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طريقة لحل المعادلات التفاضلية اللاخطية

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المستخلص:

في هذا البحث اعطينا تطبيق لطريقة كالركن باستخدام دوال والش لحل المسائل الابتدائية المرافقة للمعادلات التفاضلية الاعتيادية اللاخطية. هذه الطريقة وضحت مع بعض الامثلة العددية.

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$$\left[\left(\frac{1}{2}c_o+\frac{1}{4}c_1\right)c_o\right]-\frac{1}{4}c_oc_1+c_o+2\left[\left(\frac{1}{2}c_o+\frac{1}{4}c_1\right)^2+2\left(\frac{1}{2}c_o+\frac{1}{4}c_1\right)+\left(\frac{-1}{4}c_o\right)^2+1\right]+$$
$$\frac{3}{16}c_o+\frac{1}{2}+\frac{1}{8}c_1=2+\frac{1}{2}$$

and by multiplying eq.(4) by $\phi_1(x)$ and then integrating from 0 to 1 to give

$$\left[\left(\frac{1}{2}c_o+\frac{1}{4}c_1\right)c_1\right]-\frac{1}{4}(c_o)^2+c_1+2\left[2\left(\frac{-1}{4}c_o\right)\left(\frac{1}{2}c_o+\frac{1}{4}c_1\right)-\frac{2}{4}c_o\right]+\frac{-1}{8}c_o+\frac{-1}{16}c_1-\frac{1}{4}=-\frac{1}{4}$$

Solving the above equations to get $c_0=-7.437\times10^{-8}, c_1=3.862\times10^{-7}$, then the solution y(x)=x is the approximate solution of the initial value problem (3) which is agree with the exact solution $y^*(x)$=x.

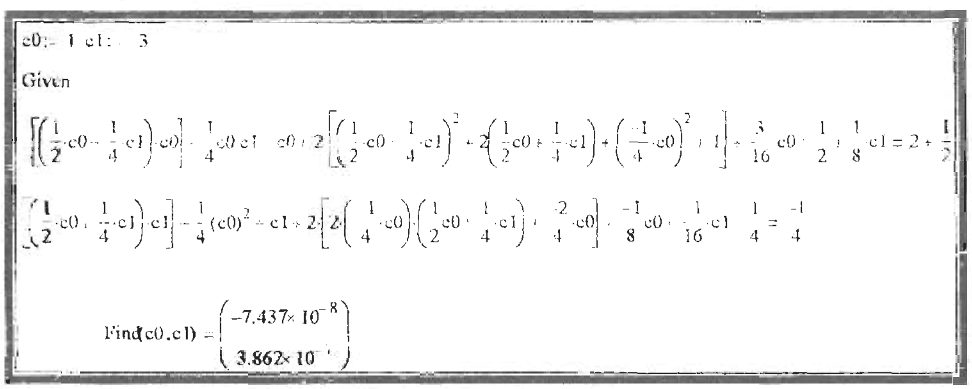

Fig.(1.2) represents the non-linear system of equations associated *with example*(2) *written in* Mathcad software with its solution for initial values of c_0=1 and c_1=-3.

Now, the solution of the above initial value problem is given by integrating the function $y'(x)$ from 0 to x. That is,

$$y(x)=\left(\frac{1}{2}c_o+\frac{1}{4}c_1\right)\int_0^x\phi_o(x)dx+\left(-\frac{1}{4}c_0\right)\int_0^x\phi_1(x)dx+\int_0^x 1\,dx+y(0)$$

$$=\left(\frac{1}{2}c_o+\frac{1}{4}c_1\right)\left(\frac{1}{2}\phi_o(x)-\frac{1}{4}\phi_1(x)\right)+\left(-\frac{1}{4}c_0\right)\left(\frac{1}{4}\phi_o\right)+x$$

$$=\left(\frac{1}{2}c_o-\frac{1}{16}c_0+\frac{1}{8}c_1\right)\phi_o(x)+\left(-\frac{1}{8}c_0-\frac{1}{16}c_1\right)\phi_1(x)+\frac{1}{2}\phi_o(x)-\frac{1}{4}\phi_1(x)$$

Hence $y(x)=\left(\frac{3}{16}c_o+\frac{1}{2}+\frac{1}{8}c_1\right)\phi_o(x)+\left(-\frac{1}{8}c_0-\frac{1}{16}c_1-\frac{1}{4}\right)\phi_1(x)$.

