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A Study of Some Nuclear Properties of a Nuclei with Mass Numbers (A=100,102) using Interacting Boson Model

[*]A.R.H Subber [**]Mohammed A. Al Sharefi [***]Ahmed M. Shuaib
[*]Department of Physics – College of Education – Basrah university
[**],[***]Department of Physics – College of Science – Babylon university

Abstract :

In this study, we have employed the Interacting Boson Model (IBM-1 and IBM-2) to determine the most appropriate Hamiltonian for the study of a nuclei in the region (A≈100). Using the best fit values of parameters to construct the Hamiltonian, we have estimated energy levels and multipole mixing ratios (δ(E2/M1)) for some doubly even (A=100,102) nuclei. The results are compared with experimental data and it is observed that they are in good agreement.

الخلاصة :

تم في هذا البحث استعمال انموذجي البوزونات المتفاعلة (الاول و الثاني) لتحديد المؤثر الهاملتوني الأكثر ملائمة لدراسة النوى في المنطقة (A≈100) باستعمال افضل تطابق للبارامترات في المؤثر الهاملتوني, وقد تم حساب مستويات الطاقة ونسب الخلط لبعض النوى ذات العدد الكتلي (A=100,102) , وتمت مقارنة النتائج مع القيم العملية وكانت النتائج في هذا البحث ذات توافق جيد .

1-Introduction :

The study of nuclear structure concerns itself with the motions of nucleons within the nucleus. In principle what is needed is the total wavefunction of the nucleus. However, for all but the lightest nuclei this becomes extremely complicated and of little practical use. As the problem becomes intractable with increasing mass, one has to resort to nuclear models which make use of simplifications and approximations [1]. These models must be simple enough to understand and calculate, but still powerful enough to predict observed phenomena. Some of the most successful nuclear models are the Collective Model of Bohr and Mottlesohn , and its group theory counterpart, the Interacting Boson Model (IBM) of Arima and Iachello . Both models have special limiting cases that form structural paradigms, and which are empirically observed in select nuclei. It is therefore important to subject these limiting cases to extreme scrutiny, as they act as benchmarks of nuclear structure.[2]

In previous years, some nuclei are calculated by (IBM) such as : Yazar et al. [3] explore the energy levels and the electric quadrupole transition probabilities B(E2;$I_i \rightarrow I_f$) and gamma-ray E2/M1 mixing ratios for selected transitions of some even-even erbium isotopes, Zamfir et al. [4] stated that Xe isotopes of the mass region of A ~ 130 appear to evolve from U(5) to O(6)-like structure in the IBM.

2- Theoretical Basis :

2-1 (IBM-1) :

The IBM-1 Hamiltonian can be written as : [5]

$$H = \epsilon'' \hat{n}_d + a_0 P^+ P + a_1 \hat{L}^2 + a_2 Q^2 + a_3 T_3^2 + a_4 T_4^2, \qquad (1)$$

Where :

$P = 1/2\ (d^2 - s^2)$

$Q = (s^+ . d + d^+ . s) + \chi\ (d^+ . d)$

$n_d = \sqrt{5}\ T_o \qquad L = \sqrt{10}\ T$

The program code PHINT written by Scholten is used within the option of specifying the parameters in the neutron proton formalism where it take care of projecting maximum symmetry states. We have used EPS, PAIR, ELL, QQ parameters and kept OCT, HEX as zero since there contribution is very small. For these calculation the experimental data are taken from .

