixed model ARMA (1,1) can be expressed in moving averages form and written as follows:

$Zt=\Sigma\psi j\ at-,0=1$ **(7)**

It can be written in the following form

$Zt = at + (\emptyset - \theta)\sum_{j=1}^{\infty}\emptyset^{j-1}\ at - 1$ **(8)**

For all string models this formula is generally different but varies in terms of ψ

Since

$Zt=(B)at$ **(9)**

$(B)=(1-\theta 1B)/(1-\phi 1B)$ **(10)**

Since

$(1-\phi B)(1+\psi 1B+\psi 2B^2+\psi 3B^3+\cdots)=(1-\theta B)$

Equating both sides of the coefficients of Bj , we get :

$\psi 1 = \phi - \theta$

$\psi 2 = \phi\psi 1 = (\phi-\theta)\phi$

$\psi j = (\phi-\theta)\phi^{j-1} \qquad , j \geq 1 \qquad (11)$

2-2 Unit root test

Unit root is a feature of a nonstationary linear random process, and occurs when an integer is the root of the characteristic equation of that random process. It is worth noting that the random process is non-stationary but does not necessarily always have a general trend. If the rest of the roots of the characteristic equation have an absolute value less than one integer (in other words, they lie outside the bounds of the unit circle), then the first difference in this process is Constant, except that the random process needs to make multiple variations to achieve stability. There are several tests used to test whether a time series has a unit root (2),

1- Expanded Dickey-Fuller test

This test was developed by the scientist Dickey and Fuller (1980). This test works to determine whether the time series is stable or not.?

The test is based on a first-order autoregressive model:

The idea of the Dickey-Feller test is based on testing the following hypothesis:

$$\mathbf{H_0}: \emptyset_1 = 1 \qquad \mathbf{V.S} \qquad \mathbf{H_1}: |\emptyset_1| < 1 \qquad (13)$$

If the null hypothesis is accepted, the time series is unstable and the first-order autoregressive model turns into a random walk model. Conversely, the roots of the characteristic equation are within the boundaries of the unary circle, that is, under the validity of the alternative hypothesis.

The criterion used to test the null hypothesis was the T-test statistic according to the following formula:

$$DF_t = \frac{\hat{\emptyset}-1}{EST.st.error(\hat{\emptyset})} \qquad (14)$$

The test statistics are not distributed according to the t-distribution, so it is not possible to rely on the tabular values of the t-distribution. Rather, there are special values for this test prepared by the scientist Dickey-Feller. The following test statistics can also be relied upon to test the null hypothesis.

$$DF_{\emptyset} = T(\hat{\emptyset} - 1) \qquad (15)$$

T: represents the number of views

This test assumes that the random error terms of the autoregressive model are pure (white noise) , that is, not self-correlated . Therefore, the scientist developed his Dickey-Feller test to take into account serial correlation . The developed test was called the extended Dickey-Feller test , which is based on estimating the following model [8].

$$y_t = \gamma + \delta_t + \emptyset y_{t-1} + \sum_{j=1}^{K} \theta_j \Delta y_{t-1} + \epsilon_t \qquad (16)$$

$$\gamma = \beta_o(1-\emptyset) + \beta_1\emptyset \qquad (18)$$

$$\delta = \beta_1(1-\alpha) \qquad (19)$$

After estimating the expanded model, the unit root is tested by adopting the same formula for the test statistics DFt or DF∅**,** as they have the same distribution of the Dickey-Feller test statistics, so the same tabular values are used for this test [(4)].

2- Test Kipss

One of the unit root tests proposed by researchers Kwiatkowski & Philips & Schmidt & Shine in (1992) to test the stability of time series. This test is an expansion of the expanded Dickey-Feller test. The KPSS test is concerned with testing the null hypothesis that states that a random walk has a variance equal to zero against the alternative hypothesis that states However, the series is stable with the differences, that is, testing the following hypothesis:

$$H_0 : \sigma_u^2 = 0 \qquad V.S \qquad H_1 : \sigma_u^2 > 0$$

The test statistics used to test the hypothesis are based on the one-sided Lakrange LM factorial test and the Locally best invariant (LBI) test.

