

Low Order Harmonics Elimination in Multilevel Inverters Using Gradient Descent Algorithm

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Abstract: *In recent years, the topic of harmonics in general and the elimination of low order harmonics in power system in particular attracts the attention of many researchers in the field of power electronics. Estimation of the values of switching angles in multilevel inverters has been of interest in reduction or elimination of low order harmonics. In this paper, the technique that is based on the gradient or steepest descent algorithm is described. This algorithm searches for the minimum value of the power harmonic distortion of the low order harmonics in the output waveform, according to the value of the modulation index set at the beginning of the optimization process. The power equations of harmonics are used for this purpose and therefore a high convergence rate is obtained with this method. The results have been verified by MATLAB software.*

Keywords: *3-phase PWM, low-order harmonics, multi-level inverters, power harmonic distortion, steepest descent algorithm.*

I. Introduction

Multilevel inverter in general has the staircase output waveform. Fig.1 shows a typical output voltage waveform of a seven- level inverter and this specific type of output waveform is generated by adding successfully the output of three voltage sources using a proper switching angle in the system. This stepped output waveform can be approximated to get a continuous sinusoidal waveform and off course more close to ideal sine wave can be obtained if more levels in the output waveform are employed and consequently a lower total harmonic distortion (THD) in the system is achieved. One of the well-known topologies used in design of multi-level inverters is called cascaded H-bridge as shown in Fig.2. This multilevel inverter can generate an output waveform with $2n+1$ level where n is the number of voltage sources in the bridges. The output voltage V_t of the inverter in Fig.2 can be expressed as in equation (1) below:

$$V_t = V_{a1} + V_{a2} + V_{a3} + V_{a4} \quad (1)$$

The output voltage of the i th H-bridge(i is any positive integer number); V_{ai} , can have any of the three values V_i , $-V_i$ or 0 Volt, depending on the states of the four semi-conductor device switches of the H-bridge. Therefore the number of H-bridges required to generate the voltage waveform in Fig. 1 are only 3. The voltage of this waveform is toggling from 0 Volt to V_1 at angle θ_1 and to $V_1 + V_2$ at angle θ_2 and finally to $V_1 + V_2 + V_3$ at angle θ_3 . If we consider $V_1 = V_2 = V_3 = V_{dc}$ then the peak voltage which is $V_1 + V_2 + V_3$ will be $3V_{dc}$. Selecting the proper values of switching angles using Fourier series, a designated unwanted low-order harmonics amplitudes of the inverter output waveform including the fundamental frequency component can be controlled.

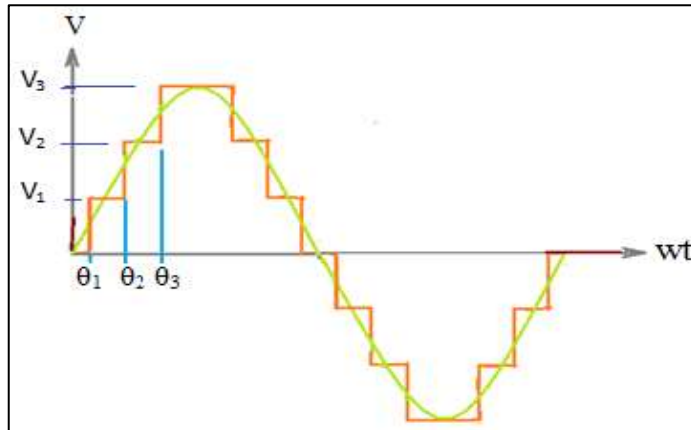


Figure 1 Typical seven-level voltage output waveform of a multilevel inverter with three switching angles

II. Low-Order Harmonics Elimination

Because of adverse impacts of presence of low-order harmonics in the electrical power systems, It is highly recommended to eliminate the low-order harmonics that are near the fundamental frequency rather than minimizing THD[1]. The waveform depicted in Fig.1 can be analyzed by the Fourier series expansion as follows:

$$f(\theta) = \sum_{n=1}^{\infty} [a_n \sin n\theta + b_n \cos n\theta] \quad (2)$$

