



Noise Removal of ECG Signal Using Recursive Least Square Algorithms

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Abstract

This paper shows an approach for Electromyography (ECG) signal processing based on linear and nonlinear adaptive filtering using Recursive Least Square (RLS) algorithm to remove two kinds of noise that affected the ECG signal. These are the High Frequency Noise (HFN) and Low Frequency Noise (LFN). Simulation is performed in Matlab. The ECG, HFN and LFN signals used in this study were downloaded from <ftp://ftp.ieee.org/uploads/press/rangayyan/>, and then the filtering process was obtained by using adaptive finite impulse response (FIR) that illustrated better results than infinite impulse response (IIR) filters did.

Keywords: Adaptive filters, RLS algorithms, noise, ECG signals, signal processing.

1. Introduction

Adaptive digital filters are successfully used in many practical systems such as echo, noise canceling, line enhancers, speech coding, and equalizers etc. It operates by adjusting its coefficients in response to its input so as to effectively process that input. Thus the filter coefficients are a function of the actual data they process. RLS adaptive algorithms operate by adjusting the filter coefficient vector so as to converge to the optimum Wiener coefficient vector. This is one of the primary advantages of an adaptive filter. Another primary advantage of adaptive filters is that they can be used in nonstationary environments. Since an adaptive filter is continuously self-designing, it can adapt to statistical changes in the data. [1]

Kevin M. Buckley illustrated the application of filtering to biomedical signals such as ECG, in fetal heart monitor to remove power-line (50 or 60 Hz) interference noise. [1]

Kiyoshi Nishikawa and Hitoshi Kiya proposed a method for increasing the throughput of recursive least squares (RLS) adaptive filters

when only a few processing elements (PEs) are available.[3]

This paper shows method to remove two types of noise that corrupted the ECG signal, which are the HFN and LFN downloaded as HFN-ECG and LFN-ECG from internet with sampling frequency of 1000 Hz.[2] The RLS algorithm is used because it has much faster convergence rate than the least mean square (LMS) algorithm.

2. Adaptive Filter for Noise Cancelling

Adaptive signal processing system has been used in several biomedical applications. The RLS learning algorithm that are used in both linear and nonlinear adaptive filters. When the input process of an adaptive system is (quasi-) stationary, the best steady state performance results from slow adaptation. However, when the input statistics are time-variant (nonstationary), the best performance is obtained by a compromise between fast adaptation (necessary to track variations in the input process) and slow adaptation (necessary to limit the noise in the adaptive process). The LMS adaptation algorithm is a simple and efficient

approach for adaptive noise canceling (ANC); however, it is not appropriate for fast-varying signals due to its slow convergence, and due to the difficulty in selecting the correct value for the step size μ . An alternative approach based on the exact minimization of the least-squares criterion is the RLS method. The RLS algorithm has been widely used in real-time system identification and noise cancellation because of its fast convergence, which is about an order of magnitude higher than that of the LMS method.

An important feature of the RLS algorithm is that it utilizes information contained in the input data, and extends it back to the instant of time when the algorithm was initiated. Given the least-squares estimate of the tap-weight vector of the filter at time $n - 1$, the updated estimate of the vector at time n is computed upon the arrival of new data.[4].

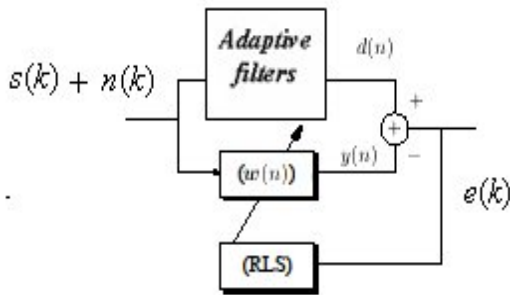


Fig. 1. General Structure of the Adaptive RLS Filter[2]

In the derivation of the RLS algorithm, the instantaneous error $e[n]$ can be calculated from the following equation:

$$e[n] = x[n] - \mathbf{h}'[n] \mathbf{y}[n] = x[n] - \mathbf{y}'[n] \mathbf{h}[n] \quad \dots(1)$$

where $\mathbf{y}[n]$ is the output of the filter, $\mathbf{y}'[n]$ is the output of the filter vector, and $\mathbf{h}[n]$ is the filter parameter, $\mathbf{h}'[n]$ is the filter parameter vector.