The test statistic formula is as follows:

$$kpss = \frac{1}{T^2}\frac{\sum_{t=1}^{T} S_t^2}{S_\in^2} \qquad (20)$$

Since S_t^2: represents the squares of the partial sums of the estimated series of residuals S_t, which are calculated as follows [(10)].

$$S_t = \sum_{i=1}^{t} e_i \qquad t = 1, 2, \ldots . T \qquad (21)$$

Since

e_i **:** The residuals represent random error

$S_\in^2$**:** Estimating the random error variance of the time series model yt

3- Estimation method (Maximum Likelihood Method)

This method boils down to the fact that the values of the parameters to be estimated are chosen in accordance with the principle of maximizing the maximum possibility function.

$$L(\theta, \vartheta, \mu, \sigma_a^2) = (2\pi\sigma_e^2)^{\frac{-T}{2}} \exp\left(-\frac{1}{2\sigma_e^2}\sum_{i=1}^{t} a_t^2\right) \qquad (22)$$

When you take the logarithm, it will be

$$LnL(\theta, \vartheta, \mu, \sigma_a^2 / Y) = -(\frac{T}{2})Ln(2\pi) - (\frac{T}{2})Ln\sigma_a^2 + -\left[\frac{S(\phi, \mu, \varphi)}{2\sigma_a^2}\right] \qquad \textbf{(23)}$$

whereas

$$S(\phi, \mu, \varphi) = \sum_{t=P+1}^{T} (a_t(\phi, \mu, \varphi)/Y)^2 \qquad \textbf{(24)}$$

By deriving the maximization equation for ϕ**,** ϕ setting it equal to zero , and simplifying it, we obtain the required estimates [(8)]

Laplace distribution and estimation of its parameters.

According to Equation (10), it is shown that yt follows the ARMA(1,1) model and that the error $a_t$follows the Laplace distribution with a mean equal to zero and a variance equal to σ^2, so the probability density function for a_t is:

$$f(a_t) = \frac{1}{\sigma\sqrt{2}} exp\left[-\sqrt{2}\left|\frac{a_t}{\sigma}\right|\right] \quad , \quad -\infty < a_t < \infty \qquad (25)$$

It is the same as the probability density function for yt Taking the natural logarithm, it becomes:

$$l_t = lnf(y_t \backslash F_{t-1}) = -\frac{\ln(2)}{2} - \ln(\sigma) - \sqrt{2}\left|\frac{a_t}{\sigma}\right| \qquad (26)$$

To estimate the parameters of the ARMA model **(p,q)** defined by equation (10), we find the derivatives of the logarithm of the probability density function of the variable y_t **with respect to ϕ_0, ϕ_i and θ_j are** [3].

$$\frac{\partial l_t}{\partial \phi_0} = \frac{-a_t\sqrt{2}}{|a_t|\sigma}\frac{\partial y_t}{\partial \phi_0} \qquad (27)$$

$$\frac{\partial l_t}{\partial \phi_i} = \frac{-a_t\sqrt{2}}{|a_t|\sigma}\frac{\partial y_t}{\partial \phi_i} \qquad (28)$$

$$\frac{\partial l_t}{\partial \theta_j} = \frac{-a_t\sqrt{2}}{|a_t|\sigma}\frac{\partial y_t}{\partial \theta_j} \qquad (29)$$

$t = 1, 2, \dots, n \; ; \; i = 1, 2, \dots, p \; ; \; j = 1, 2, \dots, q$

4- The Kolmogorov-Smirnov test

Is one of the nonparametric distributions that is used to determine the appropriate distribution for the sample. The test is carried out according to the following hypotheses.

$Ho: F(x) = F(y)$

$H1: F(x) \neq F(y)$

Whereas **F(x)** represents the aggregate distribution of the totals of the first sample of the population , **F(y)** represents the aggregate distribution of the totals of the second sample of the population.

The test statistics are in the following format

***D*=[|F1(x)−F2(x)|]** **(30)**

Since

F1(x): Represents the sum of frequencies for the first sample

F2(x): Represents the sum of frequencies for the second sample

The value of **D** is compared with its corresponding value in the Kolmogorov-Smirnov tables with a significance level of **α** . If the calculated value is greater, the null hypothesis is rejected and vice versa [6,2].

5- ACI Akaike Information standard

Akaike's Information Criterion is one of the criteria used to choose the best model, which is the model whose variance is weak and whose variance decreases with the increase in the number of estimated features and the sum of the squares of the residuals is small, as in the following equation:

$$\mathrm{AIC(p)} = \ln(\sigma^2) + \frac{2(p+q)}{n} \qquad \textbf{(31)}$$

Where σ^2 : represents the model variance, and (p+q) represents the number of estimated features.

