The Complete Solution of some Kinds of Linear Third Order Partial Differential Equations with Three Independent Variables

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ABSTRACT

Our aim of this research is to find the complete solution of some kinds of linear third order partial differential equations of constant coefficients which have the general form

$$A_1 Z_{xxx} + A_2 Z_{yyy} + A_3 Z_{ttt} + A_4 Z_{xxy} +$$
$$A_5 Z_{xxt} + A_6 Z_{yyx} + A_7 Z_{yyt} + A_8 Z_{ttx} +$$
$$A_9 Z_{tty} + A_{10} Z_{xyt} + A_{11} Z_{xx} + A_{12} Z_{yy} +$$
$$A_{13} Z_{tt} + A_{14} Z_{xy} + A_{15} Z_{xt} + A_{16} Z_{yt} +$$
$$A_{17} Z_x + A_{18} Z_y + A_{19} Z_t + A_{20} Z = 0,$$

where $A_i \quad i=1,2,...20$ are constants.

By use the assumption
$$Z(x, y,t) = e^{\int u(x)dx + \int v(y)dy + \int w(t)dt}$$ will
transform the above equation to the nonlinear second order ordinary differential equation with three independent functions $u(x)$, $v(y)$ and $w(t)$.

1. INTRODUCTION

The differential equations are very importance in the plenty of the fields of the science as Physics, Chemistry and other sciences, therefore plenty of the scientists are studied this subject and they are tried to find modern methods for getting rid up the difficulties that facing them in the solving of some of these equations.

In [7] used the assumption

$$y(x) = e^{\int Z(x)\, dx}$$

to find the general

solution of equation

$$A_1 Z_{xxx} + A_2 Z_{yyy} + A_3 Z_{ttt} + A_4 Z_{xxy}$$
$$+ A_5 Z_{xxt} + A_6 Z_{yyx} + A_7 Z_{yyt} + A_8 Z_{ttx}$$
$$+ A_9 Z_{tty} + A_{10} Z_{xyt} + A_{11} Z_{xx} + A_{12} Z_{yy}$$
$$+ A_{13} Z_{tt} + A_{14} Z_{xy} + A_{15} Z_{xt} + A_{16} Z_{yt}$$
$$+ A_{17} Z_x + A_{18} Z_y + A_{19} Z_t + A_{20} Z = 0,$$
$$y'' + P(x)\, y' + Q(x)\, y = 0$$

where $P(x)$ and $Q(x)$ are either constants or functions of x .

In [1] used the assumption

$$Z(x, y) = e^{\int u(x)\, dx + \int v(y)\, dy}$$

to find the complete solution ofequation are constants

$$AZ_{xx} + BZ_{xy} + CZ_{yy} + DZ_x + EZ_y + FZ = 0$$

Where A , B , C , D , E and F are arbitrary constants .

In [4] used the assumption

$$Z(x, y,t) = e^{\int u(x)dx + \int v(y)dy + \int w(t)dt}$$

to find the complete solution of the equation

$$AZ_{xx} + BZ_{xy} + CZ_{xt} + DZ_{yy} + EZ_{yt}$$
$$+ FZ_{tt} + GZ_{x} + HZ_{y} + IZ_{t} + JZ = 0$$

Where A,B,C,…,I and J are arbitrary constants .

In [5] used the assumptions

$$Z(x,y) = e^{\int \frac{u(x)}{x} dx + \int v(y) dy} \quad,$$

$$Z(x,y) = e^{\int u(x) dx + \int \frac{v(y)}{y} dy} \quad and$$

$$Z(x,y) = e^{\int \frac{u(x)}{x} dx + \int \frac{v(y)}{y} dy}$$

to find the complete solution of the equation

$$A(x, y)Z_{xx} + B(x, y)Z_{xy} + C(x, y)Z_{yy}$$
$$+ D(x, y)Z_{x} + E(x, y)Z_{y} + F(x.y)Z = 0$$

Where some of A(x,y),B(x,y),C(x,y),D

(x,y),E(x,y) and F(x,y)

are functions of x or y or both x and y .

