Al-Khwarizmi Engineering Journal

Al-Khwarizmi Engineering Journal, Vol. 5, No. 3, PP 38 - 50 (2009)

# Stress Analysis of Guide Rails of Elevators

**Hayder F. Neamah*          Wisam H. Ali****

** Department of Mechanical Engineering/ College of Engineering/University of Baghdad*

Email:haider_neama@yahoo.com

** Department of Mechanical Engineering/ College of Engineering/Al-Moustenseria University*

(Received 16 January 2008; accepted 28 July 2009)

## Abstract

The mechanical design of elevator elements is always performed by international standards. The engineer selects the appropriate elements of elevator according to catalogues without knowing scientific details. Therefore, a theoretical analysis is achieved at two operating conditions for guide rails (1) safety gear operation, and (2) running condition with the loads unevenly distributed on the elevator car. The guide rail is considered a continuous beam with variable supports. Then the British code is listed showing the equations used in it.

The theoretical equations showed that guide rails are never subjected to stress in simultaneous combined buckling and bending in the plane, where the bending moment is exerted. It is always a combination of pressure and bending. Consequently, it is wrong to consider a simultaneous effect of buckling and bending. The equations in the catalogues oppose the theoretical results concerning buckling of guide rails. Therefore, a recommended calculation method for guide rails is presented to be an acceptable method for analysis of guide rail.

***Keywords:*** *Guide rail, stress, deflection, safety gear, buckling, continuous beam, standard codes.*

## 1. Introduction

The functions of guide rails are as follows: (1) to guide the car and the counterweight in their vertical travel and to minimize their horizontal movement, (2) to prevent tilting of the car due to eccentric load, and (3) to stop and hold the car on the application of the safety gear. Fig.1 shows the components of the elevator and the cross-section of the guide-rail [1].

Both the car and the counterweight must be guided by at least two rigid steel guide rails, which are manufactured from a structural steel having a tensile strength of no less than 370 MPa and not greater than 520 MPa [2]. In the U.S., a suitable nonmetallic material may be used for guide rails where steel may present an accident hazard, as in chemical and explosive plants, provided the rated speed of the car does not exceed 0.76 m/s.

In recent years, round guide rails have been successfully used for hydraulic elevators and counterweights without safety gears [3]. Through investigating the studies on the elevator components, the calculations of guide rails depend mainly upon USA or European codes.

Utsunomiya et. al. [4] invented a guide device for an elevator in which a pair of corresponding actuators were controlled in accordance with information from acceleration sensors, and the force with which guide members were pressed against guide rails was adjusted.

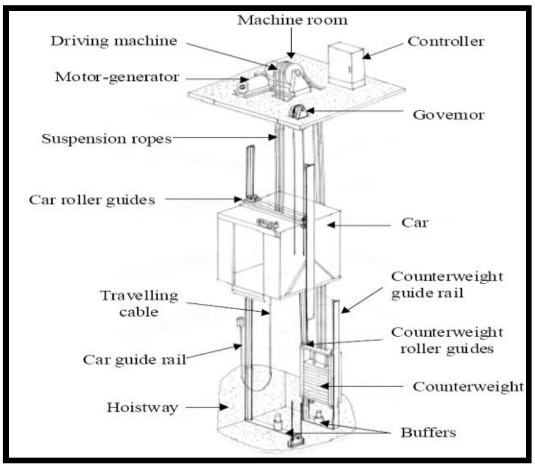


**Fig. 1. Components of Elevators**

Recep et. al.[5] performed the calculation and selection of guide rails according to international standard and compared the results with FEA. Penn et. al. [6] uses ANSYS to improve the manufacturing and design of a neoprene elevator roller guide. The study uses 2d and 3d hyperelastic and contact elements to model neoprene material tests and also highly deformed roller guide proof tests. The ANSYS analysis succeeded in modeling the neoprene material performance. Geometry changes in the shape of the neoprene were studied to reduce adhesion stresses between the neoprene and aluminum center hub, and yet to maintain the spring stiffness of the current roller guide design.

Clem Skalski and Barker Mohandas [7] performed a procedure to control the vibration of the guide rail. Zhu et. al. [8] analyzed the vibration of elevators depending on the present theory of dynamic analysis of elevator systems. The vibration models of elevator systems in the horizontal and vertical direction are established. The seismic responses of the building are used as excitation and input into the model. Differential equations of the system are set up and the time-history of the dynamic responses of the main parts are worked out. Finally, some earthquake protective measures for elevators are proposed. The current study proved that there are problems with the used codes, and a preferred method was invented quoting from the theoretical and international standard to calculate and select the best guide rails.

## 2. Guide Rails Calculations

In the calculation of the guide rails, two operating conditions should be taken into consideration (1) safety gear operation, (2) running conditions with the load unevenly distributed on the car floor.

In most national standards, the calculation of stress in guide rails is carried out for (2), while the calculation of deflection concerns quite different operating conditions, namely(1) [8].

Three stages of calculations are performed in the current paper: 1) theoretical analysis, 2) the British codes, and 3) an acceptable method for design of guide rails is introduced after completing the previous two stages.

### 2.1. Theoretical Analysis

#### 2.1.1. During Safety Gear Operation (Without Taking Buckling Into Consideration)

The aim of the theoretical analysis is to find the nature of relation between buckling and bending moments during safety gear operation. The following will be studied:

- The bending moment distribution without the effect of buckling and the guide rail is considered as a continuous beam.
- The effect of buckling is considered with the bending moment at the section of guide rail that is subjected to compression force.

In the first stage, the guide rails will be analyzed by calculating the maximum bending moment produced by the braking force without taking buckling into consideration. We will assume a simultaneous effect of buckling and bending moments in the second stage.

The guide rails will be considered a continuous beam with a variable number of supports. The Theorem of Three Moments and the Finite Element Method may be used as methods of solution. The first method is used for the derivation of the related equations.

The guide rail is subjected to a combined effect of the braking $F_b$, acting parallel to the longitudinal axis of the guide rail, and the outer moment $F_b \times e$. The outer moment is induced due to the eccentric position of the braking force $F_b$, which is represented by the distance e, as shown in Fig. 2. The bending moment $M$ (z) depends on the number of beam fields and on the outer moment $F_b \times e$(both value and location).