This test was developed into a Bayesian Information Criterion test, symbolized by (BIC) and formula [2].

$$BIC = 2\,Ln\left(\sigma_u^2\right) + M\,Ln(N) \qquad (32)$$

Since

N, M : The number of views of the series and the total number of model parameters The best model according to the two criteria gives the lowest value .

5- Forecasting

It is one of the final stages of studying and analyzing time series. After diagnosing the appropriate model and estimating the parameters of that model, that model is used to know the values of the future phenomenon and for periods (L).

$$\hat{Y}_{t+l} = E(Y_{t+l} \backslash Y_t, Y_{t-1}, Y_{t-2}, \ldots\ldots] \quad for\ L \geq 1 \qquad (33)$$

If the model is (1)AR, the best prediction for the number of steps (L) is:

$$\hat{Y}_{t+l} = \emptyset^l Y_{t-1+l} \qquad for\ l \geq 1 \qquad (34)$$

In the case of moving media MA(q), the best prediction for the number of steps (L) is:

$$\hat{Y}_{t+l} = a_{t+l} - \theta_1^l a_{t-1+l} - \theta_2^l a_{t-2+l} - \cdots - \theta_q^l a_{t-q+l} \qquad (35)$$

In the case of the mixed model (ARMA(p, q), the best prediction is the number of steps (L) [5]

$$\hat{Y}_{t+l} = \emptyset_1^l Y_{t-1+l} + \emptyset_2^l Y_{t-2+l} + a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t+l-q} \quad (36)$$

The applied aspect

The data was analyzed to learn more about the new data for the years 2015 to 2022, a mixed model for the autosomal hemodynamics of the P rows and the moving medians of the q score, and the data are shown in Table (1).

Table (1) Number of recorded traffic accidents by months from 2015-2022

السنة شهر	2015	2016	2017	2018	2019	2020	2021	2022
1	624	601	584	696	709	743	836	785
2	754	716	658	748	833	859	810	884
3	701	676	711	868	854	657	797	939
4	719	713	695	838	909	359	901	912
5	712	727	770	749	880	450	926	928
6	742	672	664	767	883	522	875	943
7	621	703	663	857	995	547	792	899
8	801	678	668	872	885	674	829	904
9	827	816	854	880	1025	786	984	1165
10	827	734	807	903	1122	888	918	1099
11	685	1024	952	868	882	842	991	1081
12	823	703	798	806	776	859	1001	984

Using the Easy Fit 15.6 program, the data was analyzed and it was concluded that the data follows the Laplac distribution through the results of the Kolmikrov-Smirnov test in Table (2), and the graph was also inserted according to Figure No. (1).

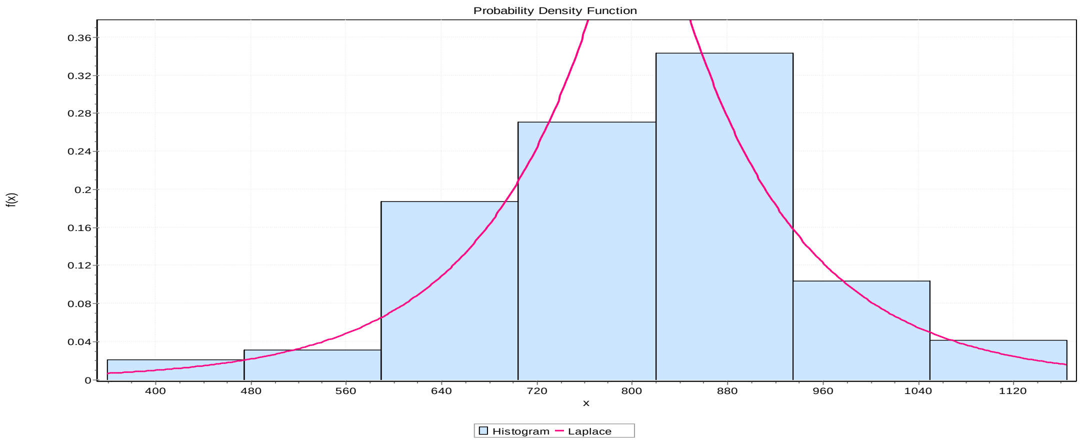

Figure (1) Lablas distribution probability density function table

Table (2) Goodness-of-fit tests

Laplace [#32]					
Kolmogorov-Smirnov					
Sample Size	**96**				
Statistic	**0.09799**				
P-Value	**0.2956**				
Rank	**35**				
⍰	**0.2**	**0.1**	**0.05**	**0.02**	**0.01**
Critical Value	**0.10777**	**0.12312**	**0.13675**	**0.15291**	**0.16408**
Reject?	**No**	**No**	**No**	**No**	**No**

We note from the results in Table (2) that the null hypothesis, which states that the data follows a Laplacs distribution, was not rejected at a significance level of 0.05.

