# Design an Algorithm to Calculate the Inverse Permutations Of the Symmetric Group $S_n$ In Computer

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## Abstract

The search is an application for one of the problems of mathematics in the computer; as providing construction and design of a major program to calculate the inverse permutations of the symmetric group $S_n$ , where $1 \leq n \leq 13$; using some of the methods used in the Number Theory by computer . Also the research includes design flow chart for the main program and design flow chart for the program inverse permutations and we give some illustrative examples for different symmetric groups and their inverse permutations.

**Keywords: Symmetric group, permutations, inverse permutations, identity permutation, length of cycle.**

# Introduction

Number Theory is concerned with properties of the integers:

$$. . . ,−4,−3,−2,−1, 0, 1, 2, 3, 4, . . . .$$

The great mathematician Carl Friedrich Gauss called this subject arithmetic and he said: (1) "Mathematics is the queen of sciences and arithmetic the queen of Mathematics"

Symmetry property can be described as many things, such as objects of engineering and mathematical equations, etc., and it is characterized by symmetry recipe Human Man convicted him, two hands, two legs, two eyes and ears, this means that half of it is similar to the right half of the left form.

In mathematics, a permutation group is a group G whose elements are permutations of a given set M, and whose group operation is the composition of permutations in G (which are thought of as injective functions from the set M to itself); the relationship is often written as (G,M). Note that the group of all permutations of a set is the symmetric group; the term permutation group is usually restricted to mean a subgroup of the symmetric group. The symmetric group of n elements is denoted by $S_n$; if M is any finite or infinite set, then the group of all permutations of M is often written as Sym(M).(2)

The application of a permutation group to the elements being permuted is called its group action; it has applications in both the study of symmetries, combinatory and many other branches of mathematics, physics and chemistry.

The degree of a group of permutations of a finite set is the number of elements in the set.

At first blush one might think that of all areas of mathematics certainly Arithmetic should be the simplest, but it is a surprisingly deep subject. Mathematics is of abstract science, but do not touch reality only when we see its applications. So one of the Mathematics problems we applied in the computer. This is guaranteed by our research.

The inverse of permutation for $S_n$ has been given by Morris (3) for $1 \le n \le 13$. It seems to be that there are no-ready-programs available to calculate those tables. We give algorithms of programs to calculate the inverse for $S_n$ $1\le n \le 13$. In this paper we adopt the properties of inverse and permutations for $S_n$.

## *Definition:* (1) (4)

Let X is a non-empty set, a permutation of X is a function $\alpha$: X $\longrightarrow$ X that is a one-to-one correspondence and the symmetric group on X, denoted by $S_X$, is the group whose elements are the permutation of X and whose binary operation is composition of function, of particular interest is the special case when X is finite. If X = {1,2,3,…,n}, we write $S_n$ instead of $S_X$ and we call $S_n$ the symmetric groups of degree n, or the symmetric groups on n letters.

Note that $|S_n| = n!$

***Definition:*** **(2) (5)**

A permutation $\alpha = \begin{bmatrix} x_1 & x_2 & x_3 & \dots & x_k \\ x_2 & x_3 & x_4 & \dots & x_1 \end{bmatrix}$ of a set X is called a cycle of length k or a k-cycle written $\alpha = (x_1x_2x_3\dots x_k)$ and $\alpha(x) = x$ for all $x \in X$ but $x \notin \{x_1,x_2,x_3,\dots,x_k\}$ k is called the length of the cycle.

***Definition:*** **(3) (3)**

The product of two cycles need not again be a cycle. Thus $\alpha = (x_1x_2 \dots x_k)$ then $\alpha^{-1} = (x_1x_k \dots x_2)$ such that $\alpha.\alpha^{-1} = \alpha^{-1}.\alpha = i$ (identity permutation).

From the previous definitions we can deduce the following properties:

(1) A cycle of length 2 has inverse that same cycle.

(2) The inverse of identity permutation is itself.

(3) A cycle has two kinds joint and disjoint.

**The Algorithms:**

This part contains a collection of the computer algorithms for many standard methods of Number Theory installed in our main program.

**Algorithm (1): The Length of Cycle**

This algorithm is designed for determining the length of Cycle of permutation for $S_n$.

**Input:** $S_n$ (The symmetric group of degree n)

**Step 1:** To evaluate k

If $\alpha = (a_1\ a_2\ \ldots\ a_k)$

$\ni \alpha(a) = a$ for $a \in a_i$

But $a \notin \{a_1\ a_2\ \ldots\ a_k\}$

Then k is length of cycle

**Output:** The length of cycle (k)

End.

**Algorithm (2): The Inverse of Joint Cycle**

This algorithm is designed for determining the inverse of joint cycle.