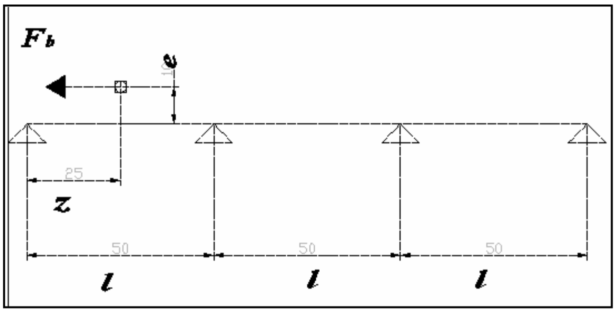


**Fig. 2. Location of the Braking Force $F_b$**

Equations for the maximum bending moment $M$(z) as a function of z, and values and locations of the extremes of individual functions are reviewed in Table 1 for $F_b \times e$ acting in field I of the beam and in Table 2 for $F_b \times e$ acting in field II. The maximum bending moment always occurs at the point of application of the outer moment $F_b \times e$. We will derive only two equations which are listed in the mentioned tables.

### *a)* Assumptions

***i.*** $F_b \times e$ in field I(span I).

***ii.*** No. of *fields* is two, as shown in Fig 3.

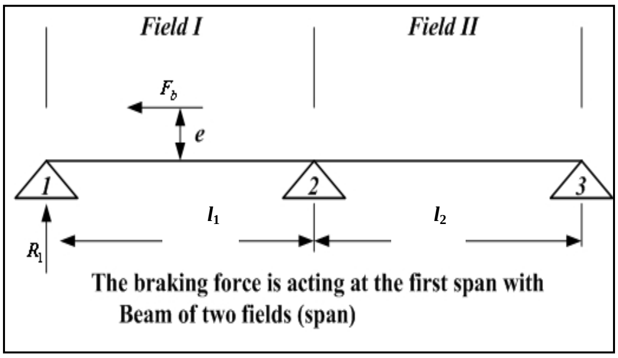


**Fig. 3. The Location of the Braking Force at Field I**

By using Theorem of three moments [6]**,

$$M_1 l_1 + 2M_2(l_1 + l_2) + M_3 l_2 + \frac{6A_1\bar{a}_1}{l_1} + \frac{6A_2\bar{b}_2}{l_2} = (\frac{h_1}{l_1} + \frac{h_3}{l_2}) * 6EI \qquad \text{… (1)}$$

where $h_1$, $h_3$ are the deflections at supports 1 and 3 respectively. There are no moments at the edge supports,

$$M_1 = M_3 = 0, \quad l_1 = l_2 = l \qquad \text{... (1a)}$$

$$\frac{6A_1\bar{a}_1}{l_1} = -\frac{F_b e}{l}(3z^2 - l^2) \qquad \text{… (1b)}$$

$$\frac{6A_2\bar{b}_2}{l_2} = 0 \qquad \text{… (1c)}$$

By substituting eqns.(1a,1b and 1c) into eqn.(1), it results in:

$$2M_2(l + l) = \frac{F_b e}{l}(3z^2 - l^2) \qquad \text{… (1d)}$$

$$M_2 = \frac{F_b e}{4l^2}(3z^2 - l^2) \qquad \text{... (1e)}$$

By taking moments about support (2),

$$R_1(l) + \frac{F_b e}{4l^2}(3z^2 - l^2) - F_b e = 0 \Rightarrow$$

$$R_1(l) = -\frac{F_b e}{4l^2}(3z^2 - l^2 - 4l^2) = -\frac{F_b e}{4l^2}(3z^2 - 5l^2)$$

$$\Rightarrow R_1 = -\frac{F_b e}{4l^2}(3z^2 - 5l^2) \qquad \text{…(1f)}$$

$$M(z) = R_1 z \qquad \text{… (1j)}$$

To find the maximum moment that produces through the guide rail,

$$\frac{dM(z)}{dz} = 0 = -\frac{F_b e}{4l^3}(5l^2 - 9z^2) \Rightarrow$$
$$z = 0.7454l \qquad \text{… (1i)}$$

By substituting eqn.(1i) into eqn.(1j) to find the maximum bending moment, it results in:

$$\therefore M(z)_{\max} = -0.6211 F_b e \qquad \text{... (2)}$$

This value is listed in Table 1.

### *b)* Assumption

***i.*** $F_b \times e$ in field II(span II).

***ii.*** No. of fields is two.

***iii.*** z will be taken from the left support of field II, as shown in Fig. 4.

The equations are the same as (case a), by replacing z with (*l*-*z*), therefore, from eqn.(1j),

$$M(z) = \frac{F_b e}{4l^3}(5l^2 - 3(l - z)^2)\,(l\text{-}z)$$

$$M(z) = \frac{F_b e}{4l^3}(3z^3 - 9lz^2 + 4l^2 z + 2l^3) \qquad \text{… (3)}$$

This value is listed in Table 2.

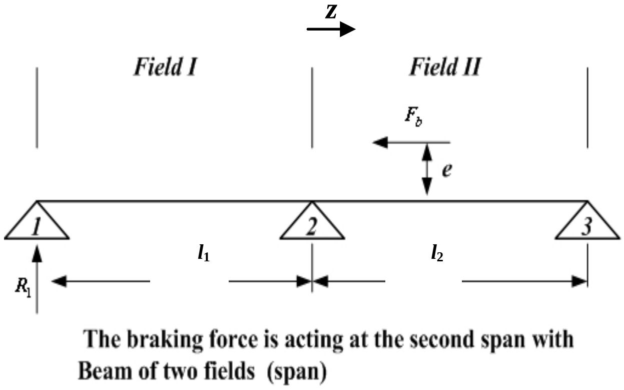


**Fig. 4. The Location of the Braking Force at FieldII**

**Table 1,**
**Bending Moment M*(z)* and its Maximum Values $M_{max}$ ($F_b \times e$ in field I) (spanI)**

| Number of fields (spans) | M(z) | Maximum value $M(z)_{max}$ | Location of the extreme $z_m$ | Equation No. |
|---|---|---|---|---|
| 2 | $-\frac{F_b \times e}{4l^3}(5l^2 \times z - 3z^3)$ | $-0.6211 F_b \times e$ | *0.7454 l* | *1&2* |
| 3 | $-\frac{F_b \times e}{15l^3}(19l^2 \times z - 12z^3)$ | $-0.6135 F_b \times e$ | *0.7265 l* | |
| 4 | $-\frac{F_b \times e}{56l^3}(71l^2 \times z - 45z^3)$ | $-0.6130 F_b \times e$ | *0.7252 l* | |
| 5 | $-\frac{F_b \times e}{209l^3}(265l^2 \times z - 168z^3)$ | $-0.6129 F_b \times e$ | *0.7251 l* | |