In [8] used the assumptions

$$Z(x, y) = e^{\int u(x)dx + \int v(y)dy} \quad,$$

$$Z(x, y) = e^{\int \frac{u(x)}{x}dx + \int v(y)dy}$$

$$Z(x, y) = e^{\int u(x)dx + \int \frac{v(y)}{y}dy} \quad and$$

$$Z(x, y) = e^{\int \frac{u(x)}{x}dx + \int \frac{v(y)}{y}dy}$$

to find the complete solutions of the equation

$$A\big(x, y, Z, Z_x, Z_y, Z_{xx}, Z_{xy}, Z_{yy}\big)Z_{xx} +$$
$$B\big(x, y, Z, Z_x, Z_y, Z_{xx}, Z_{xy}, Z_{yy}\big)Z_{xy} +$$
$$C\big(x, y, Z, Z_x, Z_y, Z_{xx}, Z_{xy}, Z_{yy}\big)Z_{yy} +$$
$$D\big(x, y, Z, Z_x, Z_y, Z_{xx}, Z_{xy}, Z_{yy}\big)Z_{x} +$$
$$E\big(x, y, Z, Z_x, Z_y, Z_{xx}, Z_{xy}, Z_{yy}\big)Z_{y} +$$
$$F\big(x, y, Z, Z_x, Z_y, Z_{xx}, Z_{xy}, Z_{yy}\big)Z = 0$$

Where A,B,C,D,E and F are functions of dependent variable Z and partial derivatives of dependent variable with respect to the independent variables x and y.

In [6] used the assumption

$$Z(x, y) = e^{\int u(x)dx + \int v(y)dy}$$

to find the complete solutions of the equation

$$AZ_{xxx} + BZ_{yyy} + CZ_{xxy} + DZ_{xyy} +$$
$$EZ_{xx} + FZ_{yy} + GZ_{xy} + HZ_{x} + IZ_{y} +$$
$$JZ = 0$$

Where A,…,I and J are arbitrary constants .

In this paper, we will find the complete solution of the equation

$$A_1 Z_{xxx} + A_2 Z_{yyy} + A_3 Z_{ttt} + A_4 Z_{xxy} +$$
$$A_5 Z_{xxt} + A_6 Z_{yyx} + A_7 Z_{yyt} + A_8 Z_{ttx} +$$
$$A_9 Z_{tty} + A_{10} Z_{xyt} + A_{11} Z_{xx} + A_{12} Z_{yy} +$$
$$A_{13} Z_{tt} + A_{14}Z_{xy} + A_{15} Z_{xt} + A_{16} Z_{yt} +$$
$$A_{17} Z_{x} + A_{18} Z_{y} + A_{19} Z_{t} + A_{20} Z = 0,$$

by using the assumption

$$Z(x, y, t) = e^{\int u(x)dx + \int v(y)dy + \int w(t)dt}$$

.Where $A_i \quad i = 1,2,...,20$ are constants

2. DESCRIPTIONOFTHESUG GESTEDMETHOD

Let us consider the equation

$$A_1 Z_{xxx} + A_2 Z_{yyy} + A_3 Z_{ttt} + A_4 Z_{xxy} +$$
$$A_5 Z_{xxt} + A_6 Z_{yyx} + A_7 Z_{yyt} + A_8 Z_{ttx} + \quad …(1)$$
$$A_9 Z_{tty} + A_{10} Z_{xyt} + A_{11} Z_{xx} + A_{12} Z_{yy} +$$
$$A_{13} Z_{tt} + A_{14}Z_{xy} + A_{15} Z_{xt} + A_{16} Z_{yt} +$$
$$A_{17} Z_{x} + A_{18} Z_{y} + A_{19} Z_{t} + A_{20} Z = 0,$$

The equation (1) is linear third order partial differential equation (P.D.E.) with three independent variables and $A_i; \quad i = 1,2,...20$ are arbitrary constants.