The time series was plotted in Figure (2) to show whether the data series is stable

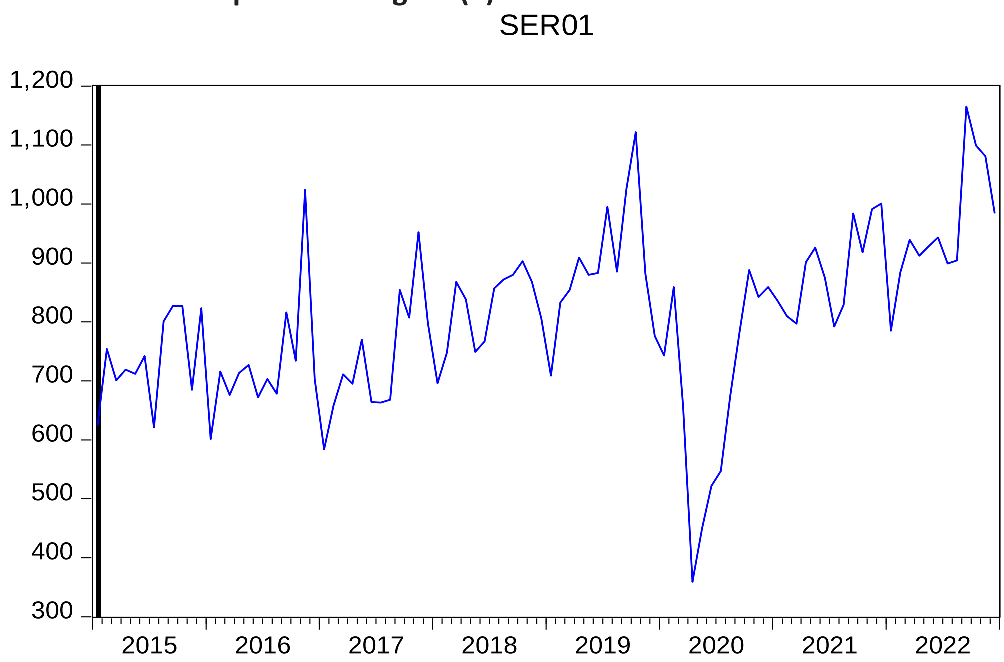

Figure (2) Time series plot

Figure (2) shows the instability of the time series. To verify this, we use unit root tests, including the expanded Dickey-Feller test.

Table (3) Expanded Dickey-Feller test

Teast	ADF
Statistic	- 0.3120
P-Value	0.5714

From Table (3) , it was concluded that the series is unstable because the P-Valu is equal to 0.5714.Greater than 0.05 , this means accepting the null hypothesis, which states that there is a unit root, meaning that the series is unstable.

For the stability of the series, the first difference was taken as shown in Table (4).

Table (4) shows the first difference for the stability of the time series

Teast	ADF
Statistic	- 12.11
P-Value	0.000

Through the results of Table (4), it was concluded that the series stabilized after taking the first difference