The RLS algorithm considers all the available data for determining the filter parameters. The filter should be optimum with respect to all the available data in certain sense.

Minimizes the cost function

$$\varepsilon[n] = \sum_{k=0}^n \lambda^{n-k} e^2[k] \quad \dots(2)$$

with respect to the filter parameter vector

$$\mathbf{h}[n] = \begin{bmatrix} h_0[n] \\ h_1[n] \\ \vdots \\ h_{M-1}[n] \end{bmatrix} \quad \dots(3)$$

where λ is the weighing factor known as the forgetting factor, recent data is given more weightage, for stationary case $\lambda = 1$ can be taken, $\lambda=0.99$ is effective in tracking local nonstationarity, the minimization problem is Minimize

$$\varepsilon[n] = \sum_{k=0}^n \lambda^{n-k} (x[k] - \mathbf{y}'[k] \mathbf{h}[n])^2 \quad \text{with respect}$$

to $\mathbf{h}[n]$ the minimum is given by $\frac{\partial \varepsilon(n)}{\partial \mathbf{h}(n)} = 0$

$$2 \sum_{k=0}^n \lambda^{n-k} (x[k]y[k] - \mathbf{y}'[k] \mathbf{h}[n]) = 0 \quad \dots(4)$$

$$\mathbf{h}[n] = \left(\sum_{k=0}^n \lambda^{n-k} \mathbf{y}[k] \mathbf{y}'[k] \right)^{-1} \sum_{k=0}^n \lambda^{n-k} x[k] \mathbf{y}[k] \quad \dots(5)$$

Let us define

$$\widehat{\mathbf{R}}_{yy}[n] = \sum_{k=0}^n \lambda^{n-k} \mathbf{y}[k] \mathbf{y}'[k] \quad \dots(6)$$

which is an estimator for the autocorrelation matrix \mathbf{R}_{yy} .

similarity

$$\widehat{r}_{xy}[n] = \sum_{k=0}^n \lambda^{n-k} x[k] \mathbf{y}[k] \quad \dots(7)$$

is the estimator for the crosscorrelation vector $\widehat{r}_{xy}[n]$ hence

$$\mathbf{h}[n] = \left(\widehat{\mathbf{R}}_{xy}[n] \right)^{-1} \widehat{r}_{xy}[n] \quad \dots(8)$$

is the matrix inversion is involved which makes the direct solution difficult. We look forward for a recursive solution.[4]

2.1. Recursive Representation of $\widehat{\mathbf{R}}_{yy}[n]$

$\widehat{\mathbf{R}}_{yy}[n]$ can be written as follows

$$\begin{aligned} \widehat{R}_{yy}[n] &= \sum_{k=0}^{n-1} \lambda^{n-k} y[k]y'[k] + y[n]y'[n] \\ &= \lambda \sum_{k=0}^{n-1} \lambda^{n-1-k} y[k]y'[k] + y[n]y'[n] \\ &= \lambda \widehat{R}_{yy}[n-1] + y[n]y'[n] \end{aligned} \quad \dots(9)$$

This shows that the autocorrelation matrix can be recursively computed from its previous values and the present data vector.[4]
Similarly

$$\begin{aligned} \widehat{r}_{xy}[n] &= \lambda \widehat{r}_{xy}[n-1] + x[n]y[n] \\ h[n] &\equiv [R_{yy}[n]]^{-1} \widehat{r}_{xy}[n] \\ &= (\lambda \widehat{R}_{yy}[n-1] + y[n]y'[n])^{-1} \widehat{r}_{xy}[n] \end{aligned} \quad \dots(10)$$

For the matrix inversion above the matrix inversion lemma will be useful.

