2024, 21(3): 1095-1100
https://dx.doi.org/10.21123/bsj.2023.7959
P-ISSN: 2078-8665 - E-ISSN: 2411-7986
Baghdad Science Journal

A Comparison between Ericson's Formulae Results and Experimental Data Using New Formulae of Single Particle Level Density

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Received 15/10/2022, Revised 26/02/2023, Accepted 28/02/2023, Published Online First 20/08/2023, Published 01/03/2024

Abstract

The partial level density PLD of pre-equilibrium reactions that are described by Ericson's formula has been studied using different formulae of single particle level density g. The parameter g was used from the equidistant spacing model (ESM) model and the non- equidistant spacing model (non-ESM) and another formula of g are derived from the relation between g and level density parameter a. The formulae used to derive g are the Roher formula, Egidy formula, Yukawa formula, and Thomas –Fermi formula. The partial level density results that depend on g from the Thomas-Fermi formula show a good agreement with the experimental data.

Keywords: Exciton model, induced nuclear reaction, Level density, Pre-compound nucleus, Pre-equilibrium reactions.

Introduction

Level density is considered an important parameter in many theoretical studies like astrophysical studies and cross-section calculations reaction rates calculations [1-5].

Level density depends on the parameter a, which is called the level density parameter. This parameter measures the number of states per unit energy interval at the Fermi surface [6]

The parameter a was studied for 310 nuclei in the framework of Fermi Gas model for many isotopes between ^{18}C to ^{251}Cf by fitting the complete level scheme at low excitation energies. The obtained formula of this parameter represents useful tools to calculate the level density for nuclei that are far from stability[7]. The level density parameter was calculated for super heavy nuclei using the single particle energies obtained with the Woods-Saxon potential [8].

Excitation energy and angular momentum dependence of the parameter a were studied around the value of mass number ($A \cong 110$) using the Hauser – Feshbeach model. It is found there is an acceptable agreement with the experimental data [9].

In this paper, formulae of the parameter a will be used to find new formulae to the single particle level density g and the least will substitute in Ericson's formula then results will be compared with the experimental data.

Materials and Methods

Theory:

The pre-equilibrium reactions are introduced as the reactions that lead to emission

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before the thermal equilibrium occurs or in other words before reaching the compound nucleus stage [10].

Since some of the nucleons are excited during the pre-equilibrium region and the level density comes only from those excited nuclei, therefore, they are called partial level density (PLD).

The first formula that describes the PLD is Ericson's formula [10,11].

$$\omega\,(\mathrm{n.E}) = \frac{g^n E^{n-1}}{p!\,h!(n-1)!} \quad \text{..... 1}$$

The symbols ω (n, E) is the number of levels per unit energy, p is the particle number, h is the number of the holes $n = p + h$ is the number of exactions, E is the excitation energy and g represents the single particle level density.

The parameter g in the framework of equidistant spacing model ESM (the model considered the spacing between the energy levels to be equal)[10]

$$g = \frac{A}{d}\ (d \simeq 13\ MeV^{-1}) \quad \text{........ 2}$$

Where A is the mass number and d is the spacing between levels.

If the spacing between the levels is considered non-equal as it is supposed by the non-equidistant spacing model (non - ESM) the parameter g is given by [12]:

$$g_o = g\sqrt{\frac{\varepsilon}{F}} \quad \text{....... 3}$$

The symbol $\varepsilon = {}^{E}/_{n}$ represents the division of total energy on exciton number and F is the Fermi energy level which is equal to 38 MeV.

The parameter g is associated with the single level density parameter a by [13,14]

$$a = \frac{\pi^2}{6} g \quad \text{.......4}$$

The parameter a is given by many formulae [12,15]

Roher formula

$$a = 0.0071A + V \quad \text{.......5,}$$

Where V= 1.64 for A ≤ 38

Egidy formula

$$a = 0.18\, A^{0.9} \quad \text{........6,}$$

Thomas Fermi formula

$$a = (0.109(1 - 4.476I^2)A + 0.076(1 + 31.47I^2)A^{2/3} - 0.0024Z^2A^{-1/3}) \quad \text{......7}$$

The parameter I is the isospin and Z is the atomic number

Yukaua formula

$$a = (0.068A + 0.213A^{2/3} + 0.385A^{1/3}) \quad \text{.......8}$$

From the eq 4 and all a formulae one can get new formulae of g.

$$g = \frac{6}{\pi^2}\ (0.0071A + V) \quad \text{........9}$$

$$g = \frac{6}{\pi^2}\ (0.18A^{0.9}) \quad \text{........10}$$

$$g = \frac{6}{\pi^2}\ (0.068A + 0.213A^{2/3} + 0.385A^{1/3}) \quad \text{........11}$$

$$g = \frac{6}{\pi^2}\ (0.109(1 - 4.476I^2)A + 0.076(1 + 31.47I^2)A^{2/3} - 0.0024Z^2A^{-1/3}) \quad \text{.......12}$$

These new formulae of g will use in Ericron's formula in the next section.