In order to find the complete solution of the equation (1) , we search functions $u(x)$, $v(y)$ and $w(t)$ such that the assumption

$$Z(x, y, t) = e^{\int u(x)dx + \int v(y)dy + \int w(t)dt}$$

…(2)

Represents the complete solution of it, this assumption will transform the equation (1) to non-linear second order ordinary differential equation. By finding

$Z_x, Z_{xx}, Z_{xxx}, Z_{xy}, Z_{xt}, Z_{xxy}, Z_{xxt}, Z_y, Z_{yy},$
$Z_{yyy}, Z_{yt}, Z_{yyx}, Z_{yyt}, Z_t, Z_{tt}, Z_{ttt}, Z_{ttx}, \quad Z_{tty}$

and Z_{xyt} from the equation (2) , we get

$$Z_x = u(x)\ e^{\int(x)\ dx + \int v(y)\ dy + \int w(t)dt}$$,

$$Z_{xx} = \left[u'(x) + u^2(x)\right] e^{\int(x)\ dx + \int v(y)\ dy + \int w(t)dt}$$

$$Z_y = v(y)\ e^{\int(x)\ dx + \int v(y)\ dy + \int w(t)dt}$$,

$$Z_{yy} = \begin{bmatrix} v'(y) + \\ v^2(y) \end{bmatrix} e^{\int(x)\ dx + \int v(y)\ dy + \int w(t)dt}$$

$$Z_{xy} = u(x)\,v(y)\ e^{\int(x)\ dx + \int v(y)\ dy + \int w(t)dt}$$,

$$Z_t = w(t)\ e^{\int(x)\ dx + \int v(y)\ dy + \int w(t)dt}$$,

$$Z_{tt} = \begin{bmatrix} w'(t) + \\ w^2(t) \end{bmatrix} e^{\int(x)\ dx + \int v(y)\ dy + \int w(t)dt}$$ (1) , we get

$$Z_{xt} = w(t)u(x)\ e^{\int(x)\ dx + \int v(y)\ dy + \int w(t)dt}$$

$$Z_{yt} = w(t)\,v(y)\ e^{\int(x)\ dx + \int v(y)\ dy + \int w(t)dt}$$,

$$Z_{xxx} = [u''(x) + 3u(x)u'(x) + u^3(x)]e^{\int u(x)dx + \int v(y)dy + \int w(t)dt}$$

$$Z_{xxy} = [v(y)(u'(x) + u^2(x))]e^{\int u(x)dx + \int v(y)dy + \int w(t)dt}$$

$$Z_{ttt} = [w''(t) + 3w(t)w'(t) + w^3(t)]e^{\int u(x)dx + \int v(y)dy + \int w(t)dt}$$

$$Z_{tty} = [v(y)(w'(t) + w^2(t))]e^{\int u(x)dx + \int v(y)dy + \int w(t)dt}$$

$$Z_{ttx} = [u(x)(w'(t) + w^2(t))]e^{\int u(x)dx + \int v(y)dy + \int w(t)dt}$$

$$Z_{xxt} = [w(t)(u'(x) + u^2(x))]e^{\int u(x)dx + \int v(y)dy + \int w(t)dt}$$

$$Z_{yyx} = [u(x)(v'(y) + v^2(y))]e^{\int u(x)dx + \int v(y)dy + \int w(t)dt}$$

$$Z_{yyt} = [w(t)(v'(y) + v^2(y))]e^{\int u(x)dx + \int v(y)dy + \int w(t)dt}$$

$$Z_{xyt} = w(t)u(x)v(y)e^{\int u(x)dx + \int v(y)dy + \int w(t)dt}$$

$$Z_{yyy} = [v''(y) + 3v(y)v'(y) + v^3(y)]e^{\int u(x)dx + \int v(y)dy + \int w(t)dt}$$