2.2. Matrix Inversion Lemma

If **A**, **B**, **C** and **D** are matrices of proper orders, **A** and **C** nonsingular

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B(DA^{-1}B + C)DA^{-1} \quad \dots(11)$$

Taking $A = \lambda \widehat{R}_{yy}[n-1]$, $B = y[n]$, $C=1$ and $D = y'[n]$

we will have

$$\begin{aligned} (\widehat{R}_{yy}[n]^{-1}) &= \frac{1}{\lambda} \widehat{R}_{yy}^{-1}[n-1] - \\ &\frac{1}{\lambda} \widehat{R}_{yy}^{-1}[n-1]y[n] \left(y'[n] \frac{1}{\lambda} [\widehat{R}_{yy}^{-1}[n-1]]y[n] + 1 \right)^{-1} y'[n] \widehat{R}_{yy}^{-1}[n-1] \\ &= \frac{1}{\lambda} \left(\widehat{R}_{yy}^{-1}[n-1] - \frac{\widehat{R}_{yy}^{-1}[n-1]y[n]y'[n]\widehat{R}_{yy}^{-1}[n-1]}{\lambda + y'[n]\widehat{R}_{yy}^{-1}[n-1]y[n]} \right) \end{aligned} \quad \dots(12)$$

Rename $P[n] = \widehat{R}_{yy}^{-1}[n]$. Then

$$P[n] = \frac{1}{\lambda} (P[n-1] - K[n]y'[n]P[n-1]) \quad \dots(13)$$

where $K[n]$ is called the "gain vector" and given by

$$K[n] = \frac{P[n-1]y[n]}{\lambda + y'[n]P[n-1]y[n]} \quad \dots(14)$$

$K[n]$ important to interpret adaptation is also related to the current data vector $y[n]$ by $K[n]=P[n]y[n]$

To establish the above relation consider

$$P[n] = \frac{1}{\lambda} (P[n-1] - K[n]y'[n]P[n-1]) \quad \dots(15)$$

multiplying by λ and post-multiplying by $y[n]$ and simplifying we get

$$\begin{aligned} \lambda P[n]y[n] &= (P[n-1] - K[n]y'[n]P[n-1])y[n] \\ &= P[n-1]y[n] - K[n]y'[n]P[n-1]y[n] \\ &= \lambda K[n] \end{aligned} \quad \dots(16)$$

Therefore

$$\begin{aligned} h[n] &= \left(\widehat{R}_{yy}[n] \right)^{-1} \widehat{r}_{xy}[n] \\ &= P[n](\lambda \widehat{r}_{xy}[n-1] + x[n]y[n]) \\ &= \lambda P[n] \widehat{r}_{xy}[n-1] + x[n]P[n]y[n] \\ &= \lambda \frac{1}{\lambda} [P[n-1] - K[n]y'[n]P[n-1]] \widehat{r}_{xy}[n-1] + x[n]P[n]y[n] \\ &= h[n-1] + K[n]y'[n]h[n-1] + x[n]K[n] \\ &= h[n-1] + K[n](x[n] - y[n]h[n-1]) \end{aligned} \quad \dots(17)$$

2.3. RLS Algorithm Operation

The RLS algorithm can be motivated and derived as the *exact* solution to a well-defined estimation problem with a least-squares cost function.[5]

The RLS algorithm operation was:

For 1 to n = Final

1. Get $x[n]$, $y[n]$
2. Get $e[n] = x[n] - \mathbf{h}'[n-1]y[n]$

3. Calculate gain vector

$$K[n] = \frac{P[n-1]y[n]}{\lambda + y'[n]P[n-1]y[n]}$$

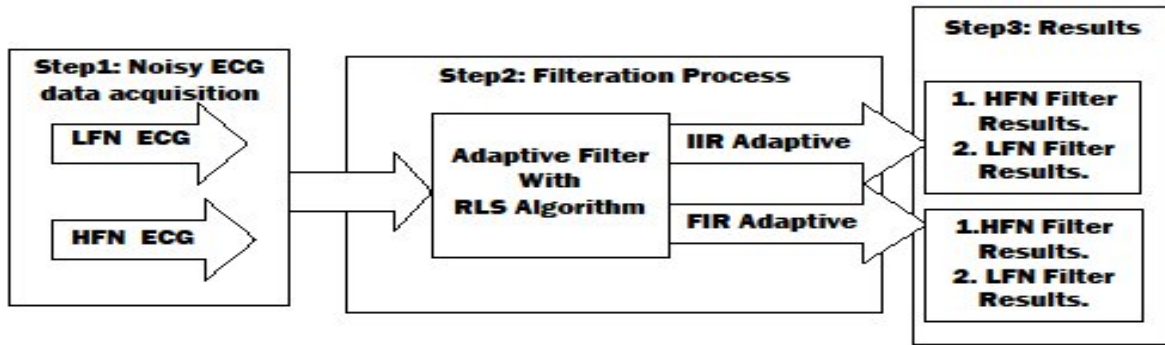
4. Update the filter parameters

$$\mathbf{h}[n] = \mathbf{h}[n-1] + K[n]e[n]$$

5. Update the **P** matrix

$$P[n] = \frac{1}{\lambda} (P[n-1] - K[n]y'[n]P[n-1])$$

end



3. Methodology

From the following bloke diagram, this section is divided in to three steps as follows:

3.1. 1st Step Noisy ECG Data Acquisition

The ECG is the electrical manifestation of the contractile activity of the heart, and can be recorded fairly easily with surface electrodes on the limbs or chest. The corrupted ECG signals with HFN and LFN are a real data down loaded from internet[1], these contaminated ECG of 500 μ sec (2000 samples) data length with sampling frequency of 1000Hz.

3.2. 2nd Step Filtration Process

The noisy ECG signals in this step were passing through the adaptive FIR and IIR filters, the adaptation process were done using RLS algorithm to remove the HFN and LFN from the ECG signals. The selected parameters were illustrated in table 1:

Table 1,
Parameters of the Proposed System.

	FIR	IIR
System order	5	10
λ	0.999	0.9995

The selected order for the FIR type is half of the IIR filter and illustrated more pleasant results with more sharpness compared with the IIR filter that required higher order to illustrated a familiar results.

The weighting factor λ (also known as the forgetting factor) with $0 < \lambda < 1$ the number of samples

< 1 give more “weight” to the more recent error values. Such weighting is desired in the case of nonstationary data, where changes in the signal statistics make the inclusion of past data less appropriate [6]. So that the smaller forgetting factor λ give an indication of the smaller contribution of previous samples. This makes the filter more sensitive to recent samples, which means more fluctuations in the filter coefficients. The $\lambda = 1$ case is referred to as the growing window RLS algorithm.[7]

3.3. Results and Discussion

A. Noise Removal

To verify the algorithm suggested in this paper, the ECG signals were filtered and evaluated the overall error of the signal and the optimal filter order. In this paper, MATLAB was used to estimate the performance of the proposed filters, and experimented with two kinds of filters.

The mean \bar{x} and standard *std* deviation were calculated. The mean which is the average value of a signal. It is found by adding all of the samples together, and divide by N . It looks like this in mathematical

$$\bar{x} = \frac{1}{N} \sum_{i=0}^{N-1} x_i \quad \dots(18)$$

std: standard deviation the standard deviation s of a data vector X :

$$std = \left(\frac{1}{N} \sum_{i=0}^{n-1} \left(x_i - \bar{x} \right)^2 \right)^{1/2} \quad \dots(19)$$

Where $\bar{x} = \frac{1}{N} \sum_{i=1}^n x_i$

and is the n number of elements in the sample. The two forms of the equation differ only in $n-1$ versus n in the divisor.

The results are shown in table (2):

**Table 2,
FIR and IIR Filter Results.**

		HFN removal		LFN removal	
		Output (RLS)	Output (RLS)	Output (RLS)	Output (RLS)
		FIR	IIR	FIR	IIR
Mean	desired signal	-0.528	-0.176	-0.798	-0.435
	output signal	-0.489	0.023	-0.713	-0.034
Std	desired signal	0.774	0.723	0.785	0.772
	output signal	0.776	0.536	0.847	0.787

From the above table the means of both FIR filter causes to remove the HFN and LFN is smaller value and approach to the desired values of the output as shown in figure 3 and 4 respectively. While IIR filter means of the output are small values and less than the desired signals values as shown in figure 5 and 6.