2-2 (IBM-2) :

The microscopic picture of the IBM is very complicated. A commonly used microscopic picture is given in terms of collective pairs of nucleons. The S and D pairs of valence nucleons have angular momenta J = 0 and J = 2, respectively. These pairs correspond intuitively to the s and d bosons, respectively. The building blocks of the IBM-2 are the proton bosons s_π, d_π and the neutron bosons s_ν, d_ν. For the analysis of excitation energies in some (A=100,102) nuclei we tried to keep to a minimum number of free parameters in the Hamiltonian [6]. We thus considered equal values for the neutron and proton d-boson excitation energy, in addition to the standard quadrupole interaction and Majorana term. We only considered the dipole neutron-proton boson interaction whose strength is characterized by a single parameter $W_{\pi\nu}$. The explicit expression of the Hamiltonian adopted in the calculations is : [5,7]

$$\hat{H} = \varepsilon(\hat{n}_{d_\pi} + \hat{n}_{d_\nu}) + K_{\pi\nu}\hat{Q}_\pi \cdot \hat{Q}_\nu + K_{\pi\pi}\hat{Q}_\pi \cdot \hat{Q}_\pi + w_{\pi\nu}\hat{L}_\pi \cdot \hat{L}_\nu + \hat{M}_{\pi\nu}\,, \qquad (2)$$

where the indexes $n_{d\pi}$ and $n_{d\nu}$ refer to neutron and proton bosons respectively, and $M_{\pi\nu}$ is the Majorana term. Moreover,

$$\hat{n}_{d_\eta} = (\hat{d}^+_\eta \cdot \hat{\tilde{d}}_\eta)\,,$$

$$\hat{Q}_\eta = [\hat{d}^+_\eta \times \hat{\tilde{s}}_\eta + \hat{s}^+_\eta \times \hat{\tilde{d}}_\eta]^{(2)} + \chi_\eta[\hat{d}^+_\eta \times \hat{\tilde{d}}_\eta]^{(2)}\,,$$

$$\hat{L}_\eta = \sqrt{10}\,[\hat{d}^+_\eta \times \hat{\tilde{d}}_\eta]^{(1)}, \qquad \eta = \pi, \nu\,,$$

$$\hat{M}_{\pi\nu} = \frac{1}{2}\xi_2[\hat{s}^+_\nu \times \hat{d}^+_\pi - \hat{s}^+_\pi \times \hat{d}^+_\nu]^{(2)} \cdot [\hat{\tilde{s}}_\nu \times \hat{\tilde{d}}_\pi - \hat{\tilde{s}}_\pi \times \hat{\tilde{d}}_\nu]^{(2)} - \sum_{k=1,3} \xi_k[\hat{d}^+_\nu \times \hat{d}^+_\pi]^{(k)} \cdot [\hat{\tilde{d}}_\nu \times \hat{\tilde{d}}_\pi]^{(k)}\,.$$

The E2 transition strengths were calculated by using the operator :

$$\hat{T}(E2) = e_\pi\hat{Q}_\pi + e_\nu\hat{Q}_\nu\,, \qquad (3)$$

where e_π and e_ν are boson effective charges. The M1 operator is given by :[7,9]

$$\hat{T}(M1) = \sqrt{\frac{3}{4\pi}}(g_\pi\hat{L}_\pi + g_\nu\hat{L}_\nu)\,, \qquad (4)$$

where g_π and g_ν are the proton and neutron boson g-factors, respectively. The numerical diagonalization has been carried out by using the computer code NPBOS of Otsuka.

The mixing ratio is given by :

$$\delta(E2/M1) = 0.835\ E_\gamma\ \Delta(E2/M1) \qquad (5)$$

3- Results and Discussions :

As a starting point we used parameters extrapolated from the IBM calculations of some (A=100,102) nuclei , the fitted values of the parameters are given in Tables (1 and 2). These parameters are consistent with those of previous calculations. In general, at using IBM-2, we find the values of ε and κ decrease with increasing mass number.
Also, we can know the limit of nuclei (vibrational, rotational or γ-unstable) by the parameters of IBM versions . Also, we have estimated multipole mixing ratios (δ(E2/M1)) of some transitions for this nuclei and then compared them with some experimental results from [9,10], The mixing ratios values are given in the tables (3-5).

Table 1 . The Hamiltonian parameters set used in the present study for the IBM-1 calculations of some nuclei with mass numbers (A=100,102). (all in MeV except CHI and SO6) .