Results and Discussion

Results:

Now, we will make a comparison between the theoretical results of PLD Ericson's formula with different values of g and the experimental data for ^{56}Fe

Fig 1. Shows a comparison between the results of PLD formula with g from ESM-model eq 2 and the experimental data. One can see the theoretical curve is in agreement with the experimental curve between 1 MeV to 2 MeV then the theoretical curve increases rapidly with increasing the excitation energy and the gap between both curves becomes as big as the excitation energy increase. This behavior can be interpreted that in ESM model the PLD increases rapidly with E then the theoretical curve becomes higher than the experimental data.

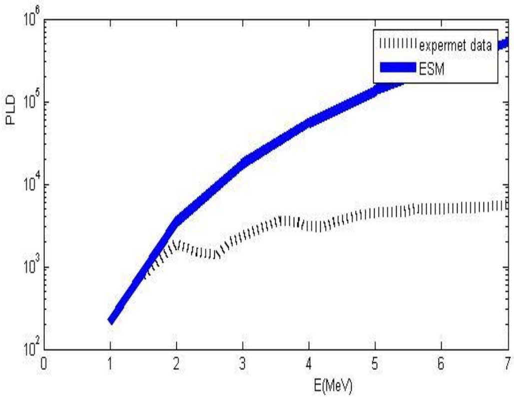

Figure 1. Comparison between theoretical curve with g from ESM and the experimental data

Fig 2. Shows a comparison between the theoretical PLD with g from non-ESM eq 3, and experimental data. As it is shown in the figure the theoretical values lower than the experimental data, the bigger difference between the theoretical and experimental curve at 1 MeV decreases with increasing energy but the theoretical curve stays lower than the experimental curve. Because in non-ESM the levels overlap with increasing energy therefore, the number of PLD will be little and this interprets why the theoretical curve is less than the experimental one.

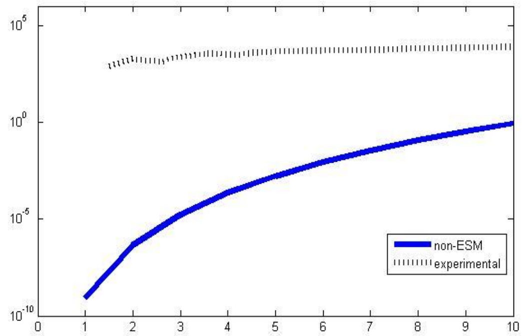

Figure 2. Comparison between theoretical curve with g from non-ESM and the experimental data

In Fig 3. A comparison was made between PLD with g from Roher eq 9, and the experimental data. The theoretical curve is very less than the experimental curve.

It is noticed that the theoretical curve is very less than the experimental curve this because g from Roher is multiplied by 0.0071 and this affects PLD.

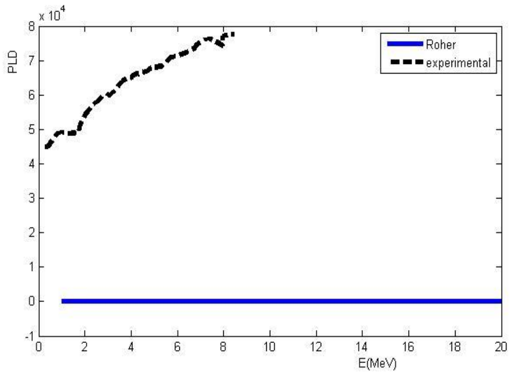

Figure 3. Comparison between theoretical curve with g from Roher and the experimental data

Fig 4. Shows a PLD results with g from Egidy eq 10, formula and experimental data. The PLD curve is less than the experimental curve. This can be interpreted that g from Egidy's formula consists A with power 0.9 this which decreases the value of g.