To compare between the standard deviation of the two outputs the results shown that std of FIR output of HFN is approximately similar to std of the desired signal and less than of IIR output std but approximately similar to std of desired signal that mean that the deviation of the results from the true curve is smaller in case of FIR filters to remove both noises than the IIR filters.

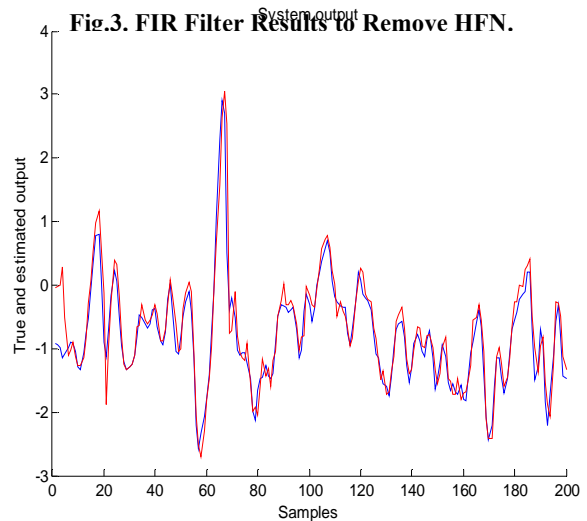


Fig.3. FIR Filter Results to Remove HFN.

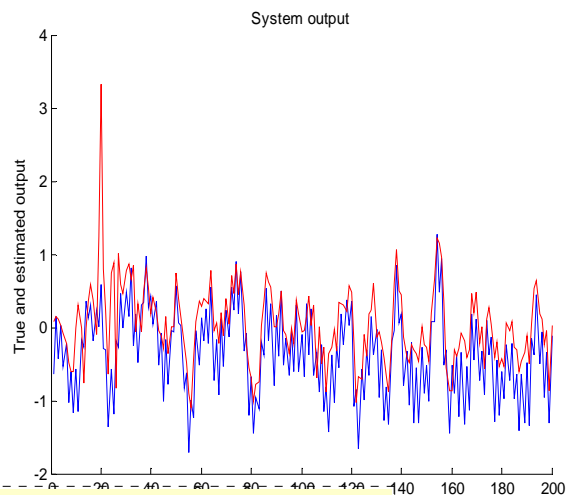
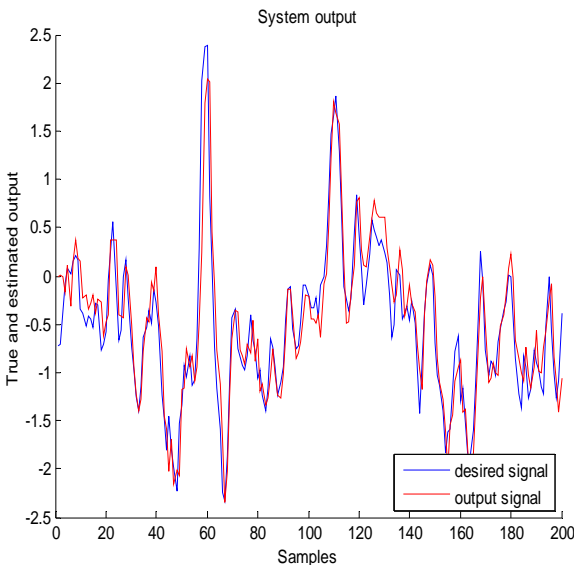


Fig.4. FIR Filter Results to Remove LFN.

Fig.5. IIR Filter Results to Remove HFN.