Due to quarter wave symmetry of the waveform, the Fourier series coefficients are as follows:

$$a_n = \begin{cases} \frac{4V_{dc}}{n\pi} \sum_{m=1}^N (-1)^{m+1} \cos(n\theta_m) & n \text{ is odd} \\ 0 & n \text{ is even} \end{cases}$$

$$b_n = 0;$$

where N is the number of the switching angles to be computed. Note that all even harmonics will disappear in this voltage waveform. From the above formula, we can derive the equations for the 1st, 5th, and 7th harmonics as:

$$A(1)=[\cos(\theta_1)-\cos(\theta_2)+\cos(\theta_3)]=3MI \quad (3)$$

$$A(5)= (1/5) [\cos (5\theta_1)..-\cos (5\theta_2)+ \cos(5\theta_3)] \quad (4)$$

$$A(7)= (1/7) [\cos(7\theta_1) - \cos(7\theta_2)+\dots \cos(7\theta_3)] \quad (5)$$

Where A(1) and A(5, and A(7)) are the amplitudes of the 1st, 5th, and 7th harmonics normalized to $(4/\pi)V_{dc}$, MI is the modulation index and V_{dc} is the voltage of the dc sources used, which are assumed to be equal. Set the value of equation (4) and (5) to zero and solving the three equations, the value of switching angles at which the fifth and seventh harmonics are eliminated can be defined. The reasons of selecting 5th and 7th harmonics in equations 4 and 5 respectively are:

- The triplen (multiple of three) harmonics are not present in balanced three-phase systems.
- Because of quarter wave symmetry of the waveform in Fig.1, so all even harmonics have been eliminated completely.

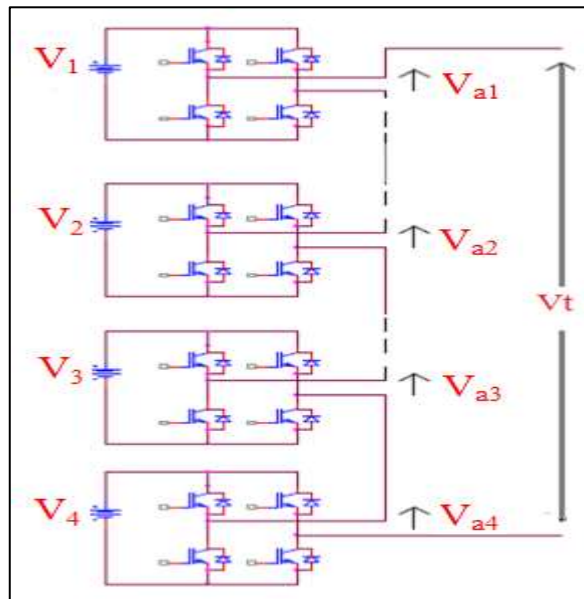


Figure 2 The power circuit-single phase- of nine- level cascaded H-bridges

Please, it is worthy to know that Fig.2 is showing only one phase of three-phase system that has been used for our study. Generally, if there are many voltage sources and m th harmonic, where m is higher than seven, the equations of the fundamental up to the m th harmonic of the multilevel inverter output with (n) switching angles $\theta_1, \theta_2 \dots \theta_n$, according to Fourier series analysis, can be given as follows :

$$A(1)=[\cos(\theta_1)- \cos(\theta_2)+\dots\cos(\theta_n)]= n MI \tag{6}$$

$$A(5)=(1/5)[\cos(5\theta_1)-\cos(5\theta_2)+\dots\cos(5\theta_n)] \tag{7}$$

$$A(7)=(1/7)[\cos(7\theta_1)- \cos(7\theta_2)+\dots \cos(7\theta_n)] \tag{8}$$

$$A(m)=(1/m)[\cos (m\theta_1)+\cos(m\theta_2)+\dots \cos(m\theta_n)] \tag{9}$$

For any specific value of MI, the number of equations that can be solved is (n) so $(n-1)$ harmonics can be eliminated by equating the