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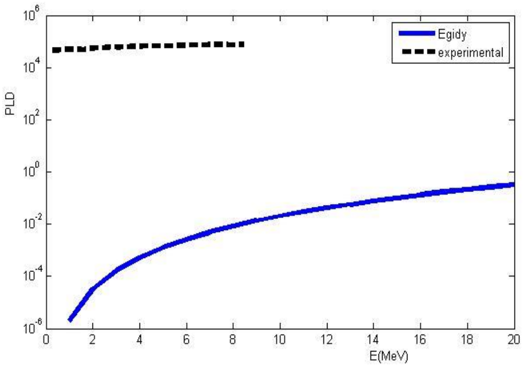

Figure 4. Comparison between theoretical curve with *g* from Egidy and the experimental data

Fig 5. Shows a comparison of PLD curve with g from the Thomas-Fermi formula eq 11 and experimental data. It is noticed the theoretical curve is less than the experimental curve up to 4 MeV and from 4 MeV to 5 MeV both curves become to agree and after 5 MeV the theoretical curve becomes higher than the experimental curve. In other words, the theoretical curve mediates the experimental curve. This behavior can be attributed to the g formula from Thomas-Fermi contains A with different power as it is shown in eq 11 and this makes the theoretical curve of PLD neither increases rapidly like the case of ESM nor decrease like Egidy or Roher but the increase is between two cases hence the curve mediates the experimental data.

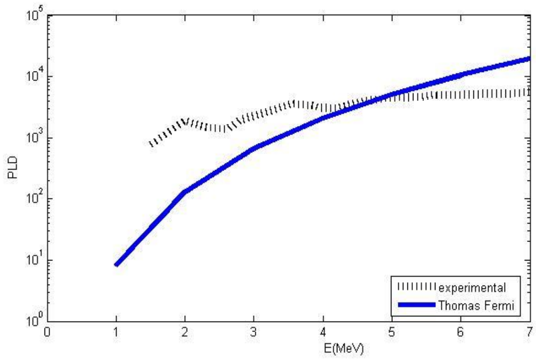

Figure 5. Comparison between theoretical curve with *g* from Thomas-Fermi and the experimental data

Fig 6. Shows a comparison between the PLD curve with g from Yukawa eq 12 and the experimental data. The theoretical curve with g from Yukawa is very higher than the experimental data. This is because g from Yukawa's formula has so big value.

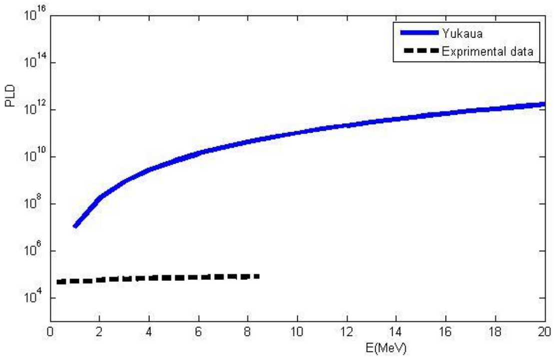

Figure 6. Comparison between theoretical curve with *g* from Yukawa and the experimental data

Conclusion

All theoretical curves increase with increasing the excitation energy. The theoretical curve that gives the best match with the experimental data depends on g from the Thomas-Fermi formula, and other theoretical curves are far from the experimental one. Therefore, we can say the results dependent on g from the Thomas-Fermi formula may be considered the most acceptable formula if it is compared with the results depending on the other formulae of g.

Authors' Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee in University of Baghdad.

Authors' Contribution Statement

Both authors proposed the topic of research and A. D. S. guidance and review and proofreading the research.