**Input:** $S_n$ (The symmetric group of degree n)

**Step 1:** $k_1$ is cycle of length $k_1$,(By using algorithm 1)

**Step2:** For I = 1 to $k_1$ ,A(I) is inverse of cycle of length I

B (1) = A (1)

B (2) = A ($k_1$)

End

**Step 3:** For J = 2 to k, L = 0

B (J) = A (k – L)

L = L + 1

Print B (J)

End J-Loop

**Output:** The inverse of joint cycle.

**Algorithm (3): The Inverse of Disjoint Cycle**

This algorithm is designed for determining the inverse of disjoint cycle.

**Input:** $S_n$ (The symmetric group of degree n)

**Step 1:** let 2 and (n – 2)! are two disjoint cycles

**Step 2:** For I = 1 to 2

$A_1$ (I)

Print $A_1$ (I)

End I-Loop

**Step 3:** For $I_2$ = 1 to (n – 2)!

$A_2$ ($I_2$)

Print $A_2$ ($I_2$)

End $I_2$-Loop

**Step 4:** $B_1$ (1) = $A_1$ (1)

$L_1 = 0$

For $J_1$ = 1 to 2

B ($J_1$) = A (2 – L)

$L_1 = L_1 + 1$

Print B ($J_1$)

End $J_1$-Loop

**Step 5:** $B_2$ (1) = $A_2$ (1), $L_2 = 0$

For $J_2$ = 1 to (n – 2)!

B ($J_2$) = $A_2$ ((n – 2)! – $L_2$)