And by substituting

$Z, Z_x, Z_{xx}, Z_{xxx}, Z_{xy}, Z_{xt}, Z_{xxy}, Z_{xxt},$
$Z_y, Z_{yy}, Z_{yyy}, Z_{yt}, Z_{yyx}, Z_{yyt},$
$Z_t, Z_{tt}, Z_{ttt}, Z_{ttx}, Z_{tty}$ and , Z_{xyt} into the equation (1) , we get

$[A_1 (u''(x)+3u(x)u'(x)+u^3(x))$

$+ A_2 (v''(y)+3v(y)v'(y)+v^3(y))$

$+ A_3 (w''(t)+3w(t)w'(t)+w^3(t))$

$+ A_4 (v(y)u'(x)+v(y)u^2(x))$

$+ A_5 (w(t)u'(x)+w(t)u^2(x))$

$+ A_6 (u(x)v'(y)+u(x)v^2(y))$

$+ A_7 (w(t)v'(y)+w(t)v^2(y))$

$+ A_8 (u(x)w'(t)+u(x)w^2(t))$

$+ A_9 (v(y)w'(t)+v(y)w^2(t))$

$+ A_{10} (v(y)w(t)u(x))$

$+ A_{11} (u'(x)+u^2(x))$

$+ A_{12} (v'(y)+v^2(y))$

$+ A_{13} (w'(t)+w^2(t)) + A_{14} u(x)v(y)$

$+ A_{15} u(x)w(t) + A_{16} w(t)v(y)$

$+ A_{17} u(x) + A_{18} v(y) + A_{19} w(t)$

$+ A_{20}]e^{\int u(x)dx+\int v(y)dy+\int w(t)dt}$

$= 0$ ... (3)

Since

$e^{\int u(x)dx+\int v(y)dy+\int w(t)dt} \neq 0$.

So,

$A_1 (u''(x)+3u(x)u'(x)+u^3(x))$

$+ A_2 (v''(y)+3v(y)v'(y)+v^3(y))$

$+ A_3 (w''(t)+3w(t)w'(t)+w^3(t))$

$+ A_4 (v(y)u'(x)+v(y)u^2(x))$

$+ A_5 (w(t)u'(x)+w(t)u^2(x))$

$+ A_6 (u(x)v'(y)+u(x)v^2(y))$

$+ A_7 (w(t)v'(y)+w(t)v^2(y))$

$+ A_8 (u(x)w'(t)+u(x)w^2(t))$

$+ A_9 (v(y)w'(t)+v(y)w^2(t))$

$+ A_{10} (v(y)w(t)u(x))$

$+ A_{11} (u'(x)+u^2(x))$

$+ A_{12} (v'(y)+v^2(y))$

$+ A_{13} (w'(t)+w^2(t)) + A_{14} u(x)v(y)$

$+ A_{15} u(x)w(t) + A_{16} w(t)v(y)$

$+ A_{17} u(x) + A_{18} v(y) + A_{19} w(t) + A_{20}$

$= 0$

The equation(3) is non-linear second order ordinary differential equation and contains three independent functions u(x) , v(y) and w(t) .
As the number of the classes of the equation (1) are very large, so, we will choose some kinds of it to find the complete solution of it , while the other kinds are solved by the same methods.

Kind (1) :
$A_1 Z_{ttt} + A_2 Z_{xxy} + A_3 Z_{xyy} = 0$.

Kind (2) :
$A_1 Z_{yyt} + A_2 Z_{yyx} + A_3 Z = 0$
.

Kind (3) :

$$A_1 Z_{tty} + A_2 Z_{xx} + A_3 Z_{tt} + A_4 Z_{xy} = 0$$

Where $A_1\ A_2\ , A_3\ and\ A_4$ are constants.