peak values of these harmonics in the above equations to zero and solving these equations. For example, with $n=4$ (four switching angles), only four equations are required for this purpose and three harmonics can be eliminated. There are many methods available to solve these transcendental non-linear trigonometric equations including iterative method, such as Newton-Raphson method [2-4] and Genetic algorithms [5-6]. Alternatively, such transcendental equations can be converted into polynomial equations; and the solutions can be obtained [7-8], or can be solved using power sums [9]. In this paper, the solution to the harmonic elimination is provided with high accuracy, high convergence rate, and with high technical feasibility in the field of multi-level power inverter systems. This can be performed by applying a search algorithm to the power equations instead of the voltage equations of the harmonics required to be eliminated and this can be done by modifying voltage equations to produce the power of the harmonics; It is obvious that the total power function that has to be minimized is equal to the sum of the power of any number of harmonics that is under investigation and has also to be zero at the point of solution, and the minimum point of the power function that is equal to zero will be considered as initial point of solution to the voltage equations. The search technique that will be used is the steepest descent technique. It will be applied to the total power function of the harmonics to be eliminated. The constraint imposed on the optimization by the value of MI gives one degree of freedom by reducing the number of angles involved in the iteration process by one.

III. The Searching Process

In applying the gradient or descent algorithm to eliminate the low order harmonics, it is required to have a performance surface of a parabolic shape (concaved up) to reach its bottom using this technique. The m th power harmonic distortion PHD(m) is defined as the following:

$$\text{PHD (m)} = [\text{A (m)}]^2 / [\text{A (1)}]^2 \quad (10)$$

At instance, we can search for the minimum of the summation of PHD(5), PHD(7), PHD(11), PHD(13)..etc., to find the minimum power distortion of these harmonics. If all of these harmonics have zero voltages at specific switching angle values, then the summation of their powers should be at a minimum with zero value at the solution point. Thus, we can use the steepest descent search technique to find this minimum power at a point located at the bottom of the parabola.

IV. The Proposed Strategy

In order to get started with this algorithm, first of all, it is necessary to consider the case of unconstrained optimization of PHD (m). In the other meanings, minimizing of PHD(m) regardless of the fundamental frequency peak value. For single variable of θ , the steepest descent technique is defined as follows[10].

$$\theta(p+1) = \theta(p) + \alpha(-\nabla(p)) \quad (11)$$

Where $\nabla(p)$ refers to the value of the gradient of the function to be minimize, i.e. PHD (m) in this case, $\theta(p)$ is the value of θ at the pth iteration, $\theta(p+1)$ is the new value of θ at the (p+1)th iteration, and α is a fixed step size (convergence) parameter that controls the rate of convergence. The negative sign in equation (11) indicates that the search is moving in the direction of the lower or minimum point of the surface of the parabola. The gradient in (11) is expressed (in the case of a single variable (θ)) as follows:

$$\nabla (p) = d[\text{PHD(m)}]/d\theta \quad | \quad \theta = \theta(p) \quad (12)$$

In the case of considering more than one angle (multidimensional), the gradient search method is expressed in a vector form as follows:

$$\theta (p+1) = \theta(p) + \alpha(-\nabla(p)) \quad (13)$$

It worthy to know that the functions $\theta(p)$, $\theta(p+1)$ and $\nabla(p)$ are vectors. This equation gives the value of the set of angles θ at the $(p+1)$ th iteration using the previous iteration values of angles and gradient. The value of the gradient in this case will be the partial derivatives of PHD(m) with respect to each angle. It is very important to consider the sign of the gradient in (13) rather than estimating its exact value for simplifying the evaluation process, and ensuring every time to follow the correct direction of movement by comparing the new value of PHD (m) with the old value. The rate of convergence in this case depends totally on the step size parameter α . Therefore, the multidimensional form of (13) can be simplified as follows:

$$\theta (p+1) = \theta(p) + \alpha (-\text{sgn}(\nabla(p))) \quad (14)$$

In the above equation, the value of α is constant for a specified whole iteration process and for every angle, but in this paper, the value of α shall be adjustable to avoid the swinging between the same two points on top of minimum point of the surface and by this way we can reach the exact minimum point of the parabola. Equation (14) can be set in the following formula:

$$\theta (p+1) = \theta(p) + \alpha(p)(-\text{sgn}(\nabla(p))) \quad (15)$$

It is obvious that the previous scalar α has been replaced by the vector quantity $\alpha(p)$ with the time index (p) and therefore has distinct values for each angle and each iteration step. This technique is applicable to minimizing single PHD (m) and a summation of the power harmonic distortions of more than harmonic.

V. Convergence Process Simulation Results

The initial value of both multidimensional angles and initial value of α have to selected for starting the search process and, as previously stated elsewhere, if any change in sign of the gradient occurs regarding an angle throughout the search path, this will

represent a critical point and the value of associated α decreases slightly but the search process continues in the other directions toward the destination point until an acceptable error margin will reach, leaving the value of α of remaining angles involved in the iteration unchanged.

As previously mentioned that the algorithm is applied with constraint of MI, or having a specific value for the amplitude of the fundamental frequency, that means the iteration process is applied to (n-1) angles, and the other angle which is the constraint of the searching process can be calculated using equation (6); The power function is then evaluated using the new values of all the angles. For instance, in case of n=3, then the iteration is applied only to (n-1) angles, and the third angle is calculated using equation (6) above, this means that only two angles will be involved in the search process leaving the constraint provides one degree of freedom. Fig.3 demonstrates the convergence process throughout using three angles. In this figure, the values of total power of 5th, 7th, and 5th+7th harmonics with MI = 0.6 have been depicted and the final value of the power ended up with zero value at the end of the search process.

Fig. 4 shows the convergence process of the three angles and the number of iterations required to reach their final values. It is clear from the figure that the initial values for the two angles θ_1 , and θ_2 , are equal to 17° , and 48° respectively and the value of θ_3 is calculated from equation (6), as mentioned earlier in this section. The final search has ended up with optimum values of 22.10° , 45.11° and 63.78° . It clear from Fig. 5 that the three angles have reached the same final values as in previous figure even though their initial values were selected randomly with each angle has three initial values rather than single initial value in the previous figure. From this, one can conclude that the searching process in this algorithm is not too sensitive to the selection of angle values starting point.

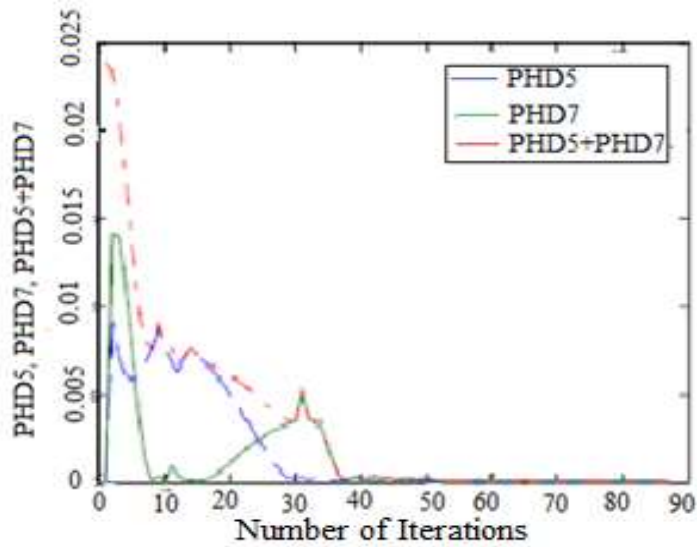


Figure 3 The normalized values of PHD (5), and PHD (5) +PHD (7), for seven level waveform, with MI=0.6