Also, write the variable x as $x=\frac{1}{2}\phi_o(x)-\frac{1}{4}\phi_1(x)$

Next, substitute the function y'', y', y & x into eq.(3) to obtain:

$$\left(\left(\frac{1}{2}c_o+\frac{1}{4}c_1\right)\phi_o+\left(-\frac{1}{4}c_0\right)\phi_1+1\right)+\left(c_o\phi_o(x)+c_1\phi_1(x)\right)$$

$$\left(\frac{3}{16}c_o+\frac{1}{2}+\frac{1}{8}c_1\right)\phi_o(x)+2\left(\left(\frac{1}{2}c_o+\frac{1}{4}c_1\right)\phi_o+\left(-\frac{1}{4}c_0\right)\phi_1+1\right)^2+$$

$$\left(-\frac{1}{8}c_0-\frac{1}{16}c_1-\frac{1}{4}\right)\phi_1(x)$$

$$=2+\frac{1}{2}\phi_o(x)-\frac{1}{4}\phi_1(x) \qquad (4)$$

and as before, by integrating both sides of eq.(4) from 0 to 1 one can get

$c0 := 1 \qquad c1 := 1$

Given

$$(c0)^2 + (c1)^2 + \frac{1}{2}c0 + \frac{1}{4}c1 - \frac{3}{2} =$$

$$2\cdot c0\cdot c1 + \frac{-1}{4}c0 + \frac{1}{4} = 0$$

$$\text{Find}(c0, c1) = \begin{pmatrix} 1 \\ -1.62\times 10^{-8} \end{pmatrix}$$

Fig.(1.1) represents the non-linear system of equations associated with example(1) written in Mathcad software with its solution for initial values of c_0=1 and c_1=1.

Example (2):

Consider the initial value problem

$$y'y'' + 2y'^2 + y = 2 + x \qquad \textbf{(3)}$$

with y(0)=0 and $y'(0) = 1$.

Approximate $y''(x)$ **as:** $y''(x) = c_0\phi_0(x) + c_1\phi_1(x)$

Therefore, $y'(x) = \int_0^1 y''(x) + y'(0)$

$$\text{Thus} \quad y'(x) = \int_0^x (c_o\phi_o(x) + c_1\phi_1(x)) + 1 = c_o\int_0^x \phi_o(x) + c_1\int_0^x \phi_1(x) + 1$$

$$= c_0\left(\frac{1}{2}\phi_o(x) - \frac{1}{4}\phi_1(x)\right) + c_1\left(\frac{1}{4}\phi_0(x)\right) + 1$$

$$= \left(\frac{1}{2}c_o + \frac{1}{4}c_1\right)\phi_o + \left(-\frac{1}{4}c_0\right)\phi_1 + 1$$

$$(c_o\phi_o(x)+c_1\phi_1(x))^2+\left(\frac{1}{2}c_o+\frac{1}{4}c_1\right)\phi_o(x)+\left(\frac{-1}{4}c_o\right)\phi_1(x)=1+\frac{1}{2}\phi_o(x)-\frac{1}{4}\phi_1(x)\ldots(3)$$

and by integrating both sides of eq.(3) from 0 to 1 and use the following facts:-

$$\int_0^1\phi_o(x)dx=1,\quad \int_0^1\phi_1(x)dx=0,\quad \int_0^1\phi_1(x)\phi_o(x)dx=0 \text{ and } \int_0^1(\phi_1(x))^2dx=1$$

one can obtain:

$$(c_o)^2+(c_1)^2+\frac{1}{2}c_o+\frac{1}{4}c_1=\frac{3}{2}$$

and by multiplying eq.(2) by $\phi_1(x)$ and then integrating from 0 to 1 to give

$$2c_oc_1-\frac{1}{4}c_o=-\frac{1}{4}$$

and by solving the above equations via the Mathcad professional software to give $c_o=1, c_1=7.139\times10^{-8}$, then the solution $y(x)=\frac{1}{2}\phi_o-\frac{1}{4}\phi_1=x$ is the approximate solution of the initial value problem (2) which is agree with the exact solution $y^*(x)=x$.

by any suitable method, the values of $\vec{c} = (c_1, c_2, \ldots, c_n)$ are computed.

To illustrate this approach, we give the following example:-

Example(1):

Consider the first order non-linear ordinary differential equation:

$$(y')^2 + y = 1 + x \qquad (2)$$

with y(0)=0.

Approximate $y'(x)$ as: $y'(x) = c_0\phi_0(x) + c_1\phi_1(x)$

Therefore, the solution of the above initial value problem is given by integrating the function $y'(x)$ from 0 to **x**. That is,

$$y(x) = \int_0^1 y'(x) + y(0) =$$

$$\int_0^x \left(c_0\phi_0(x) + c_1\phi_1(x)\right) = c_0\int_0^x \phi_0(x)dx + c_1\int_0^x \phi_1(x)dx$$

$$= c_0\left(\frac{1}{2}\phi_0(x) - \frac{1}{4}\phi_1(x)\right) + c_1\left(\frac{1}{4}\phi_0(x)\right)$$

$$= \left(\frac{1}{2}c_0 + \frac{1}{4}c_1\right)\phi_0 + \left(-\frac{1}{4}c_0\right)\phi_1$$