References

1. Pandit D, Bhattacharaya S, Mondal D, Roy P, Banerjee K, Mukhopadhyay S et al. Experimental signature of collective enhancement in nuclear level density. Phy Rev C. 2018; 97: 041301(R) DOI: https://doi.org/10.1103/PhysRevC.97.041301 .
2. Ingeberg V W, Jones P, Msebi L, Siem S, Wiedeking M, Avaa A A. Nuclear Level Density and γ-ray Strength Function of 63Ni. Phy Rev C. 2022: 054315. https://link.aps.org›pdf›PhysRevC.106.054315
3. .Okuducu S, Aktı N N, Akbas S, Kansu1 M O, Nuclear Level Density Parameters of 203−209Pb and 206−210Bi Deformed Target Isotopes Used on Accelerator-Driven Systems in Collective Excitation Modes. Sci Technol Nucl. 2012; 2012: 1-9. https://doi.org/10.1155/2012/915496 .
4. Zelevinsky V, Karampagia S. Nuclear level density and related physics. EPJ Web Conf. 2018; 194: 01001. https://doi.org/10.1051/epjconf/201819401001 .
5. Alwan T A, Hamed B S. Study the Nuclear Structure of Some Even-Even Ca Isotopes Using the Microscopic Theory. Baghdad Sci J. 2023; 20(1): 235-244. http://dx.doi.org/10.21123/bsj.2022.6924 .
6. Mamun Md A. Thermal Properties of Nuclei and Their Level Densities. M.Sc. Ohio. Ohio University. 2015.
7. Bucurescu D, Egidy T. Systematic of nuclear level density parameters. J Phys G: Nucl Part Phys. 2005; 31(10): S1675. https://doi.org/10.1088/0954-3899/31/10/052 .
8. Rahmatinejad A, Bezbakh A N, Shneidman T M, Adamian G, Antonenko N V, Jachimowicz P, et al. Level-density parameters in super heavy nuclei. Phys Rev C. 2021; 103(3): 1-10. https://doi.org/10.1103/PhysRevC.103.034309 .
9. Roy S, Mukhopadhyay S, Aggarwal M, Pandit D, Rana T K, Kundu S, et al. Excitation energy and angular momentum dependence of the nuclear level density parameter around A=110. Phys Rev C. 2021; 103: 024602. https://doi.org/10.1103/PhysRevC.103.024602
10. Salloum A D, Mohammed, J F Theeban A M. Effect of energy and mass number on energy transition rates in pre-equilibrium region. Dig J Nanomater Bios. 2018, 13(4): 1055-1061. https://doi.org/10.24996/ijs.2022.63.5.12
11. Abdulla A M, Salloum A D. A Comparison Between the Theoretical Cross Section Based on the Partial Level Density Formulae Calculated by the Exciton Model with the Experimental Data for $^{197}_{79}Au$ nucleus , Baghdad Sci J. 2020; 18(1): 2021. https://doi.org/10.21123/bsj.2021.18.1.0139 .
12. Karampagia S, Zelevinsky V. Nuclear shell model and level density. Int J Mod Phys. 29(6): 2030005 https://doi.org/10.1142/S0218301320300052
13. Jasim M H, Dakhil Z A, Ahmed R S. The nuclear level density parameter. Iraq J Phy. 2013. 11(20): 35-46. https://doi.org/10.30723/ijp.v11i20.380 .
14. Magner A G, Sanzhur A I, Fedotkin S N, Levon A I, Shlomo S. Level density within a micro-macroscopic approach. Nucl Phys A. 2022; 1021: 122423: https://doi.org/10.1016/j.nuclphysa.2022.122423 .
15. Shil R, Banerjee K, Roy P, Sadhukhan J, Rana T K, Mukherjee G et al. Isospin dependence of nuclear level density at A≈ 120 mass region. Phys Lett B. 2022; 831: 1-9. https://doi.org/10.1016/j.physletb.2022.137145 .

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P-ISSN: 2078-8665 - E-ISSN: 2411-7986

Baghdad Science Journal

مقارنة بين نتائج صيغة اريكسون والقيم العملية باستخدام صيغ جديدة لكثافة المستويات لجسيم منفرد

هدير كاظم محمد ، علي داود سلوم

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الخلاصة

تم دراسة كثافة الحالات الجزئية PLD الموصوفة بصيغة اركسون لتفاعلات قبل التوازن باستخدام صيغ مختلفة لكثافة الحالات لجسيم منفرد g. تم استخدام المتغير g من نموذج المسافات المتساوية و نموذج المسافات غير المتساوية و الصيغ الأخرى للمتغير g التي تم اشتقاقها باستخدام العلاقة بين g ومعلم كثافة الحالات a. صيغ المتغير a المستخدمة في اشتقاق g هم كل من صيغة روهر و صيغة اكدي وصيغة يوكاوا و صيغة ثوماس-فيرمي. وقد تبين ان نتائج كثافة الحالات الجزئية التي تعتمد على كثافة الحالات لجسيم منفرد من صيغة ثوماس-فيرمي لها اتفاق جيد مع القيم العملية.

الكلمات المفتاحية: الاكسايتون موديل، التفاعل النووي المحتث، كثافة الحالات, النواة قبل المركبة، تفاعلات قبل التوازن.