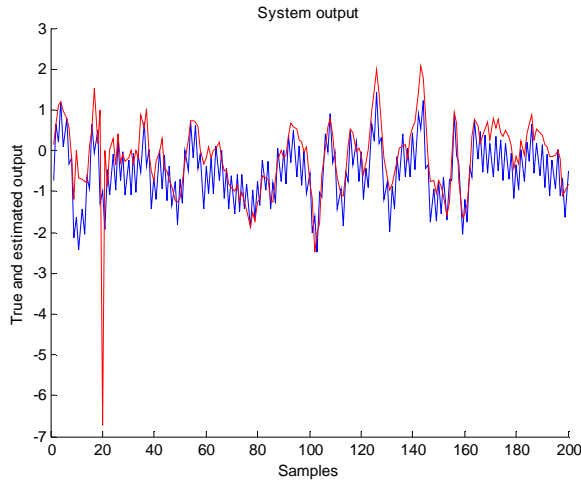


Fig.6. IIR Filter Results to Remove LFN.

B. Error Calculations

The error curves were calculated as and plotted as shown in figures 6,7,8 and 9, respectively. (The logarithmic scale is used to display better the minor differences between the spectrograms.) It is seen that the frequency components of the HFN have been suppressed by using FIR filter rather than the IIR filter according to the following calculations of mean and std that shown in table 3.

Table 3, Error Curves Calculations.

	HFN removal		LFN removal	
	Error (RLS)	Error (RLS)	Error (RLS)	Error (RLS)
	FIR	IIR	FIR	IIR
Mean	-0.547	-0.1283	-0.0592	-0.4115
Std	0.2879	0.4973	0.2609	0.6174

Fig.7. Error Curve of HFN Removal Using FIR Filter.

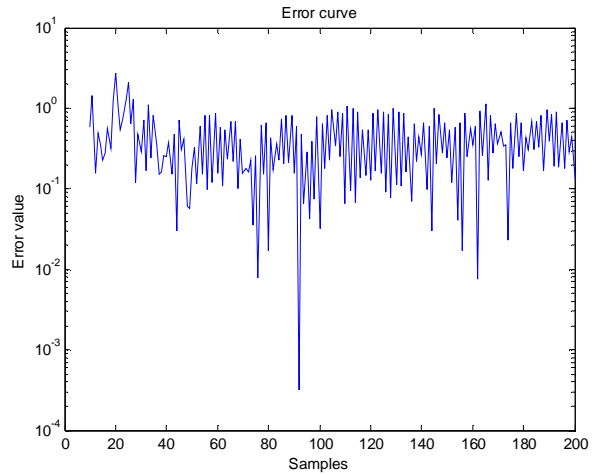


Fig.8. Error Curve of HFN Removal Using IIR Filter.

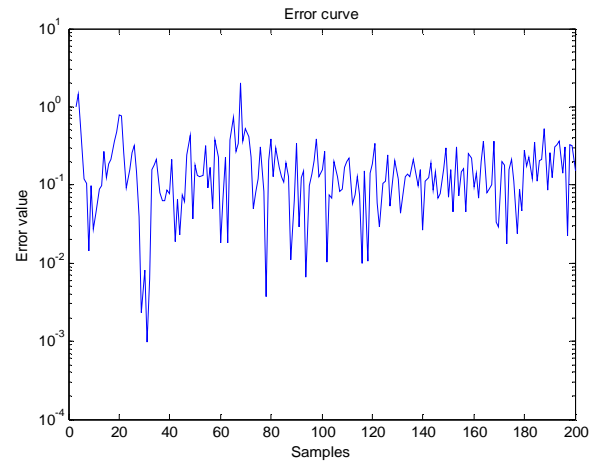


Fig.9. Error Curve of LFN Removal Using FIR Filter.