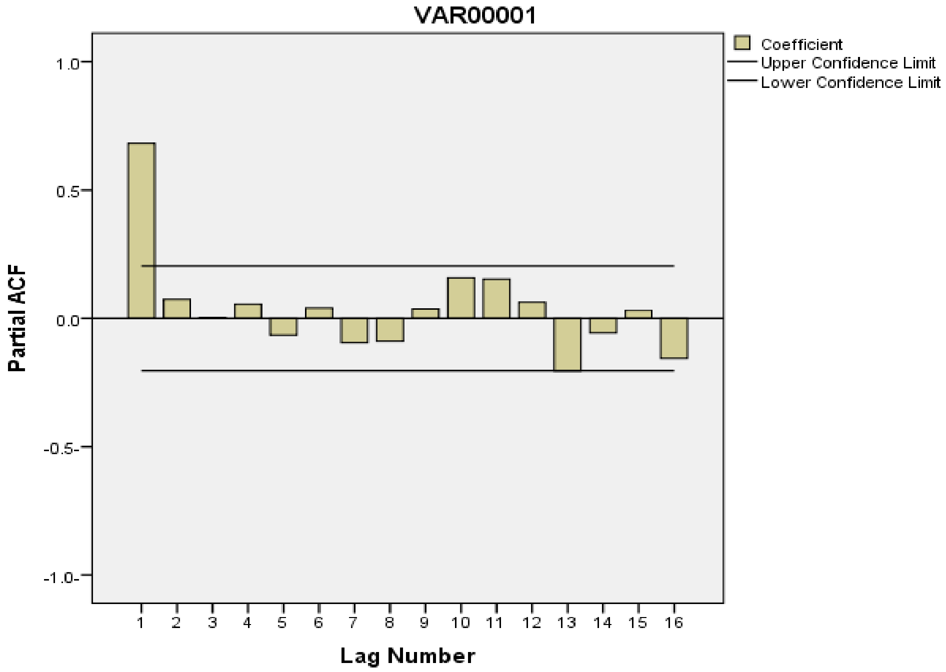

Figure (3) Autocorrelation function

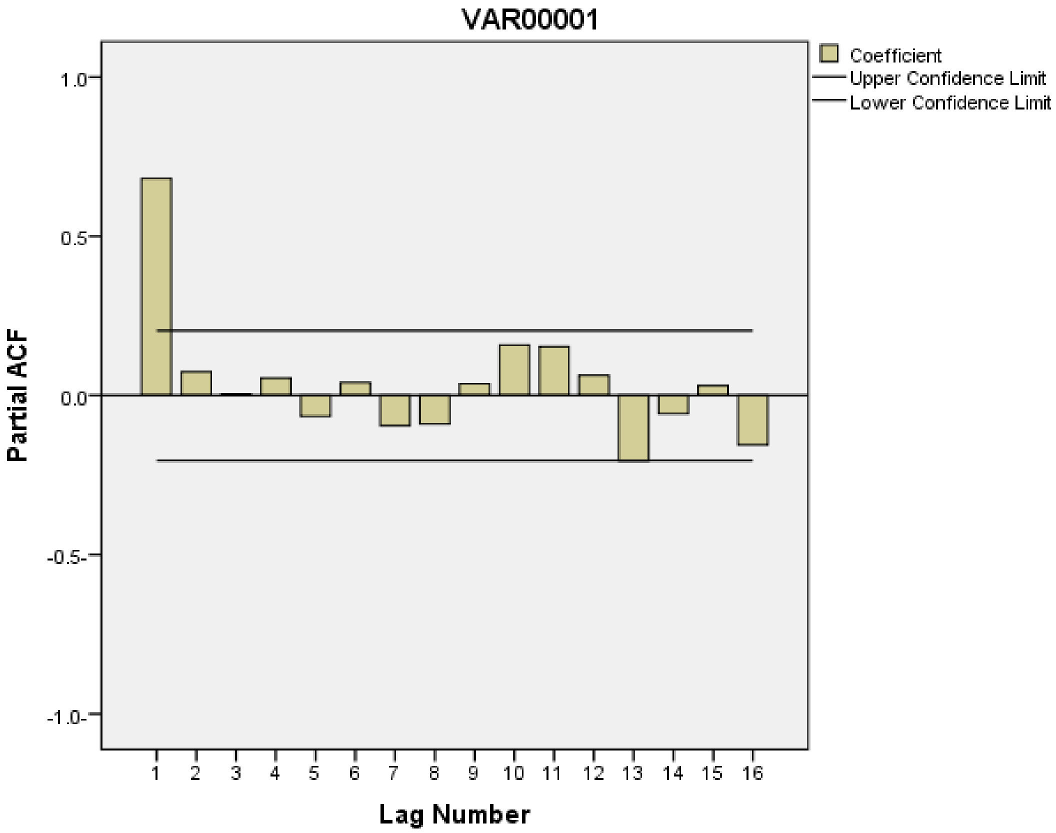

Figure (4) Partial autocorrelation function

From Figures (3) and (4) it turns out that the data is a component of ARMA when the error follows the Laplace distribution, and it turns out that the best model is ARMA(1,1) based on the results of the two criteria AIC and BIC shown in Table (5) and with the following details.