Also, write the variable x as $x = \frac{1}{2}\phi_0(x) - \frac{1}{4}\phi_1(x)$

Next, substitute the functions $y', y \ \&\ x$ into eq.(2) **to** obtain:

$$\int_{o}^{x}\phi_o(x)dx = x = b_o\phi_o(x) + b_1\phi_1(x) + \ldots + b_n\phi_n(x)$$

where $b_o, b_1, \ldots, b_n$ are known parameters that can be easily found from:

$$b_n = \int_{0}^{1}\phi_n(x)xdx$$

Continue in this manner to find the integrals of the Walsh functions $\int_0^x\phi_1(x)dx, \int_0^x\phi_2(x)dx, \ldots, \int_0^x\phi_n(x)dx$ in terms of the linear combinations of the Walsh functions themselves. Thus

$$y^{(n-1)}(x) = c_0[b_{oo}\phi_o + b_{o1}\phi_1 + \cdots + b_{on}\phi_n] + c_1[b_{1o}\phi_0 + b_{11}\phi_1 + \cdots + b_{1n}\phi_n] + c_n[b_{no}\phi_o + b_{n1}\phi_1 + \cdots + b_{nn}\phi_n]$$

$$= \sum_{i=0}^{n} c_{(i+1)}\left(\sum_{j=0}^{n} b_{ji}\right)\phi_i(x)$$

where $\int_0^x\phi_i(x)dx = \sum_{j=0}^{n} b_{ij}\phi_i(x)$ and $\{b_{ij}\}_{i=o,\ j=0}^{n}$ are known parameters that can be found similar to the above .

Repeat the above procedure to find $y^{(n-2)}(x), \ldots, y(x)$ in terms of the Walsh functions. Then by substituting the above functions $y^{(n)}(x)$, $y^{(n-1)}(x)$,...y(x) and **x** into eq.(1) and taking the scalar product with $\phi_i(x)$, for all i=0,1,...,n, yields a non-linear system of n equations with **n** variables $\vec{c} = (c_1, c_2, \ldots, c_n)$. Hence by solving this system where $q = [\log_2 n] + 1$, in which [.] means taking the greatest integer of ".". Therefore

$$n = b_q . 2^{q-1} + b_{q-1} . 2^{q-2} + \ldots + b_1 . 2^0$$

where $b_q b_{q-1} \ldots b_1$ is the binary expression of n.

3- The Approach

Consider the general the initial value non-linear ordinary differential equation:

$$F(x, y(x), y'(x), y''(x), \ldots, y^{(n)}(x)) = 0, \qquad 0 \le x \le 1$$

(1)

associated with *nth* initial conditions. As in [], the approach is based on approximating the highest order derivative of the unknown solution $y^{(n)}(x)$ into a Walsh series:

$$y^{(n)}(x) = c_o \phi_o(x) + c_1 \phi_1(x) + \cdots + c_n \phi_n(x)$$

where c_n are the unknown coefficients of the Walsh series of $y^{(n)}(x)$ that must be determined. The th (n-1)th derivative of y(x0 is obtained by:

$$y^{(n-1)}(x) = \int_0^x y^{(n)}(x) + y^{(n-1)}(0)$$

$$\Rightarrow y^{(n-1)}(x) = c_0 \int_0^x \phi_o(x) dx + c_1 \int_0^x \phi_1(x) dx + \cdots + c_n \int_0^x \phi_n(x) dx + y^{(n-1)}(0)$$ But

$\int_0^x \phi_o(x) dx$ can be also approximate by a Walsh series:

On the other hand, [1] devoted this method for solving the linear differential equations which depends on expanding the solution of them as a Walsh series.

2. Rademacher and Walsh Functions, [1]:

Rademacher's function $r_k(t)$ is a set of square waves of unit height with periods equal to 1, $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots, 2^{(1-k)}$, respectively. Alternatively, we state that the number of cycles of the square waves of $r_k(t)$ is 2^{k-1}. It is noted that the set is not complete since, except for $r_o(t)$, the set involves only functions which are odd about $t=\frac{1}{2}$.

In 1923, Walsh independently developed a complete set known as Walsh functions. The set of Walsh functions $\phi_n(t)$ and the set of Radamacher functions have the following relation:

$$\phi_o(t) = r_o(t),$$

$$\phi_1(t) = r_1(t),$$

$$\phi_2(t) = [r_2(t)]^1 [r_1(t)]^0,$$

$$\phi_3(t) = [r_2(t)]^1 [r_1(t)]^1,$$

$$\phi_4(t) = [r_3(t)]^1 [r_2(t)]^0 [r_2(t)]^0,$$

.

.

$$\phi_n(t) = [r_q(t)]^{b_q} [r_{q-1}(t)]^{b_{q-1}} [r_{q-2}(t)]^{b_{q-2}}$$

An Approach For Solving Nonlinear Differential Equations

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Abstract:

In this paper, an implementation of the Galerkin's method via the Walsh functions is presented to solve the initial value problem associated with the non-linear ordinary differential equations. This method is illustrated with some examples.

1. Introduction:

The Galerkin's method is an approach used to solve the linear integral equations, [2] as well as the linear differential equations, [5] and the linear eigenvalue problems, [3].

In [4], this method is described to solve the non-linear eigenvalue problems associated with the non-linear integral equations which is based on the polynomial basis.

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