**Table 2,**
**Bending Moment M*(z)* and its Maximum Values $M_{max}$ ($F_b \times e$ in field II) (span II)**

| Number of fields | M(z) | Maximum value $M(z)_{max}$ | location of the extreme $z_m$ | Equation No. |
|---|---|---|---|---|
| *2* | $-\frac{F_b \times e}{4l^3}(2l^3 + 4l^2 \times z - 9l \times z^2 + 3z^3)$ | $-0.6210 F_b \times e$ | *0.2546 l* | *3* |
| *3* | $-\frac{F_b \times e}{15l^3}(7l^3 - 14l^2 \times z + 45l \times z^2 - 30z^3)$ | $-0.6161 F_b \times e$<br>$+0.6161 F_b \times e$ | *0.1927 l*<br>*0.8073 l* | |
| *4* | $-\frac{F_b \times e}{56l^3}(26l^3 - 52l^2 \times z + 171l \times z^2 - 117z^3)$ | $-0.6162 F_b \times e$<br>$+0.6060 F_b \times e$ | *0.1885 l*<br>*0.7858 l* | |
| *5* | $-\frac{F_b \times e}{209l^3}(97l^3 - 194l^2 \times z + 693l \times z^2 - 438z^3)$ | $-0.6162 F_b \times e$<br>$+0.6057 F_b \times e$ | *0.1882 l*<br>*0.7844 l* | |

### 2.1.2. During Safety Gear Operation (Taking Buckling Into Consideration)

Safety gear location is of prime significance. When the safety gear is located under the car floor, the gripping of the gide rails may take place in field I. If the safety gear is mounted above the car roof the guide rails may be gripped in field II only, as shown in Fig.5. The calculation will be carried out in case of the combined bending and buckling (simultaneous bending and buckling).

### (A) Field I

Fig. 5. shows the guide rail (as a beam) with the braking force. The derivation of the equation of the bending moment at any point on the left side of $F_b \times e$ is listed below,
From the fundamentals of statics,

$$R_1 = \frac{F_b e + M_1}{l} \quad \text{... (4a)}$$

$$M(z) = \frac{(F_b e + M_2)}{l} z + F_b y \quad \text{... (4b)}$$

$$EI \frac{d^2 y}{dz^2} = -M(z) = -F_b y - \frac{(F_b e + M_2)}{l} z$$

$$\frac{d^2 y}{dz^2} + \alpha^2 y = -\frac{(F_b e + M_2)}{EIl} z$$

… (4C1)

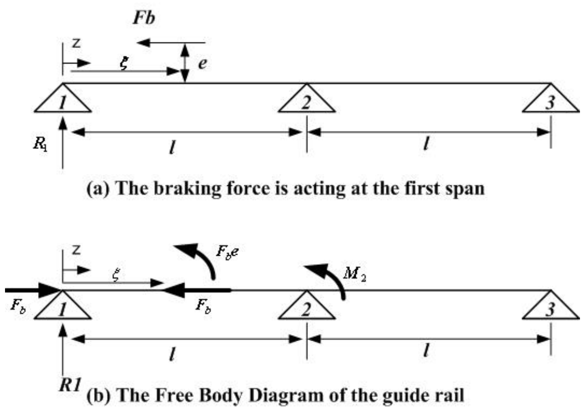


**Fig. 5. F.B.D. of Guide Rail (at Field I)**

The solution of this non-homogeneous second order D.E. has a combined particular and homogeneous solution shown as follows:

$y = y_h + y_p$, where $y_h$ and $y_p$ are the homogeneous and particular solutions respectively.

For homogeneous solution,

$$\frac{d^2y}{dz^2} + \alpha^2 y = 0$$

$$\therefore y_h = c_1 \cos \alpha z + c_2 \sin \alpha z$$

While for particular solution, we will assume $y_p=Az+B$

$$y' = A \Rightarrow y'' = 0 \qquad \ldots (4C_2)$$

By substituting eqn. ($4C_2$) into eqn.( $4C_1$), it results in:

$$0 + \alpha^2 (Az + B) = -\frac{(F_b e + M_2)}{EIl} z,$$

where $\alpha^2 = \frac{F_b}{EI}$ by comparing the factors of z , it results in,

$$A = -\frac{(F_b + M_2)}{F_b l}$$

$$\therefore y_p = -\frac{(F_b + M_2)}{F_b l} z$$

$$y = C_1 \cos \alpha z + C_2 \sin \alpha z - (F_b + M_2)\frac{z}{F_b l} \qquad \ldots (4cc)$$

The boundary conditions from Fig.5,

$$\text{at } z = 0, y = 0 \Rightarrow C_1 = 0 \qquad \ldots (4d)$$

$$z = \xi, y = 0 \Rightarrow C_2 = \frac{(F_b e + M_2)}{F_b \sin \alpha \xi l} \xi \qquad \ldots (4e)$$

Substituting eqns.(4d&4e) into eqn.(4cc), it results in,

$$y = \frac{(M_2 + F_b e)}{F_b}\left[\frac{\sin \alpha z}{\sin \alpha \xi} \cdot \frac{\xi}{l} - \frac{z}{l}\right] \qquad \ldots (4f)$$

Substituting eqn.(4f) into eqn.(4b), it results in,

$$M(z) = \frac{(F_b e + M_2)}{l} z + F_b \left(\frac{M_2 + F_b e}{F_b}\right) * \left(\frac{\sin \alpha z}{\sin \alpha \xi}\frac{\xi}{l} - \frac{z}{l}\right)$$

$$\therefore M(z) = \frac{F_b e + M_2}{l \sin \alpha \xi} \xi \times \sin \alpha z \qquad \ldots (5)$$

The extreme is located at $z_m = \pi /2\alpha$ and its value is given by the following formula

$$M(z) = \frac{F_b \times e + M_2}{l \times \sin \alpha \xi} \times \xi \qquad \ldots (6)$$

The extreme moment is located at $z_m = \pi / 2\alpha$ and its value is given by the following formula

$$M(z) = \frac{F_b e + M_2}{l \sin \alpha \xi} \times \xi \qquad \ldots (6a)$$

The graphical illustration of M(z)will be shown in the results.