Now we will discuss the above kinds ,

kind (1) :

$$A_1 Z_{ttt} + A_2 Z_{xxy} + A_3 Z_{xyy} = 0$$

this equation becomes after using (2) as follows

$$A_1(w''(t) + 3w(t)w'(t) + w^3(t)) + A_2(v(y)u'(x)$$
$$+ v(y)u^2(x)) + A_3(u(x)v'(y) + u(x)v^2(y)) = 0$$

Here we can't separate the variables, so we

suppose that $\quad w(t) = \lambda_1 \quad and \quad v(y) = \lambda_2$

Where $\lambda_1 \quad and \quad \lambda_2$ are arbitrary constants ,

then

$$A_1\lambda_1^3 + A_2\lambda_2(u'(x) + u^2(x)) + A_3\lambda_2^2 u(x) = 0$$

$$\Rightarrow u'(x) + u^2(x) + \frac{A_3\lambda_2^2}{A_2\lambda_2}u(x) + \frac{A_1\lambda_1^3}{A_2\lambda_2} = 0$$

$$\lambda_2 \neq 0$$
$$u'(x) + u^2(x) + B_1 u(x) + B_2 = 0 \qquad \dots(5)$$

$$; \quad B_1 = \frac{A_3\lambda_2^2}{A_2\lambda_2}\ and\ B_2 = \frac{A_1\lambda_1^3}{A_2\lambda_2}$$

$$u'(x) + (u(x) + \frac{B_1}{2})^2 + B_2 - \frac{B_1^2}{4} = 0$$

If $\quad B_2 \neq \frac{B_1^2}{4}$,we get

$$\int \frac{du}{\left(u(x) + \dfrac{B_1}{2}\right)^2 + f_1^2} + \int dx = \int 0$$

$$\frac{1}{f_1}\tan^{-1}\left(\frac{u(x) + \dfrac{B_1}{2}}{f_1}\right) = C - x$$

; C is constant

$$\Rightarrow u(x) = f_1\tan(f_1 c - f_1 x) - \frac{B_1}{2} \quad \dots(4)$$

then the complete solution of equation (4)is

$$Z(x, y,t) = e^{\int(f_1\tan(f_1 c - f_1 x) - \frac{B_1}{2})dx + \int \lambda_2 dy + \int \lambda_1 dt}$$

$$= e^{\ln(\cos(f_1 c - f_1 x)) - \frac{B_1}{2}x + \lambda_2 y + \lambda_1 t + c_1}$$

$$; 0 < \cos(f_1 c - f_1 x) < 1$$

$$; \frac{\pi}{2} < f_1 c - f_1 x < 2\pi$$

$$\Rightarrow \quad \frac{f_1 c - 2\pi}{f_1} < x < \frac{f_1 c - \dfrac{\pi}{2}}{f_1}$$

$$= e^{\lambda_2 y + \lambda_1 t - \frac{B_1}{2}x}(d_1\cos\frac{\sqrt{4A_2 A_3\lambda_2^2 - (A_1\lambda_1^3)^2}}{2A_2\lambda_2}x +$$

$$d_2\sin\frac{\sqrt{4A_2 A_3\lambda_2^2 - (A_1\lambda_1^3)^2}}{2A_2\lambda_2}x)$$

$$; d_1 = e^{c_1}\cos\frac{\sqrt{4A_2 A_3\lambda_2^2 - (A_1\lambda_1^3)^2}}{2A_2\lambda_2}c$$

$$and \qquad d_2 = e^{c_1}\sin\frac{\sqrt{4A_2 A_3\lambda_2^2 - (A_1\lambda_1^3)^2}}{2A_2\lambda_2}c$$

$$2)\ If\ B_2 = \frac{B_1^2}{4}\ ,we\ get$$
$$; f_1^2 \neq B_2 - \frac{B_1^2}{4}$$

$$\int \frac{du}{(u + \dfrac{B_1}{2})^2} + \int dx = \int 0$$

$$\Rightarrow u = \frac{1}{x - c} - \frac{B_1}{2}$$

then the complete solution of equation (4)is

$$Z(x, y, t) = e^{\int (\frac{1}{x-c} - \frac{B_1}{2})dx + \int \lambda_2 dy + \int \lambda_1 dt}$$