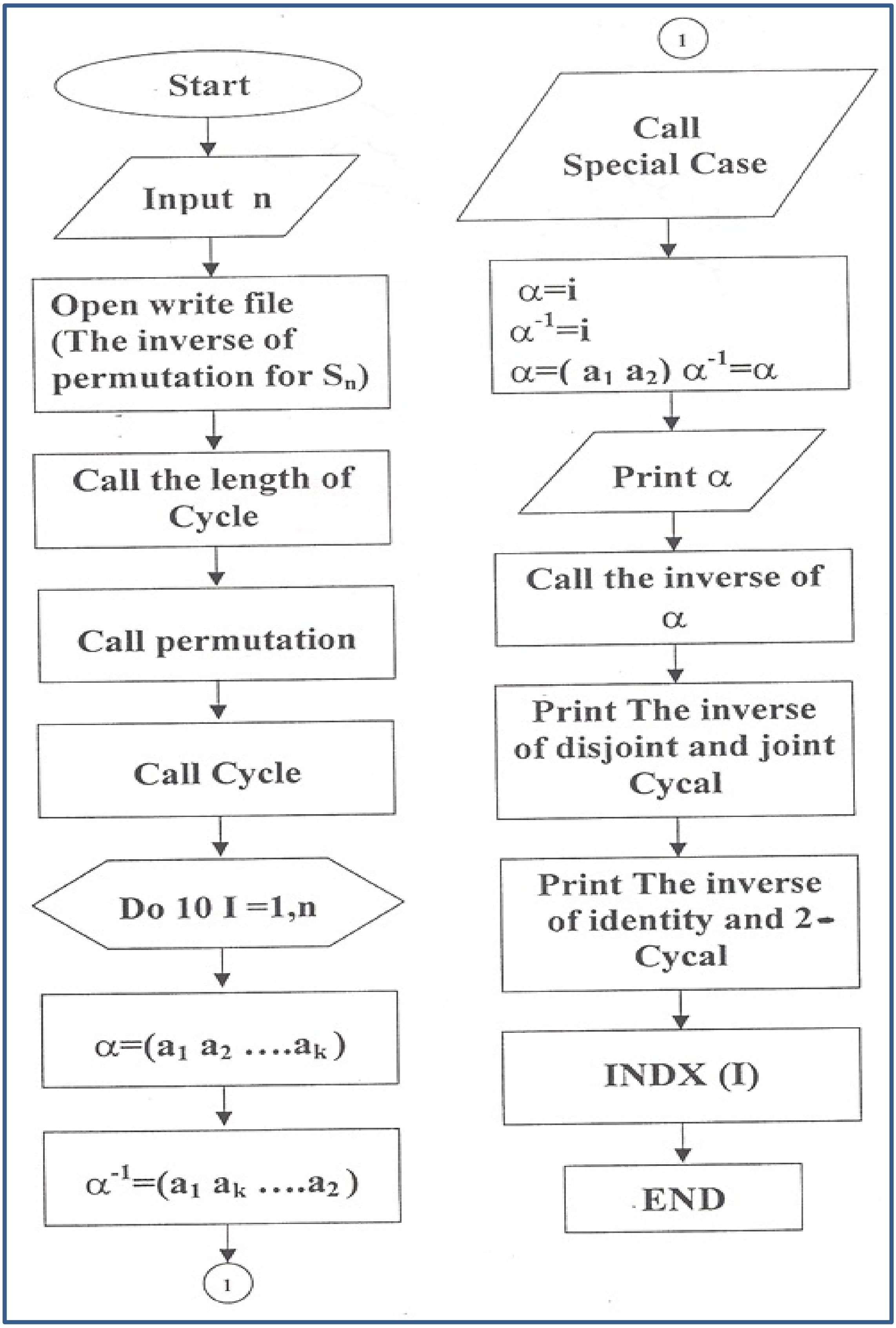
$L_2 = L_2 + 1$

Print $B_2$ ($J_2$)
End $J_2$-Loop
**Step 6:** $B_1$ ($J_1$) $B_2$ ($J_2$)
The inverse of cycle of length 2 is $B_1$ ($J_1$) and the inverse of cycle of length (n – 2)! Is $B_2$ ($J_2$), that the inverse of disjoint cycle is $B_1$ ($J_1$) $B_2$ ($J_2$).
**Step 7:** If $k_1$ = 3 and $k_2$ = (n – 3)!
Then in general
k & ( n – k)!
**Output:** The inverse of k & (n – k)!

**Algorithm (4): Special Case the Degree of the $S_n$ .**
**Input:** $S_n$ (The symmetric group of degree n)
**Step 1:** For I = 1 to n
Print A (I)
If A (I) = I then A (I) is inverse of cycle of length I
Else
End I-Loop
**Step 2:** For J = 1 to n
Print A (J)
If (J.GE.2) then B(J) = INDEX(I)
End J-Loop
**Output:** The inverse of identity and cycle of length 2
End

**Algorithm (5): The main Algorithm 1 ≤ n ≤ 13.**
**Input:** $S_n$ (The symmetric group of degree n)
**Step 1:** Call algorithm 1
**Step 2:** Call algorithm 2
**Step 3:** Call algorithm 4
**Step 4:** Call algorithm 3
**Output:** (IN (k)) (To evaluate the inverse of permutation for $S_n$)
End

**Flow Chart of the Main Program**



The following will give some illustrative examples of the different symmetric groups and their inverse permutations.

***Example* : (1)**

The elements of $S_4$ and the inverse permutations of the $S_4$ elements

| | **Elements of $S_4$** | **Inverse permutations of the $S_4$ elements** |
|---|---|---|
| 1 | (1234) | (1432) |
| 2 | (2341) | (2143) |
| 3 | (1432) | (1234) |
| 4 | (1324) | (1423) |
| 5 | (1423) | (1324) |
| 6 | (1342) | (1243) |
| 7 | (123) | (132) |
| 8 | (342) | (432) |
| 9 | (143) | (134) |
| 10 | (243) | (234) |
| 11 | (132) | (123) |
| 12 | (134) | (143) |
| 13 | (142) | (124) |
| 14 | (12) | (21) |
| 15 | (23) | (32) |
| 16 | (34) | (43) |
| 17 | (41) | (14) |
| 18 | (24) | (42) |
| 19 | (13) | (31) |
| 20 | (14) | (41) |
| 21 | (43) | (34) |
| 22 | (21) | (12) |
| 23 | (31) | (13) |
| 24 | i | i |