Nucleus	EPS	P.P.	L.L.	Q.Q.	$T_3.T_3$	$T_4.T_4$	CHI	SO6
^{100}Sr	0.000	0.000	0.0160	- 0.0091	0.000	0.000	- 1.320	1.000
^{100}Mo	0.3610	0.000	0.0099	0.000	0.0581	0.0112	0.000	1.000
^{102}Zr	0.000	0.000	0.0200	- 0.0078	0.000	0.000	- 1.320	1.000
^{102}Pd	0.5780	0.000	0.0115	- 0.020	0.000	0.000	0.000	1.000
^{102}Ru	0.1057	0.000	0.0191	0.000	0.0513	0.1016	0.000	1.000
^{102}Mo	0.1978	0.0850	0.0181	0.000	0.0636	0.000	0.000	1.000

Table 2 . The Hamiltonian parameters set used in the present study for the IBM-2 calculations of some nuclei with mass numbers (A=100,102)

Nucleus	N_π	N_ν	ε (MeV)	κ (MeV)	χ_ν	χ_π	$C_{L\nu}$ (L=0,2,4) (MeV)	$C_{L\pi}$(L=0,2,4) (MeV)
^{100}Sr	6	6	0.684	- 0.178	0.18	- 1.05	0.93,-0.67,0	0.82,0,0
^{100}Mo	4	4	0.824	- 0.058	- 1.00	- 1.00	- 0.55,- 0.1,0.1	0,-0.5,0
^{102}Zr	5	6	0.523	- 0.121	0.32	- 0.26	0,- 0.48,0	0.6,0,0
^{102}Pd	2	3	0.750	- 0.084	- 1.10	0.60	0.3,- 0.4,0	0.2,-0.65,0
^{102}Ru	3	4	0.741	- 0.079	- 0.50	- 0.60	0.2, 0,0	-0.85,-0.66,0
^{102}Mo	4	5	0.620	- 0.060	-1.00	- 1.00	-0. 1,0.05,0.1	-0.3,-0.1,0

Table 3 . The mixing ratios values of ^{100}Sr and ^{100}Mo

$I_i \rightarrow I_f$	^{100}Sr		$I_i \rightarrow I_f$	^{100}Mo	
	Exp.	**IBM-2**		**Exp.**	**IBM-2**
$2_1^+ \rightarrow 2_2^+$	----	6.3172	$2_1^+ \rightarrow 2_2^+$	----	1.2470
$2_1^+ \rightarrow 2_3^+$	----	0.1219	$2_1^+ \rightarrow 2_3^+$	----	0.4259
$4_1^+ \rightarrow 4_2^+$	----	14.8295	$4_1^+ \rightarrow 4_2^+$	----	2.9090
$3_1^+ \rightarrow 2_1^+$	----	6.4731	$3_1^+ \rightarrow 2_1^+$	----	0.2161
$1_1^+ \rightarrow 2_1^+$	----	0.1972	$1_1^+ \rightarrow 2_1^+$	----	0.2856

Table 4 . The mixing ratios values of ^{102}Mo and ^{102}Zr

$I_i \rightarrow I_f$	^{102}Mo		$I_i \rightarrow I_f$	^{102}Zr	
	Exp.	**IBM-2**		**Exp.**	**IBM-2**
$2_1^+ \rightarrow 2_2^+$	----	1.0777	$2_1^+ \rightarrow 2_2^+$	----	2.3913
$2_1^+ \rightarrow 2_3^+$	----	0.0781	$2_1^+ \rightarrow 2_3^+$	----	0.8948
$4_1^+ \rightarrow 4_2^+$	----	0.1074	$4_1^+ \rightarrow 4_2^+$	----	1.4056
$3_1^+ \rightarrow 2_1^+$	----	0.1943	$3_1^+ \rightarrow 2_1^+$	----	0.4474
$1_1^+ \rightarrow 2_1^+$	----	0.1753	$1_1^+ \rightarrow 2_1^+$	----	0.0059