$$= e^{\lambda_2 y + \lambda_1 t - \frac{A_1 \lambda_1^3}{2(A_2 \lambda_2)} x} (b_1 x - b_2)$$

$$; b_1 = e^{c_1} \quad and \quad b_2 = ce^{c_1}$$

$$\Rightarrow \quad \frac{1}{B_1} \tan^{-1}(\frac{v}{B_1}) = c - y$$

$$v = B_1 \tan(B_1 c - B_1 y)$$

$$Z = e^{\int \frac{\lambda_2^2}{A_2} dx + \int B_1 \tan(B_1 c - B_1 y) dy + \int \frac{\lambda_1^2}{A_1} dt}$$

kind (2) :

$$A_1 Z_{yyt} + A_2 Z_{yyx} + A_3 Z = 0$$

this equation becomes after using (2) as follows
.

$$A_1 w(t)(v'(y) + v^2(y)) + A_2 u(x)(v'(y) + v^2(y)) + A_3 = 0 \qquad ...(7)$$

Here we can separable the variables [3],[9] , hence

$$A_1 w(t) + A_2 u(x) + \frac{A_3}{(v'(y) + v^2(y))} = 0$$

let $\quad A_1 w(t) = \lambda_1^2, \quad A_2 u(x) = \lambda_2^2$

$$and \quad \frac{A_3}{(v'(y) + v^2(y))} = -(\lambda_1^2 + \lambda_2^2)$$

$$\Rightarrow w(t) = \frac{\lambda_1^2}{A_1}, \quad u(x) = \frac{\lambda_2^2}{A_2} \quad and$$

$$v'(y) + v^2(y) = -\frac{A_3}{(\lambda_1^2 + \lambda_2^2)}$$

$$\Rightarrow \quad v'(y) + v^2(y) + B_1^2 = 0 \qquad ...(8)$$

$$; B_1^2 = \frac{A_3}{(\lambda_1^2 + \lambda_2^2)} \qquad ; \lambda_1^2 \neq -\lambda_2^2$$

Equation (8) becomes

$$Z = e^{\frac{\lambda_2^2}{A_2} x + \frac{\lambda_1^2}{A_1} t + \ln(\cos(B_1 c - B_1 y)) + c_1} \quad ...(6)$$

$$; 0 < \cos(B_1 c - B_1 y) < 1$$

$$\Rightarrow \quad \frac{\pi}{2} < B_1 c - B_1 y < 2\pi$$

$$\Rightarrow c - \frac{2\pi}{B_1} < y < c - \frac{\pi}{2B_1}$$

$$Z = e^{\frac{\lambda_2^2}{A_2} x + \frac{\lambda_1^2}{A_1} t} (d_1 \cos \sqrt{\frac{A_3}{\lambda_1^2 + \lambda_1^2}} y + d_2 \sin \sqrt{\frac{A_3}{\lambda_1^2 + \lambda_1^2}} y)$$

$$; \quad d_1 = e^{c_1} \cos \sqrt{\frac{A_3}{\lambda_1^2 + \lambda_1^2}} c \quad and$$

$$d_2 = e^{c_1} \sin \sqrt{\frac{A_3}{\lambda_1^2 + \lambda_1^2}} c \; ; \; \lambda_1^2 \neq -\lambda_1^2$$

kind (3) :

$$A_1 Z_{tty} + A_2 Z_{xx} + A_3 Z_{tt} + A_4 Z_{xy} = 0$$

this equation becomes after using (2) as follows
.