***Example:*** **(2)**The elements of $S_5$ and the inverse permutations of the $S_5$ elements

| Se. | Elements of $S_5$ | Inverse Permutations of the $S_5$ elements | Se. | Elements of $S_5$ | Inverse permutations of the $S_5$ elements |
|---|---|---|---|---|---|
| 1 | i | i | 41 | (143) | (134) |
| 2 | (12) | (21) | 42 | (153) | (134) |
| 3 | (13) | (31) | 43 | (154) | (145) |
| 4 | (14) | (41) | 44 | (243) | (234) |
| 5 | (15) | (51) | 45 | (253) | (235) |
| 6 | (23) | (32) | 46 | (254) | (245) |
| 7 | (24) | (42) | 47 | (345) | (354) |
| 8 | (25) | (52) | 48 | (35)(142) | (53)(124) |
| 9 | (34) | (43) | 49 | (12)(345) | (21 )(354) |
| 10 | (35) | (53) | 50 | (13)(245) | (31 )(154) |
| 11 | (45) | (54) | 51 | (14)(235) | (41)(253) |
| 12 | (12)(34) | (21)(43) | 52 | (15)(234) | (51)(432) |
| 13 | (12) (35) | (21) (53) | 53 | (23)(145) | (32)(154) |
| 14 | (12) (45) | (21) (54) | 54 | (24)(135) | (42)(153) |
| 15 | (13)(45) | (31)(54) | 55 | (12)(354) | (21)(345) |
| 16 | (23)(45) | (32)(54) | 56 | (13)(254) | (31 )(245) |
| 17 | (13)(24) | (31)(42) | 57 | (14)(253) | (41)(235) |
| 18 | (13)(25) | (31)(52) | 58 | (15)(243) | (51)(234) |
| 19 | (14)(25) | (41)(52) | 59 | (23)(154) | (32)(145) |
| 20 | (14)(35) | (41 )(53) | 60 | (24)(153) | (42)(135) |
| 21 | (24)(35) | (42)(53) | 61 | (25)(134) | (52)(143) |
| 22 | (14)(23) | (41)(32) | 62 | (34)(125) | (43)(152) |
| 23 | (15)(23) | (51)(32) | 63 | (35)(125) | (53)(142) |
| 24 | (15)(24) | (51)(42) | 64 | (35)(124) | (53)(142) |
| 25 | (15)(34) | (51)(34) | 65 | (45)(132) | (54)(123) |
| 26 | (25)(34) | (52)(43) | 66 | (1254) | (1452) |
| 27 | (123) | (132) | 67 | (1354) | (1453) |
| 28 | (124) | (142) | 68 | (2354) | (2453) |
| 29 | (125) | (152) | 69 | (1342) | (1243) |
| 30 | (134) | (143) | 70 | (1352) | (1235) |
| 31 | (135) | (153) | 71 | (1452) | (1254) |
| 32 | (145) | (154) | 72 | (1453) | (1354) |
| 33 | (243) | (234) | 73 | (2453) | (2354) |
| 34 | (235) | (253) | 74 | (1432) | (1234) |
| 35 | (245) | (254) | 75 | (1532) | (1235) |
| 36 | (132) | (123) | 76 | (1542) | (1245) |
| 37 | (142) | (124) | 77 | (2543) | (2345) |
| 38 | (152) | (125) | 78 | (1543) | (1345) |
| 39 | (143) | (134) | 79 | (12345) | (15432) |
| 40 | (152) | (152) | 80 | (12354) | (14532) |

| Se. | Elements of $S_5$ | Inverse permutations of the $S_5$ elements | Se. | Elements of $S_5$ | Inverse permutations of the $S_5$ elements |
|---|---|---|---|---|---|
| 81 | (12435) | (15342) | 101 | (34)(152) | (43)(125) |
| 82 | (12453) | (13542) | 102 | (12543) | (13542) |
| 83 | (1243) | (1342) | 103 | (12543) | (15342) |
| 84 | (]253) | (1325) | 104 | (13254) | (13542) |
| 85 | (1253) | (1352) | 105 | (13425) | (14352) |
| 86 | (1254) | (1452) | 106 | (13452) | (13452) |
| 87 | (]354) | (1453) | 107 | (13524) | (13452) |
| 88 | (2354) | (2453) | 108 | (13542) | (14523) |
| 89 | (1342) | (1243) | 109 | (14235) | (12543) |
| 90 | (1352) | (1235) | 110 | (14253) | (14253) |
| 91 | (1452) | (1254) | 111 | (14325) | (12453) |
| 92 | (1453) | (1354) | 112 | (14352) | (15324) |
| 93 | (2453) | (2354) | 113 | (14523) | (13524) |
| 94 | (1432) | (1234) | 114 | (14532) | (15234) |
| 95 | (1532) | (1235) | 115 | (15234) | (12534) |
| 96 | (1542) | (1245) | 116 | (15243) | (13254) |
| 97 | (2543) | (2345) | 117 | (15324) | (14325) |
| 98 | (1543) | (1345) | 118 | (15432) | (13425) |
| 99 | (1534) | (1435) | 119 | (15342) | (14235) |
| 100 | (25)(143) | (52)(134) | 120 | (15423) | (13245) |

# Conclusions

1. Algorithms contained in the search provide the time and effort in the calculation of the inverse permutations of the symmetric group, as well as they give us the accuracy in the calculation.

2. The interval proposed in the search for the symmetric group $S_n$ is $1 \leq n \leq 13$; Therefore, the study the symmetric group $S_n$ when n>13 analog is not without complexity.

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# تصميم خوارزمية لحساب معكوس التباديل للزمرة التناظرية ($S_n$) الحاسوب

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## الخلاصة

يمثل البحث تطبيقا" لإحدى مسائل علوم الرياضيات في الحاسوب ؛ إذ نقدم بناء و تصميم برنامج رئيس لحساب معكوس التباديل للزمرة التناظرية $S_n$ ؛ إذ $1 \leq n \leq 13$ ؛ باستعمال بعض الطرائق المعتمدة من نظرية الأعداد في الحاسوب. كذلك تضمن البحث تصميم تخطيط للبرنامج الرئيس و تصميم مخطط لبرنامج معكوس التباديل؛ وبعض الأمثلة التوضيحية لزمر تناظرية مختلفة ومعكوسات تباديلها.

**الكلمات المفتاحية: الزمرة التناظرية ؛ التباديل؛ معكوس التباديل؛ التبديل المحايد؛ طول الدورة.**