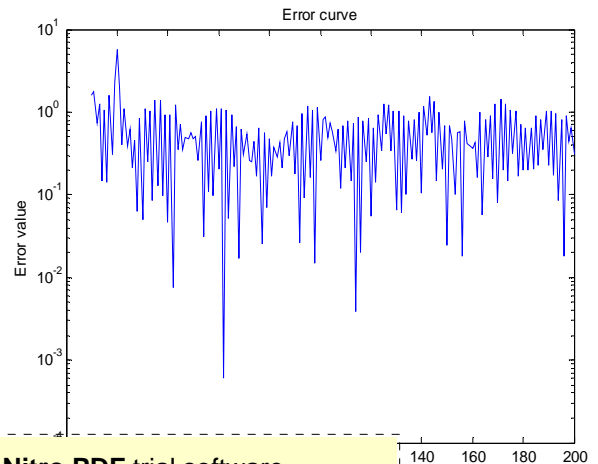
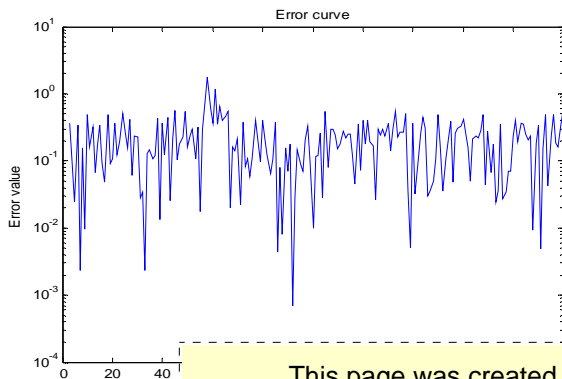


Fig.10. Error Curve of LFN Removal Using IIR Filter.

C. Comparison of the Filter Weights and Estimated Weights

According to the order of the filter the following figures shows a comparison between the filter weights and estimated weights for both FIR and IIR filters. As illustrated in section 3.1 and table 1 the FIR filter order (5) while IIR filter order (10) because FIR filter output is calculated using the current and past input, but not previous values of the output and produce more placental results at the selected order but IIR filters are recursive filters that use the surrounding input and previous output values together and the selected order introduce good results.

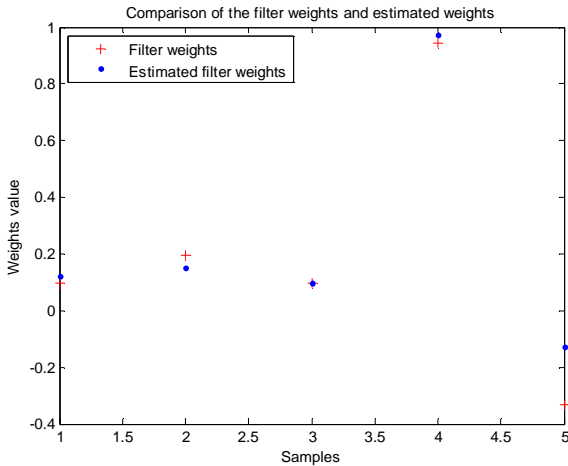


Fig.11. Comparison of the FIR Filter Weights and Estimated Weights (HFN Removal).

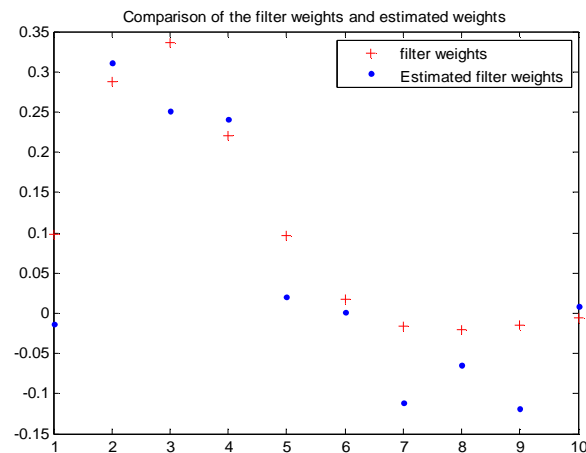


Fig.12. Comparison of the IIR Filter Weights and Estimated Weights (HFN Removal).