$$A_1 v(y)(w'(t) + w^2(t)) + A_2 (u'(x) + u^2(x)) + A_3 (w'(t) + w^2(t)) + A_4 u(x)v(y) = 0$$

$$\int \frac{dv}{B_1^2((\frac{v}{B_1})^2 + 1)} + \int dy = \int 0$$

let $v(y) = \lambda_1$

$A_1 \lambda_1 (w'(t) + w^2(t)) + A_2 (u'(x) +$

$u^2(x)) + A_3 (w'(t) + w^2(t)) +$

$A_4 \lambda_1 u(x) = 0 \qquad \cdots(10)$

equation (10) is variableseparableequation[2] , we cansolve it asfollows:-

$A_1 \lambda_1 (w'(t) + w^2(t)) + A_3 (w'(t)$

$+ w^2(t)) = -\lambda_2^2$

$A_1 \lambda_1 (w'(t) + w^2(t)) + A_3 (w'(t) +$

$w^2(t)) + \lambda_2^2 = 0$

$(A_1 \lambda_1 + A_3) w'(t) + (A_1 \lambda_1 +$

$A_3) w^2(t) + \lambda_2^2 = 0$

$w'(t) + w^2(t) + B_1^2 = 0 \qquad \qquad ...(11)$

$; B_1^2 = \dfrac{\lambda_2^2}{(A_1 \lambda_1 + A_3)} \quad and \quad \lambda_1 \neq \dfrac{-A_3}{A_1}$

The equation (11) is similar to the equation (8) and by the same method

$w = B_1 \tan(B_1 c - B_1 t)$

Also

$let \quad \dfrac{A_4 \lambda_1}{2A_2} = D_1$

$and \quad \dfrac{\lambda_2^2}{A_2} + \dfrac{A_4^2 \lambda_1^2}{4A_2^2} = D_2^2$

$u'(x) + (u(x) + \dfrac{A_4 \lambda_1}{2A_2})^2 - \dfrac{\lambda_2^2}{A_2} - \dfrac{A_4^2 \lambda_1^2}{4A_2^2} = 0$

1) $if \quad \dfrac{\lambda_2^2}{A_2} \neq -\dfrac{A_4^2 \lambda_1^2}{4A_2^2}$

s

$u'(x) + (u(x) + D_1)^2 - D_2^2 = 0$

$\int \dfrac{du}{D_2^2 (\dfrac{u(x) + D_1}{D_2})^2 - 1} + \int dx = \int 0$

$\dfrac{-1}{D_2} \tanh^{-1}(\dfrac{u(x) + D_1}{D_2}) = c - x$

$; \quad -D_2 - D_1 \leq u(x) \leq D_2 - D_1$

$u(x) = D_2 \tanh(D_2 x - D_2 c) - D_1$

if $\quad \dfrac{\lambda_2^2}{A_2} = -\dfrac{A_4^2 \lambda_1^2}{4A_2^2}$

$\Rightarrow u'(x) + (u(x) + D_1)^2 = 0$

$A_1 \lambda_1 (w'(t) + w^2(t)) + A_3 (w'(t) +$

$w^2(t)) = -A_2 (u'(x) + u^2(x)) -$

$A_4 \lambda_1 u(x) = -\lambda_2^2$

$$\int \frac{du}{(u(x)+D_1)^2} + \int dx = \int 0$$

$$\Rightarrow \frac{-1}{u(x)+D_1} = c - x$$

$$\Rightarrow u(x) = \frac{1}{c-x} - D_1$$

Now

if $\quad \dfrac{\lambda_2^2}{A_2} \neq -\dfrac{A_4^2 \lambda_1^2}{4A_2^2}$

Then the complete solution of the equation (9) is

$$Z = e^{\int (D_2 \tanh(D_2 x - D_2 c) - D_1) dx + \int \lambda_1 dy + \int B_1 \tan(B_1 c - B_1 t) dt}$$

$$Z = e^{\ln(\cosh(D_2 x - D_2 c)) - D_1 x + \lambda_1 y + \ln(\cos(B_1 c - B_1 t)) + c_1}$$

$$; c - \frac{2\pi}{B_1} < t < c - \frac{\pi}{2B_1}$$

$$Z = e^{\lambda_1 y - \frac{A_4 \lambda_1}{2A_2} x} (d_1 \cosh \sqrt{\frac{\lambda_2^2}{A_2} + \frac{A_4^2 \lambda_1^2}{4A_2^2}} x -$$