## (B) Field II

The formulae are obtained from the same initial equations by means of the same mathematical methods which are more complicated than those in field I. Fig. 6 shows this case; the moment at any section through the beam(guide rail),

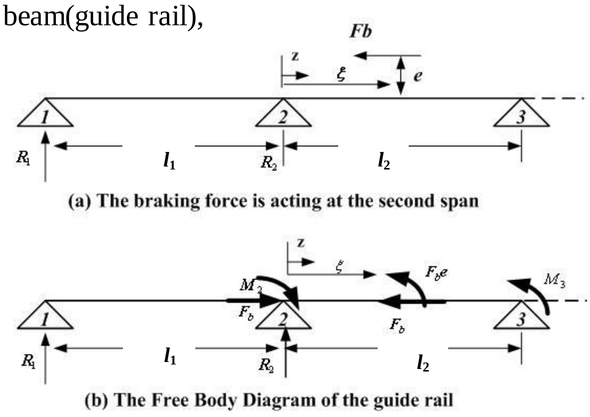


**Fig. 6. F.B.D. of Guide Rail (at Field II)**

$$M = R_2 z + M_2 + F_b y \qquad ... (7)$$

$$\frac{dM}{dz} = R_2 + F_b \frac{dy}{dz}$$

$$\frac{d^2M}{dz^2} = F_b \frac{d^2y}{dz^2} = -F_b \frac{M}{EI} = -\alpha^2 M$$

$$\frac{d^2M}{dz^2} + \alpha^2 M = 0 \qquad \ldots (8)$$

And the solution of this D.E.,

$$M = C_1 \sin \alpha z + C_2 \cos \alpha z \qquad \ldots (9)$$

The boundary conditions,

at $z=0,\ M=M_2 \Rightarrow C_2=M_2$ …(10)

at $z = \xi \Rightarrow M = M_2 - F_b e + R_2 \xi$

$$M = M_2 - F_b e + \left( \frac{M_3 + F_b e - M_2}{l} \right)$$

$$C_2 = \frac{(F_b e + M_3)\xi + M_2 (l - \xi - l \cos \alpha \xi)}{l \sin \alpha \xi} \qquad \ldots (11)$$

$M_3$ is moment at the right support of field II (support 3).

The location of the extreme is

$$z_m = \frac{1}{\alpha} \times \tan^{-1} \frac{C_2}{C_1} \qquad ... (12)$$

Constants of integration $C_1$ and $C_2$ are dependent upon the moments at supports, i.e., on the location of the moment $F_b \times e$. Consequently, in contrast to field I, the location of the maximum bending moment $(z_m)$ is a function of the location of $F_b \times e$ $(\xi)$ in this case. The maximum value of the bending moment,

$$M_{\max} = M_2 \times \cos \alpha zm + \frac{(F_b \times e + M_3) \times \xi}{l \times \sin \alpha \xi} \times \sin \alpha z_m + \frac{M_2 \times (l - \xi - l \times \cos \alpha \xi)}{l \times \sin \alpha \xi} \times \sin \alpha \xi \qquad \ldots (13)$$

$F_b \times e$ is applied at the right support (2) of field II. This case is decisive when the safety gear is mounted above the car roof.

### 2.1.3. During Normal Operation

Under normal operating conditions, the load may be unevenly distributed in two perpendicular directions. In Fig.7, a pictorial diagram of guide rails and all forces exerted upon them due to uneven car loading are shown. Forces $F_y$ are exerted in the plane of guide rails $(y-y)$, in which $Fx_1$ and $Fx_2$ are acting in $x-x$ planes at right angles to the $y-y$ plane. Each guide rail is subjected to bending due to $Fy$ and combined bending and torsion.

By taking moments about the axes x,y and z respectively, as shown in Fig. 7.

$$F_y = \frac{Q \times g \times e_y}{h} \qquad ... (14)$$

$$F_{x1} = \frac{Q \times g \times e_x \times (b + 2e_y)}{2h \times b} \qquad ... (15)$$

$$F_{x1} = \frac{Q \times g \times e_x \times (b - 2e_y)}{2h \times b} \qquad ... (16)$$

where Q is rated load(kg), g is standard acceleration of free fall (m/s$^2$), $e_y$ and $e_x$ are eccentricity of the load in the car(mm), b is width of the car (mm), c is depth of the car (mm), h is vertical distance between the centerlines of car guide shoe(mm).

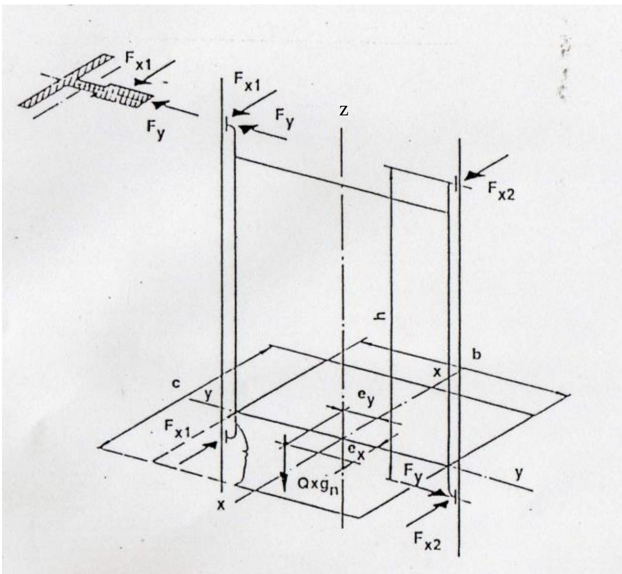


**Fig. 7. Forces on guide rails during normal operation**

## 2.2. Code Calculations (British Standard BS 5655: Part 9) [9]

The stress in guide rail during the safety gear operation $\sigma$ is given by the equation,

$$\sigma = \frac{F_b}{A} + \frac{F_b \times e}{2Z_x}\left[\cos^{-1}\left(\frac{l_k}{2}\sqrt{\frac{F_b}{E \times I_x}}\right) + 1\right] \quad \ldots (17)$$

We should note that the second term of the above equation is combined of bending and buckling moment,i.e., there is a silmultaneous effect of buckling and bending moments.