Table 5 . The mixing ratios values of ^{102}Pd and ^{102}Ru

$I_i \rightarrow I_f$	^{102}Pd		$I_i \rightarrow I_f$	^{102}Ru	
	Exp.[11]	**IBM-2**		**Exp.**[12]	**IBM-2**
$2_1^+ \rightarrow 2_2^+$	10.4	6.0107	$2_1^+ \rightarrow 2_2^+$	----	10.5100
$2_1^+ \rightarrow 2_3^+$	8.1	0.4380	$2_1^+ \rightarrow 2_3^+$	0.25	0.2085
$4_1^+ \rightarrow 4_2^+$	----	4.7389	$4_1^+ \rightarrow 4_2^+$	----	3.3664
$3_1^+ \rightarrow 2_1^+$	----	0.9183	$3_1^+ \rightarrow 2_1^+$	- 8.4	0.3467
$1_1^+ \rightarrow 2_1^+$	----	1.9234	$1_1^+ \rightarrow 2_1^+$	----	0.3504

4- Conclusion :

In this work , Interacting Boson Models IBM-1and IBM-2, have been used to calculate energy levels and nuclear properties of some even-even nuclei with (A=100) and (A=102). We observed that IBM-2 is the best in study of this nuclei, also we conclude that (^{100}Mo , ^{102}Pd , ^{102}Ru) in U(5) limits and (^{100}Sr , ^{102}Zr) in SU(3) limit, (^{102}Mo) in O(6) limits. energy levels of the low lying states of these nuclei were produced, Mixing ratios $\delta(E2/M1)$ for transitions with $\Delta I = 0$, $I \neq 0$ were calculated. the results are compared with available experimental data, Satisfactory agreements were produced.

5- References :

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The energy levels which calculated in the present study are in the following figures :

Fig.(1): Comparison between expermental,IBM-1 and IBM-2 energy levels of ^{100}Sr

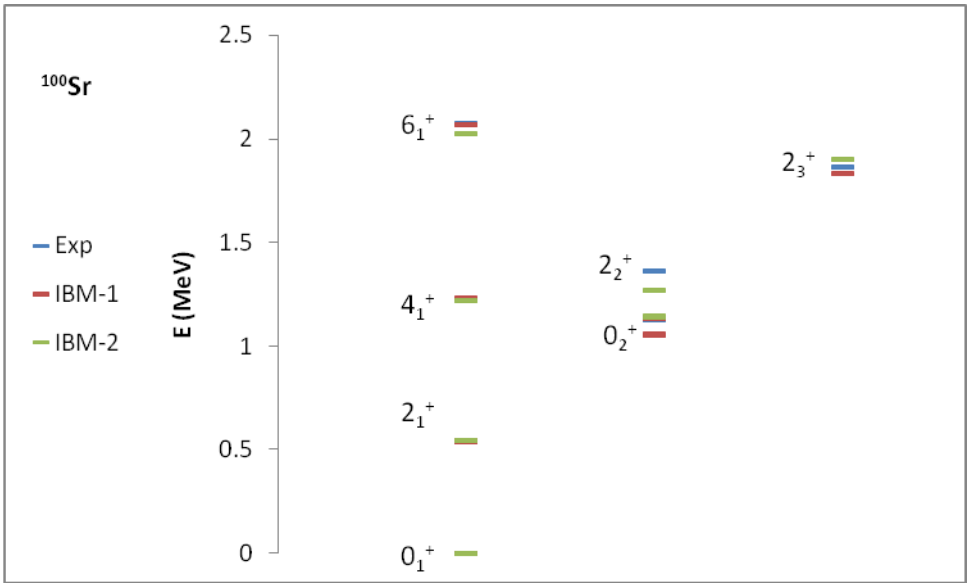

Fig.(2): Comparison between expermental,IBM-1 and IBM-2 energy levels of ^{100}Mo