$\hat{\phi}_0 = 1.760438$, $\hat{\phi}_1 = 0.5354301$ $\hat{\vartheta}_1 = 0.95166$

Ln (Likelihood) = -567.8267 , $\hat{\sigma}_a^2 = 9907.174$

AIC = 12.0174 , BIC=12.09805

After achieving the stability of the time series, the appropriate model is then diagnosed. In such a case, the use of statistical criteria is used to diagnose the appropriate model, as shown in Table (5).

Table (5): Results of ARMA model estimates for the number of traffic accidents series

MODEL	AIC	BIC
AR (1)	12.12515	12.17891
AR (2)	12.82481	12.90546
MA(1)	12.08481	12.13858
MR (2)	13.68794	13.76858
ARMA(1,1)	12.0174	12.09805

Forecasting

After determining the type of appropriate model, and estimating the parameters of that model, we can predict the future values of the time series by using the appropriate model that was obtained as shown in Table (6).

Table (6) shows the prediction of future values of the time series

Residual	Forecast	Data	Period
1.31641	622.684	624.0	1
129.4	624.6	754.0	2
-26.75	727.75	701.0	3
11.3682	707.632	719.0	4
-5.61203	717.612	712.0	5
27.814	714.186	742.0	6
-116.173	737.173	621.0	7
154.607	646.393	801.0	8
57.5041	769.496	827.0	9
10.9733	816.027	827.0	10
-140.847	825.847	685.0	11
107.31	715.69	823.0	12
-200.509	801.509	601.0	13
71.836	644.164	716.0	14
-25.8457	701.846	676.0	15
30.5942	682.406	713.0	16
19.4542	707.546	727.0	17
-51.9167	723.917	672.0	18
19.0974	683.903	703.0	19
-21.9582	699.958	678.0	20
132.412	683.588	816.0	21
-55.2015	789.202	734.0	22
277.317	746.683	1024.0	23
-263.911	966.911	703.0	24
-175.689	759.689	584.0	25
36.0984	621.902	658.0	26
59.6932	651.307	711.0	27
-4.40151	699.402	695.0	28
73.0934	696.907	770.0	29
-91.6563	755.656	664.0	30
-21.2786	684.279	663.0	31
-0.423558	668.424	668.0	32
184.971	669.029	854.0	33
-9.16206	816.162	807.0	34
141.931	810.069	952.0	35
-125.384	923.384	798.0	36
-129.585	825.585	696.0	37
23.6712	724.329	748.0	38
123.944	744.056	868.0	39
-5.06189	843.062	838.0	40
-91.2471	840.247	749.0	41
-2.31178	769.312	767.0	42
88.4332	768.567	857.0	43
32.4589	839.541	872.0	44
13.6241	866.376	880.0	45
24.6376	878.362	903.0	46
-31.0703	899.07	868.0	47
-69.7785	875.779	806.0	48
-112.861	821.861	709.0	49
99.1828	733.817	833.0	50
40.7614	813.239	854.0	51
62.4017	846.598	909.0	52
-17.1084	897.108	880.0	53
-1.84865	884.849	883.0	54
110.368	884.632	995.0	55
-88.1058	973.106	885.0	56

120.16	904.84	1025.0	57
120.919	1001.08	1122.0	58
-216.058	1098.06	882.0	59
-152.841	928.841	776.0	60
-66.3495	809.349	743.0	61
100.951	758.049	859.0	62
-181.902	838.902	657.0	63
-337.316	696.316	359.0	64
19.3026	430.697	450.0	65
75.441	446.559	522.0	66
40.188	506.812	547.0	67
134.712	539.288	674.0	68
139.484	646.516	786.0	69
130.333	757.667	888.0	70
-19.7415	861.741	842.0	71
11.6491	847.351	859.0	72
-21.7497	857.75	836.0	73
-31.7663	841.766	810.0	74
-20.8438	817.844	797.0	75
98.4797	802.52	901.0	76
44.5173	881.483	926.0	77
-42.9069	917.907	875.0	78
-93.2865	885.287	792.0	79
16.1973	812.803	829.0	80
157.252	826.748	984.0	81
-34.1951	952.195	918.0	82
64.4917	926.508	991.0	83
22.2174	978.783	1001.0	84
-212.719	997.719	785.0	85
53.0002	831.0	884.0	86
64.9426	874.057	939.0	87
-14.6143	926.614	912.0	88
11.6327	916.367	928.0	89
16.1498	926.85	943.0	90
-41.9179	940.918	899.0	91
-5.11157	909.112	904.0	92
258.65	906.35	1165.0	93
-13.0492	1112.05	1099.0	94
-22.3	1103.3	1081.0	95
-103.227	1087.23	984.0	96