The guide rail deflection during the safety gear is limited to a maximum of 0.25 $\times$length of the machine face of the guide rail in order to avoid the risk of guide shoe disengagement from the guide rail. For this condition, the maximum permissible braking force is given by the equation,

$$F_b = \frac{4E \times I_{xx}}{l_k} \cos^{-2}\left(\frac{e}{2y_{\max} + e}\right) \quad \ldots (18)$$

In general, the braking force in the event of two guide rails being employed is given by the formula

$$F_b = \frac{Q + K}{2} \times (a + g) \quad \ldots (19)$$

where K is the car mass, (kg). Stress in guide rails calculated from eqn(17) must not exceed the values: 140 Mpa for steel of 370 Mpa tensile grade, 170 Mpa for steel of 430 Mpa tensile grade and 210 Mpa for steel of 520 Mpa tensile grade. The Young's modulus of elasticity is specified E=2.07 ($10^5$) Mpa.

Performance criteria based on stress and deflection in guide rails during normal operation are as follows: the guide rail is considered a simple beam with a certain degree of constraints on the fixing points and the lateral force is assumed to be imposed midway between the guide rail fixings.

Then the maximum stress in bending is given by

$$\sigma_y = \frac{F_y (l_k)}{6Z_x} \quad \ldots (20)$$

$$\sigma_x = \frac{F_x (l_k)}{6Z_y} \quad \ldots (21)$$

The constant factor in denominators of the above equations would be 4 for pin-jointed supports and 8 for fixed ends. Horizontal deflections at the midpoints of the beam in two perpendicular deflection are given by the formula

$$y_y = \frac{F_y (l_k{}^3)}{96EI_x}, y_x = \frac{F_x (l_k)^3}{96EI_y} \quad \ldots (22), (23)$$

The constant factors in the above equations would be 48 for pin-jointed supports and 129 for fixed ends. The maximum permissible deflection in compliance with eqn.(21) is 3mm in the pane of guide rails (yy) and 6mm in the perpendicular directions ($y_x$).

The problems of standard codes is shown in the results.

## 2.3. Recommended Calculation Method for Guide Rails

From the theoretical analysis, it can be concluded that there is no simultaneous buckling and bending (as we will see in the results), therefore the current procedure of design of guide rails includes this note. Later a case study is performed to achieve the current procedure.

### A. Safety Gear Operation

**(1)** Stress in combined bending and pressure (axial stress) is [4]

$$\sigma = F_b\left(\frac{1}{A} + C_1 \frac{e}{Z_x}\right) \quad \ldots (24)$$

Bending moment is induced by the eccentrically located braking force $F_b$; the outer moment is $F_b$×e. The calculation is carried out for a continuous beam. Coefficient $C_1$ is given in Table.3 depending upon the number of fields of continuous beam $C_1$ is concluded from Table.(1,2). The braking force for all cases is calculated from eqn.(19) .

**Table 3,**
**Coefficient $C_1$**

| Number of field | $C_1$ |
|---|---|
| 2 | *0.621* |
| 3 or more | *0.616* |

**(2)** Stress in buckling

The guide rail is assumed as a simple beam with two pinn-jointed supports, subjected to the braking force $F_b$ in its longitudinal axis. The procedure of buckling analysis is as follows [10,11,12,13,14]:

A) Determine the critical slenderness ratio,

$$S_r)_D = \frac{\pi\sqrt{2E}}{\sigma_y} \qquad \text{... (25a)}$$

B) Determine the slenderness ratio of the guide rail, $S_r = \frac{l}{k}$, $k = \sqrt{\frac{I_{\min}}{A}}$ ...(25b)

C) If $S_r)_D \le S_r$ ,then use Euler' s equation to find the critical force at which the frame will fail if it exceed this force,

$$P_{cr} = \frac{\pi^2 EA}{S_r^{\,2}} \qquad \text{… (26)}$$

D) *And* if the oppose case existed, Johnson' s equation is used,

$$P_{cr} = A[\sigma_y - \frac{1}{E}(\frac{\sigma_y S_r}{2\pi})^2] \qquad \text{… (27)}$$

## B. Running Conditions (Normal Operations)

The configuration of lateral and transversal force acting upon the guide rails is shown in Fig. 7. The load is assumed to be evenly distributed on three quarters of the car floor area in the most unfavorable position, as shown in Fig.8.

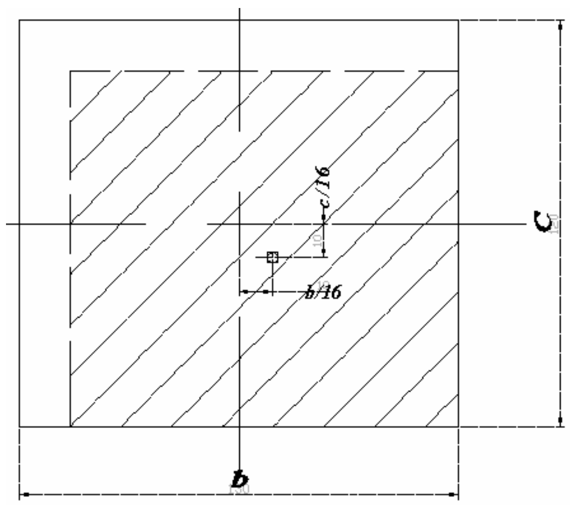


**Fig.8. Load Distribution for the Calculation of Bending Stress**

## Stress

Bending stress calculation for elevators with eccentric rails should be determined. With conventional elevators, bending stresses are low. The calculation will be done in a general case when 2-axial bending is taking place, as shown in Fig.7