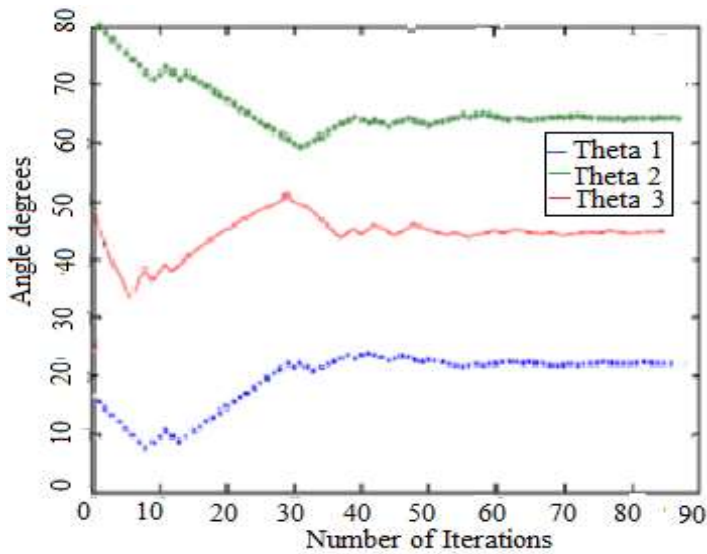


Figure 4 The three angles convergence process with one starting point for seven level voltage waveform with MI=0.6

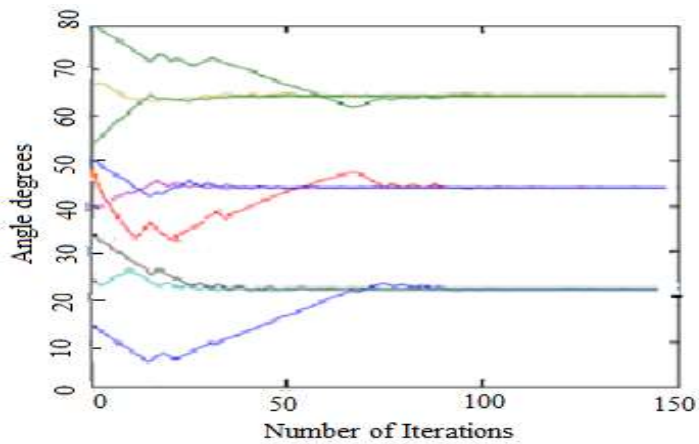


Figure 5 The three angles convergence process with three different starting points for seven level voltage waveform with $MI=0.6$

VI. Matlab Simulation Results

The parameters used in this simulation work are:

- Four batteries of 100 Volt DC have been employed as power supply for each phase of the three phase system of the nine level inverter shown in Fig. 2. The reason of selecting nine-level rather than seven level is simply to demonstrate that this methodology is applicable to all multi-level inverter - The inductive load employed for this test is 50Ω resistor with 1mH inductor.
- MATLAB software is used for this simulation work.
- The value of MI is 0.6.

Fig.6 shows the output waveform of the nine-level inverter and the angles 22.10° , 45.11° and 63.78° that found by suggested algorithm have been used to eliminate 5th and 7th harmonics and this verifies the suggested methodology employed to eliminate these low order harmonics. In plotting the harmonics spectrum of the output waveform of the inverter both 5th, 7th harmonics have been totally eliminated in addition to eliminate all triplen (multiple of three) harmonics such as 3rd, 9th..etc .as depicted in Fig.7. Comparing the results obtained from proposed gradient descent

algorithm and MATLAB simulation result, both 5th and 7th harmonics have been reduced to zero and consequently the THD of the system has been reduced significantly.