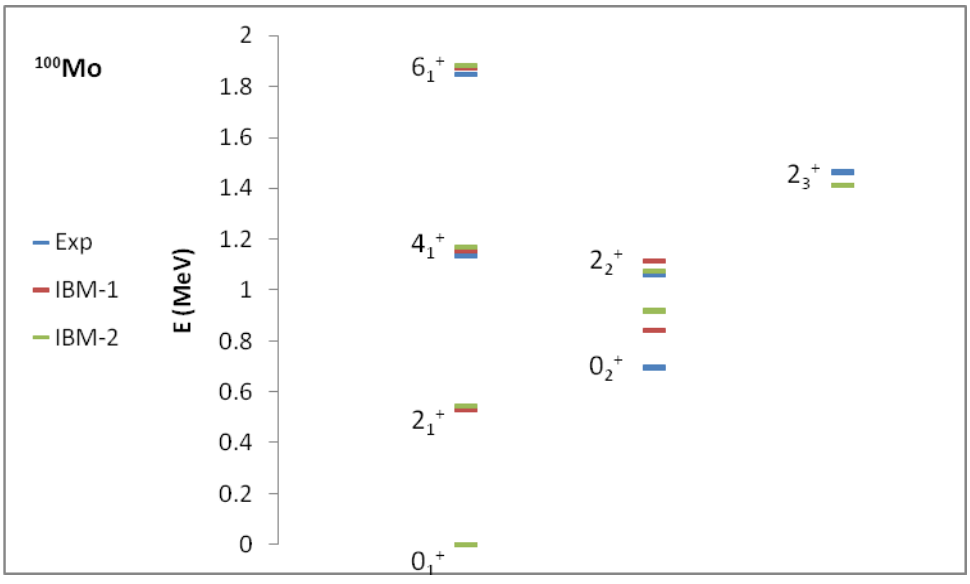

Fig.(3): Comparison between expermental,IBM-1 and IBM-2 energy levels of ^{102}Zr

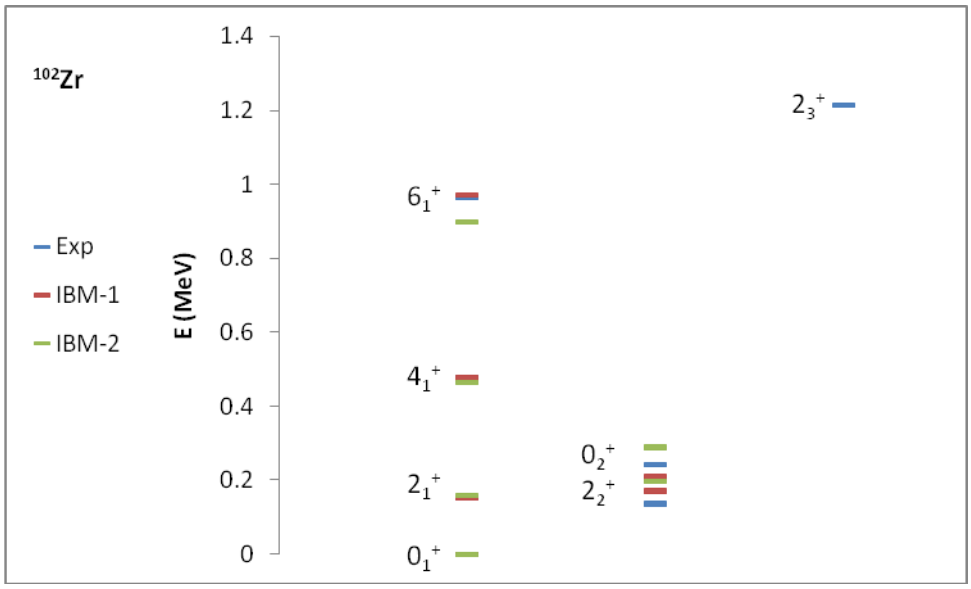

Fig.(4): Comparison between expermental,IBM-1 and IBM-2 energy levels of ^{102}Pd