Lateral force [6]:

$$F_x = \frac{5Qgc}{128h} \qquad \text{... (28a)}$$

Transversal force:

$$F_y = \frac{Q \times g \times b}{16h} \qquad \text{... (28b)}$$

The maximum bending moments may be obtained from the formulae. If $h \prec 1.5l$ (*h* is the distance between the center points of guide shoes)

$$M_x = 0.22F_x \times l$$
$$M_y = 0.2F_y \times l \qquad \text{... (29a)}$$

If $h \ge 1.5\, l$

$$M_x = 0.2Fx \times l$$
$$M_y = 0.17Fy \times l \qquad \text{... (29b)}$$

The related above equations are quoted from the theorem of three moments for continuous beam. It is found that maximum moment is about *0.2 Fl*.

Bending stresses are

$$\sigma_x = \frac{M_x}{Z_y}$$
$$\sigma_y = \frac{M_y}{Z_x} \qquad \text{...(30a, 30b)}$$

If the configuration is different from that shown in Fig. 8, forces $F_x$ and $F_y$ will be calculated accordingly. The simultaneous effect of $F_x$ and $F_y$ takes place only in the case of 2-axial bending.

## Deflection

Deflection in the plane of guide rails (*y-y*) is given by the formula [6]

$$\delta_y = C_2 \times \frac{F_y \times l^3}{E \times I_x} \quad \dots (31a)$$

where $l$ is maximum distance between the center points of adjacent guide rails brackets (mm), E is modulus of elasticity of the guide rail, and $I_x$ is the moment of inertia of guide rail cross-sectional area related to the *x-x* gravity axis. $C_2$ is quoted in Table 4 depending on the number of fields of continuous beam.

**Table 4,**
**Coefficients $C_2$**

| *Number of fields* | $C_2$ |
|---|---|
| *2* | *0.015* |
| *3* | *0.01458* |
| *4 or more* | *0.01455* |

Deflection in the x-x plane, perpendicular to the plane of guide rails, may be obtained from the formula

$$\delta_x = C_2 \frac{F_x \times l^3}{EI_x} \quad \dots (31b)$$

The maximum value of the deflection in either deflection should not exceed 3 mm [7].

Therefore , we can summarize the design procedure in Fig. 9.

## Case study

Given: Braking force $F_b$=24.525 KN, and the rated load(Q)=1000 kg.,(factor of safety) *f.s.*=4, vertical distance between the guide shoes *h=2950*mm, spacing of guide rail brackets *l=3300*mm, car width *b=1600*mm, depth of car is equal to *1400*mm, *e=27.5*mm,.

### *solution:*

We will select T 90/B with principal dimensions $90\times75\times16$ mm of 370MPa grade. $I_x=102.2\times10^4$mm$^4$, $I_y=52.0\times10^4$mm$^4$, $Z_x=20.9\times10^3$mm$^3$, $Z_y=11.9\times10^3$mm$^3$, $K_x=25$mm, $K_y=17.6$mm, $A=17.2(10^3)$mm$^2$.

**1- During safety gear operation**

**a)** check the buckling :

$$S_r = \frac{l}{k} = \frac{3300}{17.6} = 187.5$$

$$k_y = \sqrt{\frac{I_{min}}{A}} = 17.6mm$$

$$S_r)_D = \pi\sqrt{\frac{2E}{\sigma_y}} = 138.772MPa$$

$\boldsymbol{S_r \succ S_r)_D}$ $\therefore$ Euler's equation will be used.

$$P_{cr} = \frac{\pi^2 EA}{S_r^{\,2}} = \frac{\pi^2(2)(10^5)(17.2)(10^2)}{187.5^2} =$$

$$P_{cr} = 96.57KN$$

$\boldsymbol{F_b}$ is less than *96.57 KN* , the load is safe.

Factor of safety= $\frac{P_{cr}}{F_b} = \frac{96.57}{24.525} = 4$

**b)** Stress in combined load (axial stress & bending),

$$\sigma_{combined} = F_b\left(\frac{1}{A} + C_1\frac{e}{Z_x}\right) =$$

$$24525\left(\frac{1}{17.2\times10^2} + 0.616\frac{27.5}{20.9\times10^3}\right) =$$

$$\sigma_{combined} = 34.137MPa \prec 205 = \sigma_{yield}$$

**2- During Normal Operation**

2-axial bending will be assumed in conformity with Fig. 8. This configuration does not exactly comply with the specification for load distribution on 75% of the car floor area. The simplification adopted in this calculation leads to the load distribution on 76.5% of the floor area. The inaccuracy does not affect the correctness of the calculation.

Lateral forces on guide rails

$$F_y = \frac{Qgb}{16h} = \frac{1000(9.8)(1600)}{16(2950)} = 332.54N$$

$$F_x = \frac{5Qgc}{128h} = \frac{5(1000)(9.8)(1400)}{128(2950)} = 181.86N$$

$$M_x = 0.22F_x l = 132.03\times10^3 N.mm$$

$$M_y = 0.2F_y l = 219.475\times10^3 N.mm$$

$$\sigma_x = \frac{M_x}{Z_x} = \frac{132.03\times10^3}{20.9\times10^3} = 6.317MPa$$

$$\sigma_y = \frac{M_y}{Z_y} = \frac{219.475 \times 10^3}{11.9 \times 10^3} = 18.443 MPa$$

The stresses are very low compared with the yield stress (205 MPa).