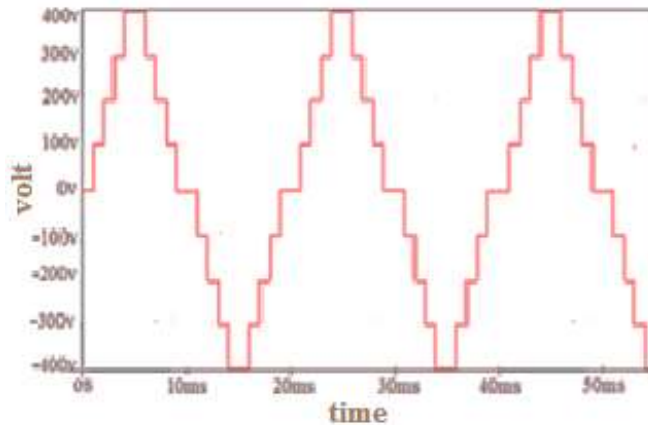


Figure 6 *Three-phase nine level output voltage waveform simulation result 50Hz, with MI=0.6*

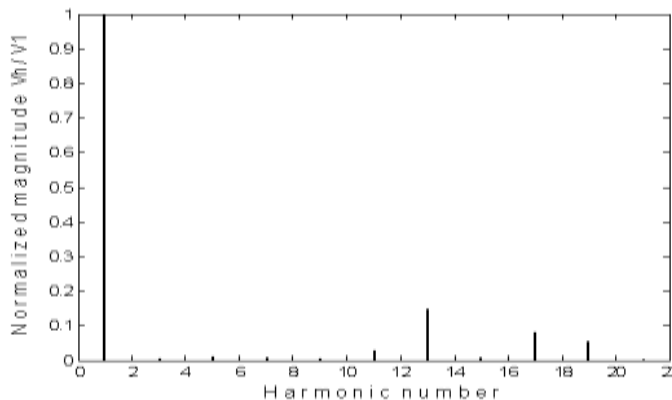


Figure 7 *MATLAB simulation result of output spectrum of 3-phase nine level voltage waveform with MI =0.6*

VII. Conclusion

The technique of employing the gradient descent algorithm to search for the minimum point of the power harmonic distortion to eliminate of low order harmonics has proved to be very accurate with high rate of convergence and high technical feasibility. This

approach calculates iteratively the switching angles required to eliminate a number of low order harmonics in a multilevel inverter output for a certain value of the modulation index. In this technique the elimination of low order harmonics and consequently reduction of THD contributes to obtain better quality of power. Finally, this methodology is not highly sensitive to the selection of initial values of the switching angles.

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ازالة التوافقيات ذات الترتيب العددي المنخفض في محولات متعددة المستويات باستعمال خوارزمية النزول التدريجي

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المستخلص:

في السنوات الاخيرة، اصبح موضوع التوافقيات بصورة عامة والحد من التوافقيات ذات الترتيب العددي المنخفض يجذب انتباه العديد من الباحثين في مجال الكترونييات القدرة. حساب قيم زوايا التحويل في محولات متعددة المستويات له فائدة كبيرة في خفض او ازالة التوافقيات ذات الاعداد المنخفضة. ففي هذا البحث تم وصف تقنية النزول التدريجي . هذه الخوارزمية تبحث عن ادنى نقطة او قيمة لطاقة توافقيات المسببة للتشويه الواطئة عدديا في الموجة الخارجة طبقا لقيمة مؤشر التعديل الموضوعة في بداية عملية تحسين الاداء. يستخدم معادلات قوة او طاقة التوافقيات لهذا الغرض وبذلك يتم الحصول على نسبة التقارب عالية بهذه الطريقة. وتم التحقق من صحة النتائج بواسطة برنامج MATLAB.

الكلمات الرئيسية: التعديل العرضي للنبرض ذو 3-اطوار، التوافقيات ذات الترتيب العددي المنخفض، محولات متعددة المستويات، طاقة توافقيات المسببة للتشويه، خوارزمية النزول التدريجي