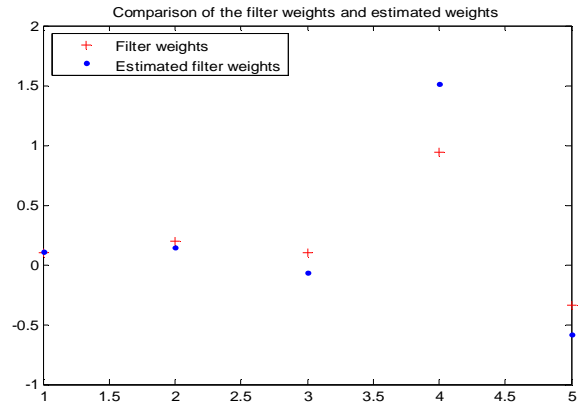


Fig.13. Comparison of the FIR Filter Weights and Estimated Weights (LFN Removal).

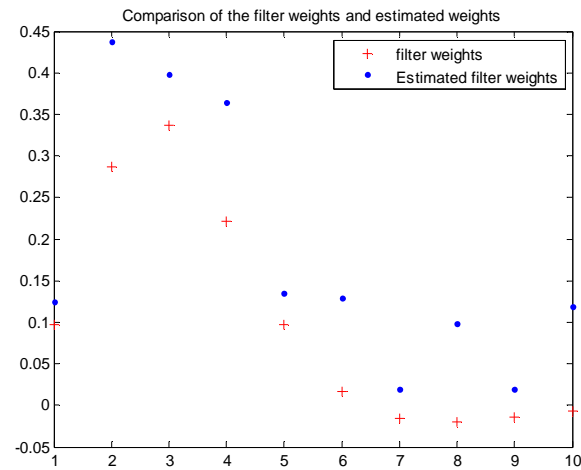


Fig.14. Comparison of the IIR Filter Weights and Estimated Weights (LFN Removal).

4. Conclusions

The ECG is perhaps the most commonly known, recognized, and used biomedical signal that corrupted with several types of noise such as HFN and LFN. Excessive filtering results in a distorted signal were evaluated the performance of the RLS algorithm for these signals.

This paper can be illustrated that the means of both FIR filter causes to remove the HFN and LFN is smaller value and approach to the desired values of the output, while IIR filter means of the output are small values and less than the desired

signals values, but the standard deviation of the two outputs

(FIR output of HFN) is approximately similar to std of the desired signal and less than of IIR output std which is similar to std of desired signal that mean that the deviation of the results from the true curve is smaller in case of FIR filters to remove both noises than the IIR filters.

The FIR adaptive filter using RLS algorithm is illustrated more perfect results compared with IIR filters.

Notation

$s[n]$	desired signal
$n[n]$	interference signal
$x[n]$	the input of the adaptive filter
$e[n]$	error signal
$y[n]$	the output of the adaptive filter
$y'[n]$	the output of the adaptive filter vector
$h'[n]$	filter parameter vector
$h[n]$	filter parameter
n,k,M	length of the signal
$k[n]$	gain vector
A,B,C,D	matrices of proper order
$\hat{R}_{yy}[n]$	an estimator for the autocorrelation matrix an estimator for the autocorrelation matrix
$P[n]$	inversion of an estimator for the autocorrelation matrix

$\hat{r}_{xy}[n]$ an estimator for the autocorrelation vector

Greek letters

λ	the forgetting factor
δ	positive number
$\varepsilon[n]$	cost function

5. References

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إزالة ضوضاء التي تستعمل أقل تكرارية الخوارزميات المربعة

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الخلاصة

هذا البحث يبحث نظرة لمعالجة إشارات تخطيط القلب مس تنده على التردد يح التكييفي اللاخطي والخطي الذي يبد تعمل خوارزمية أقل لتكرارية المربع لإزالة إثنان من أنواع الضوضاء الذي المؤثره على إشارة تخطيط القلب و المتمثله بوضوءاء التذبذب العالي و وضوءاء التذبذب ال واطي المحاكاة مؤديّة باستخدام ال وضوءاء التذبذب العالي و وضوءاء التذبذب ال واطي المسد تعمله في هذه الدراسة حملت من الإنترنت ، وبع ذلك عملية التردد يح بابد تعمل استجابة الحافز المحدود الذي صور نتائج افضل من مرشحات استجابة الحافز اللانهائي!!!

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