Deflections in individual planes:

$$\delta_x = \frac{C_1 F_x l^3}{EI_y} = 0.0193.\frac{181.86(3.3)^3 (10)^9}{2.1(10)^5 (52)(10)^4} =$$

$$\delta_x = 1.155 mm$$

$$\delta_y = \frac{C_2 F_y l^3}{EI_x} = 0.01455.\frac{332.54(3.3)^3 (10)^9}{2.1(10)^5 (102.2)(10)^4}$$

$$\delta_y = 0.81 mm$$

$$\delta_{resultant} = \sqrt{\delta_x^{\,2} + \delta_y^{\,2}} = 1.4 mm \prec 3 mm$$

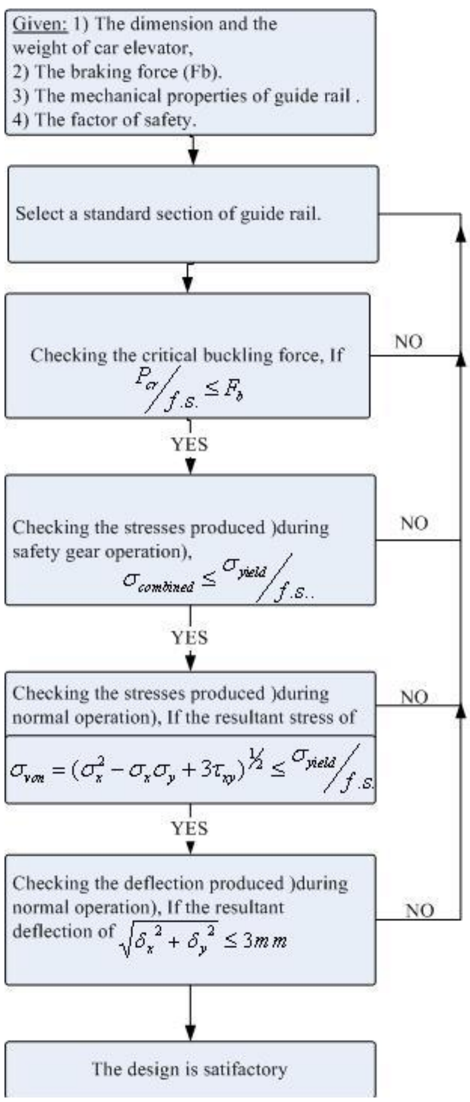


**Fig. 9. Design Diagram of Guide Rail**

## 3. Summary

It should be noted that the current study presents the following points:

- In the theoretical analysis, the maximum bending moment is investigated without the buckling effect firstly and then with the effect of the buckling. The analysis is performed during safety gear operation and the normal load operation.
- A standard code is listed to look at the equation used at the safety gear operation and the running conditions.

A simple method is introduced by depending on the results of the theoretical and standard codes.

## 4. Results and Discussion
### 4.1. Theoretical Analysis

From equation (6a), it is noted that *M* (*z*) is a sine function and is illustrated in Fig.10. For a definite location of $F_b \times e$, M (z) may be depicted in relation to the ratio *z/l*. As regards $\alpha$, three theoretical cases may take place:

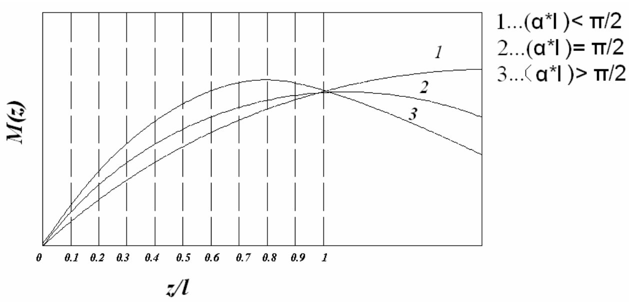


**Fig. 10. Graphical Illustration of M(z)(or $M_x$)**

**(a)** $\alpha \times l \prec \pi/2$

The extreme of the function lies beyond Field I. It means that the maximum value of the bending moment remains at the point of application of $F_b e$ and the buckling effect of the braking force $F_b$ is not decisive due to the rigidity of guide rail in the plane perpendicular to the *x-x* axis of the guide rail cross-section.

**(b)** $\alpha \times l = \pi/2$

The extreme takes place on the right support of field I(support 1).

**(c)** $\alpha \times l \succ \pi/2$

The extreme lies in field I. For $\xi \succ z_m$, it means that the bending moment left of $F_b \times e$ is greater than at the point of application of $F_b e$ and is decisive for the strength calculation of guide rail.

Through the analysis of guide rails stress during the safety gear operation, guide rails were never subjected to stress in combined buckling and bending in the plane, where the bending moment was exerted, i.e., in the direction of y-y axis of the guide rails cross-sectional area. It was always a combination of pressure and bending. Consequently, it is wrong to consider a simultaneous effect of buckling and bending. However, stress in buckling should not be overlooked to avoid the risk of the loss of guide rail static stability. Buckling would occur in the plane of the smallest rigidity in the bending of the guide rail. The specification for the braking force $F_b$ may become a matter of discussion, particularly in the case of the application of instantaneous safety factor. The load is supposed to be evenly distributed on the car floor during the safety gear operation. The simultaneous effect of the braking force due to safety gear operation and horizontal forces caused by eccentric position of the load in the car is not taken into consideration. It is highly improbable that a coincidence of safety gear operation and the most unfavorable distribution of the load on the car floor would take place with elevators of current design; however, the simultaneous effect with panoramic elevators, where the cantilevered load always creates lateral forces on guide rails, must be accounted for. Lateral forces may also be induced by the weight of the car itself, depending on the location of the center of gravity in relation to car mass and suspension point. Furthermore, due to the cantilevered position of the load, additional dynamic turning moment during safety gear operation will exert additional dynamic forces on the guide rails.

### 4.2. Code calculations (British Standard BS 5655: Part 9

In BS 5655:part9, horizontal forces on guide rails are specified for normal operation conditions as well as maximum permissible deflections. Unfortunately, a simultaneous effect of two forces acting at right angles to each other at the same point of the guide rail is not taken into consideration, though the load may be unevenly distributed in both directions at the same time. When calculating forces in the direction of x-x axis, the load is assumed to be located symmetrically to *x-x* axis and forces on both guide rails are of the same value.

Equations both for stress and deflection were derived for a simple beam with a certain degree of constraint on the fixing points. As a consequence, denominations in formula were altered. However, guide rails are actually continuous beams and it would be of interest to do comparative calculations. Torsion of guide rails caused by horizontal forces in the direction of x-x axis is not taken into consideration in any standard, though its influence should not be disregarded.