$$d_2 \sinh \sqrt{\frac{\lambda_2^2}{A_2} + \frac{A_4^2 \lambda_1^2}{4A_2^2}} x)$$

$$(d_3 \cos \sqrt{\frac{\lambda_2^2}{(A_1 \lambda_1 + A_3)}} t +$$

$$d_4 \sin \sqrt{\frac{\lambda_2^2}{(A_1 \lambda_1 + A_3)}} t)$$

$$u'(x) + u^2(x) + \frac{A_4 \lambda_1}{A_2} u(x) - \frac{\lambda_2^2}{A_2} = 0$$

$$; \quad d_1 = \cosh \sqrt{\frac{\lambda_2^2}{A_2} + \frac{A_4^2 \lambda_1^2}{4A_2^2}} c$$

$$, d_2 = \sinh \sqrt{\frac{\lambda_2^2}{A_2} + \frac{A_4^2 \lambda_1^2}{4A_2^2}} c$$

$$, d_3 = e^{c_1} \cos \sqrt{\frac{\lambda_2^2}{(A_1 \lambda_1 + A_3)}} c$$

$$and \quad d_4 = e^{c_1} \sin \sqrt{\frac{\lambda_2^2}{(A_1 \lambda_1 + A_3)}} c$$

if $\quad \dfrac{\lambda_2^2}{A_2} = -\dfrac{A_4^2 \lambda_1^2}{4A_2^2} \quad ; A_2 \neq 0$

Then the complete solution of the equation (9) is

$$Z = e^{\int (\frac{1}{x-c} - D_1) dx + \int \lambda_1 dy + \int B_1 \tan(B_1 c - B_1 t) dt}$$

$$Z = e^{\ln(x-c) - D_1 x + \lambda_1 y + \ln(\cos(B_1 c - B_1 t)) + c_1}$$

$$; x > c \text{ and } c - \frac{2\pi}{B_1} < t < c - \frac{2\pi}{B_1}$$

$$Z = e^{\lambda_1 y - D_1 x}(x-c)(d_3 \cos \sqrt{\frac{\lambda_2^2}{(A_1 \lambda_1 + A_3)}} t +$$

$$d_4 \sin \sqrt{\frac{\lambda_2^2}{(A_1 \lambda_1 + A_3)}} t)$$

$$; \quad d_3 = e^{c_1} \cos \sqrt{\frac{\lambda_2^2}{(A_1 \lambda_1 + A_3)}} c$$

$$and \; d_4 = e^{c_1} \sin \sqrt{\frac{\lambda_2^2}{(A_1 \lambda_1 + A_3)}} c$$

3. <u>REFERENCES</u>

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الملخص

هدفنا في هذا البحث هو ايجاد الحل التام لبعض انواع المعادلات

التفاضلية الجزئية الخطية من الرتبة الثالثة ذات معاملات ثابتة وصيغتها العامة

وباستخدام التعويض

$$Z(x, y, t) = e^{\int u(x)dx+\int v(y)dy+\int w(t)dt}$$ تحول

المعادلة اعلاه الى معادلة تفاضلية اعتيادية لا خطية من الرتبة الثانية مع ثلاث دوال مستقلة $u(x)$, $v(y)$ و

$$w(t)$$

$$A_1\ Z_{xxx} + A_2\ Z_{yyy} + A_3\ Z_{ttt} + A_4\ Z_{xxy}$$
$$+ A_5\ Z_{xxt} + A_6\ Z_{yyx} + A_7\ Z_{yyt} + A_8\ Z_{ttx}$$
$$+ A_9\ Z_{tty} + A_{10}\ Z_{xyt} + A_{11}\ Z_{xx} + A_{12}\ Z_{yy}$$
$$+ A_{13}\ Z_{tt} + A_{14}Z_{xy} + A_{15}\ Z_{xt} + A_{16}\ Z_{yt}$$
$$+ A_{17}\ Z_x + A_{18}\ Z_y + A_{19}\ Z_t + A_{20}\ Z = 0,$$