Upper 95.0% Limit	Lower 95.0% Limit	Forecast	Period
1218.34	796.002	1007.17	97
1277.75	739.427	1008.59	98
1326.84	693.177	1010.01	99
1369.75	653.108	1011.43	100
1408.43	617.27	1012.85	101

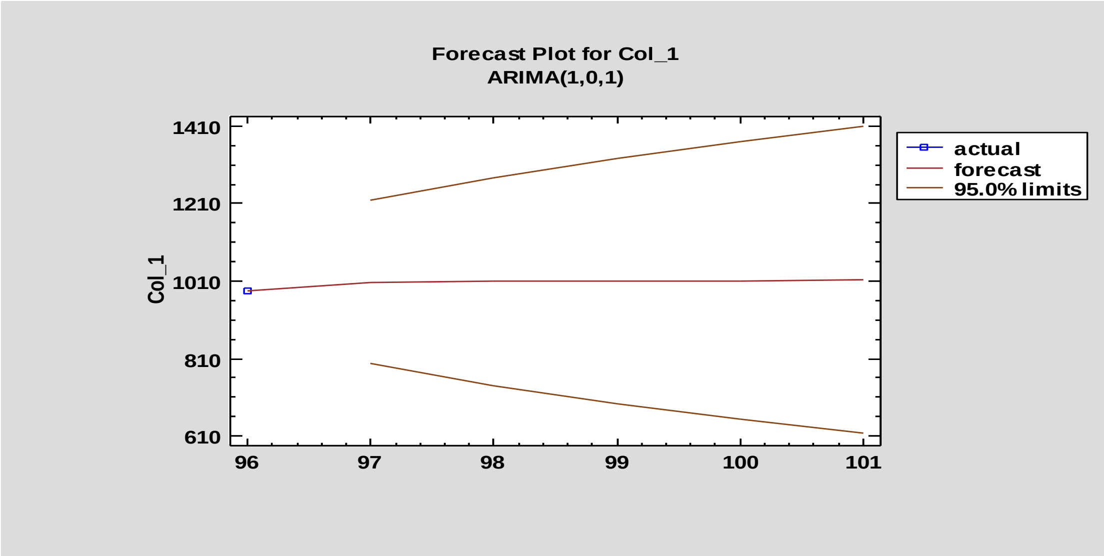

Figure (5) shows the prediction of the future time series

Conclusions

1- The model that represents the number of traffic accidents is the ARMA(1,1) model.

2- It turns out that the time series is an unstable series only after taking the first difference.

3- It turns out that the real values are close to the estimated values.

Recommendations

1. Studying models with higher orders than the model used in research in the case of continuous distributions.

2. The possibility of studying other non-continuous distributions and choosing the appropriate model for those distributions.

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تقدير أنموذج ARMA(p,q) عندما يتبع الخطأ العشوائي توزيعا Laplace بتطبيق عملي

م. د.ازهار كاظم جبارة / azkdf_2017@uomustansiriyah.edu.iq
لمى طارق عباس / باحثة / Lumaamer67@gmail.com

المستخلص

تناول هذا البحث أحد أنواع النماذج التي اقترحها بوكس جنكينز وهو النموذج المختلط ARMA (1,1). والتي يمكنها التعامل مع السلاسل الزمنية سواء كانت مستقرة أو غير مستقرة . اذ تم التطرق الى الانموذج بخطأ عشوائي يتبع التوزيع غير الطبيعي ، وتم استخدام توزيع Laplace وهو أحد التوزيعات المستمرة . والتطرق الى اهم الاختبارات المناسبة للأنموذج اذ قدرت معلمات نموذج ARMA (1,1) بطريقة MLE . وفي الجانب التطبيقي، تم تحليل مجموعة من البيانات الحقيقية التي تمثل عدد الحوادث المروريه المسجلة حسب الاشهرمن 2015-2022، واهم ما توصل اليه ملائمة توزيع البيانات والتحقق من استقرارية السلسلة ، وفي تشخيص الانموذج وقد وجد أن النموذج الملائم هو ARMA (1,1).

الكلمات المفتاحية : الانموذج المختلط ARMA(p,q) ، توزيع Laplace Maximum Likelihood Method ، Time series .