### 4.3. Recommended Calculation Method for Guide Rails

From the case study it can be seen that the method is very simple to investigate the stress analysis of the guide rail. This method can be used to design the guide rail by try and error method. First assume a guess for guide rail geometry and then check allowable stresses for it. The procedure stays until a preferred factor of safety is performed.

## 5. Conclusions

The following points may be deduced,

1) The theoretical analysis proves that there is no simultaneous effect of buckling and bending moments, and that the standard catalogues are assumed to be the opposite case.
2) The standard codes assume the guide rails as a simple beam with a certain degree of constraint on the fixing points. As a consequence, denominations in formula are altered. However, guide rails are actually continuous beams and it would be of interest to do comparative calculations.
3) The recommended method is just a design procedure to check the failure of guide rails using simple equations of strength of materials. It takes the problems of the theoretical and standard codes into consideration, and cure them.

## Notation

| | |
|---|---|
| A | Cross-sectional area of the guide rail,($mm^2$) |
| a | Maximum permissible retardation of the car,($m/s^2$) |

$\bar{a}$ Location of center of bending moment diagram from left support,mm

b Width of the car,(mm)

$\bar{b}$ Location of center of bending moment diagram from right support,(mm)

c Depth of the car, (mm)

E Young' modulus of steel, (MPa )

e Distance of braking force from the z-axis, (mm)

$e_x, e_y$ Eccentricity of the load in the car,(mm)

$F_b$ Braking force, (N)

$F_y, F_{x1}, F_{x2}$ Forces which are exerted on the guide rail during normal operation,(N)

g Gravitational acceleration,($m/s^2$)

h Vertical distance between centerlines of car guide, (mm)

I Moment of inertia of the cross-sectional area of the guide rails,($mm^4$)

K Mass of the car, (Kg)

$l$ Distance between any two successive supports(span),mm

$l_k$ Maximum distance between guide rail brackets,(mm)

$M_1$ Moment at support 1,(N.mm)

Q Rated load of elevator car, (kg).

$S_r$ Slenderness ratio of guide rail

$S_r)_D$ Critical slenderness ratio

y Deflection through the guide rail,(mm)

z Variable distance from the left support(0) to the point at which the bending moment is calculated, (mm)

## Greek letters

$\alpha$ Parameter is equal to $\alpha = \sqrt{\frac{F_b}{EI_{xx}}}$

$\delta$ Maximum deflection of guide rail during normal operation,(mm)

$\sigma$ Stress in the guide rail,(MPa)

$\xi$ Distance from the left support(1) to the point at which the outer moment $F_b e$ is acting,(mm)

## 6. References

[1] Rildova," Seismic Performance of Counterweight System of Elevator in Buildings", Research for obtaining Doctor of Philosophy, Canada, Monash University, 2004.

[2] Brockenbrough, R. L.,"Properties of structural steels and effects of steelmaking and fabrication", 3rd ed., McGraw-Hill, Inc, 2005.

[3] Caporale, R. S.,"Great Hanshin (Kobe) earthquake of 1995. *Elevator World*", JSME, June ,1995, 55-72.

[4] Utsunomiya,Ken-Ichi Okamoto, and Takashi Yumura," Active roller guide system for high speed elevator", JSME,No.60-579,pp.190-195.

[5] Recep,Honda, and Hakala," A comparative stress analysis of guide rail by FEA", JSME,No. 200-305,pp.80-88.

[6] Penn, Elsco and Robert," The study of neoprene elevator guide rails", JSME, Sept., 2006, 112-130.

[7] Clem and Barker," Active control of elevator guide rails", JSME, July, 2003, 34-50.

[8] Zhu, Wang and Xianhui," Calculation and analysis of earthquake resistance of elevator", JSME, Nov., 2000, 33-38.

[9] British Code for Elevator and Escalator, Part9 , 1995,London.

[10] R.C.Hibbeler, "Mechanics of materials", 3d edition,Prentice Hall International,Inc.,1997,

[11] Ferdinand L. Singer&Andrew Pytel, "Strength of materials", Arabic edition,1990.

[12] Allen,Alfred & Herman," Theory and problems of Machine design",Schaum's outline series,1961.

[13] Joseph E.Shigley& Charles R.Mischke, "Mechanical engineering design", 6th edition,2003.

[14] V B Bhandari, "Design of Machine elements", Tata Mc-Graw Hill,Inc.,25th reprint 2006.

# تحليل الإجهاد لموجهات سكك المصاعد

حيدرفانوس نعمة* وسام حاسم علي**
* قسم الهندسة الميكانيكية / كلية الهندسة/ جامعة بغداد
** قسم الهندسة الميكانيكية/ كلية الهندسة/ الجامعة المستنصرية

**الخلاصة**

ان التصميم الميكانيكي لأجزاء المصاعد يكون غالباً عن طريق دليل (CATALOGUE) تابع لشركات عالمية. فالمصمم سيختار الاجزاء على وفق الموجود من دون معرفة المضمون العلمي أحياناً. ولهذا تم اجراء تحليل نظري لموجهات المصاعد عند حالتين ، حالة ايقاف المصعد وحالة حركة المصعد وتم ايجاد معادلات لعزم الانحناء ولكلتا الحالتين وبحسب الظروف المذكورة في البحث.

ومن ثم تم التطرق الى الدليل البريطاني والمعادلات المستخدمة فيه. وتبين بان المعادلات النظرية افادتنا بأنه (في حالة عملية ايقاف المصعد) لا توجد محصلة أجهاد مركبة من عزم الانحناء وعزم الانبعاج، بل الإجهاد في محور الانحناء هو اجهاد ناتج من الاجهاد المحوري والاجهاد الانحنائي. وعزم الانبعاج سيحصل في محور الذي فيه أقل عزم مساحة للمقطع. أما بالنسبة للدليل فأنه يدمج عزم الانحناء مع عزم الانبعاج آنياً، في حين أن تأثيرهما غير آني كما هو مذكور في النتائج النظرية، بالاضافة الى عيوب مدرجة البحث الحالي. وبالتالي تم التطرق الى طريقة تصميمية بالاعتماد على النتائج النظرية و الدليل بالاضافة الى الطرق التصميمية المعروفة في تصميم المكائن.