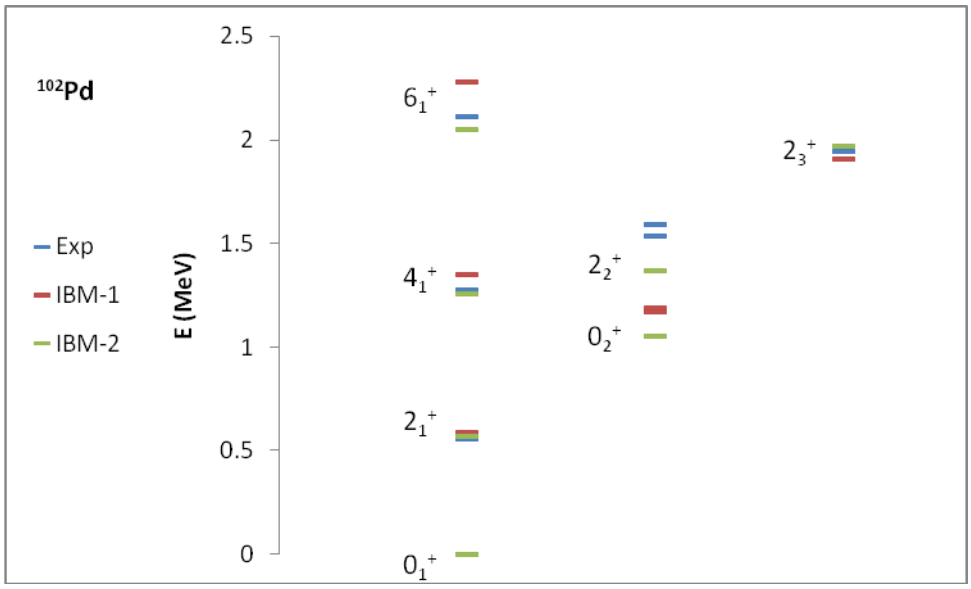

Fig.(5): Comparison between expermental,IBM-1 and IBM-2 energy levels of ^{102}Ru

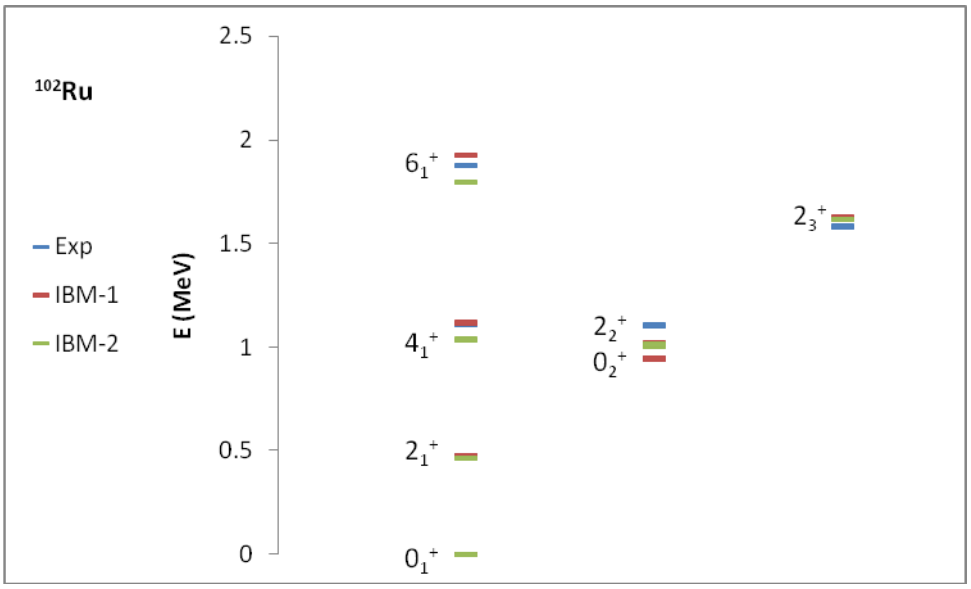

Fig.(6): Comparison between expermental,IBM-1 and IBM-2 energy levels of ^{102}Mo