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On Supra α– Connectedness in Supra Topological Spaces

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Abstract

The purpose of this paper is to introduce the concept called supra α– connectedness in supra topological spaces and study some of the properties .

Keywords: supra topological space, supra α– open set, supra α– connected.

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حول الترابط الفوقي من النمط α في الفضاءات التبولوجية الفوقية

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الملخص

الغرض من هذا البحث هو تقديم مفهوم الترابط الفوقي من النمط **α** في الفضاء التبولوجي الفوقي ودرسنا بعض الخصائص.

الكلمات الدالة: الفضاء التبولوجي الوفقي، المجموعة المفتوحة الفوقية من النمط α، الترابط الفوقي من النمط α.

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1. Introduction:

The concept of supra topology was introduced by Mashhour AS et al. [1] in the year 1983. Also they studied about $S-$ continuous functions and S^*- continuous functions. In 2008, Devi R et al [2] introduced the concept of supra $\alpha-$ open sets and supra $\alpha-$ continuous maps. Ghufran A and Taha H [3] introduced the concept of supra $\alpha-$ compactness in supra topological spaces.

In this paper, we introduces the concept of supra $\alpha-$ connectedness and investigate about their relationships using the concept of continuity.

2. Preliminaries:

The aim of this paper is to study what is called the Supra $\alpha-$ connectedness as well as the effect of some kinds of mapping on its. So we will need the following results and definitions.

Definition 2.1.[1]. Let X be a non-empty set. Let $\mu \subseteq P(X) = \{A : A \subseteq X\}$. Then μ is called a supra topology on X if $\emptyset \in \mu, X \in \mu$. If $\Upsilon_\lambda \in \mu$ for every $\lambda \in \Lambda$ where Λ is an arbitrary set, then $\bigcup_{\lambda\in\Lambda} \Upsilon_\lambda \in \mu$. The pair (X,μ) is called a supra topological space. Each element $A \in \mu$ is called a supra open set in (X,μ). The complement of A is denoted by A^c =X – A, which is called a supra closed set in (X,μ).

Definition 2.2.[1]. Let (X,μ) be a supra topological space. The supra closure of a set A which is defined by supra– $cl(A) = \cap\,\{B \subseteq X : B$ is a supra closed set in X such that $A \subseteq B\}$.

The supra interior of a set A is denoted by supra– $Int(A)$ and is defined by supra– $Int(A) = \cup\,\{U \subseteq X : U$ is a supra open set in X such that $U \subseteq A\}$.

Definition 2.3.[1]. Let $(X,\mathcal{T})$ be a topological space and μ be a supra topology on X. We call μ a supra topology associated with $\mathcal{T}$ if $\mathcal{T} \subseteq$.

Definition 2.4.[2]. Let (X,μ) be a supra topological space. A subset A of X is called a supra $\alpha-$ open set in X if $A \subseteq \text{supra int}\left(\text{supra cl}(\text{supra int}(A))\right)$ if A is supra open set. The complement of supra $\alpha-$ open set is called a supra $\alpha-$ closed set.

Definition 2.5.[2]. Let (X,μ) be a supra topological space. The supra $\alpha-$ closure of a set A is denoted by supra– $\alpha-$ $cl(A)$, and is defined as follows:

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Supra$-$ $\alpha-$ $cl(A) = \cap \{B \subseteq X : B$ is supra $\alpha-$ closed set in X such that $A \subseteq B\}$.

The supra $\alpha-$ interior of a set A is denoted by supra$-\alpha-$ $Int(A)$, and is defined by supra$-$ $\alpha-$ $Int(A) = \cup \{U \subseteq X : U$ is supra $\alpha-$ open set in X such that $U \subseteq A\}$. Clearly it is obvious that supra$-$ $\alpha-$ $cl(A)$ is a supra $\alpha-$ closed set. In the same way, supra$-$ $\alpha-$ $Int(A)$ is supra $\alpha-$ open set.

Throughout this paper, $(X, \mathcal{T})$ and $(Y, \mathcal{T}^*)$ will denoted topological spaces by the researchers. Where μ and μ^* will be their associated supra topologies with $\mathcal{T}$ and $\mathcal{T}^*$ respectively is that $\mathcal{T} \subseteq \mu$ and $\mathcal{T}^* \subseteq \mu^*$.

Theorem 2.6.[2]. Let (X, μ) be a supra topological space. Then every supra open set in X is supra $\alpha-$ open set in X.

The converse of the theorem (2.6) need not be true as shown by the following example.

Example 2.7.[2]. Suppose $X = \{a, b, c\}$ and have the supra topology $\mu = \{\varphi, X, \{a\}\}$. The set $\{a, b\} \notin \mu$, so the set $\{a, b\}$ is not supra open set in (X, μ). Now since it clearly follows that supra$-$ $Int[supra{-}\ cl[supra{-}\ Int(\{a, b\})]] =$ supra$-$ $Int[\text{supra} - cl(\{a\})] =$ supra$-$ $Int[(X)] = X$ Therefore it follows that $\{a, b\}$ is a supra $\alpha-$open set in (X, μ) .

Definition 2.8.[2]. A function $f: (X, \mu) \to (Y, \mu^*)$ is called a supra $\alpha-$ continuous function if the inverse image of each supra open set in Y is a supra $\alpha-$ open set in X .

Definition 2.9 [3] A function $f: (X, \mu) \to (Y, \mu^*)$ is called $i-$ supra $\alpha-$ continuous if $f^{-1}(V)$ of each supra $\alpha-$ open subset of Y is supra $\alpha-$ open subset of X .

Definition 2.10 [3] A function $f: (X, \mu) \to (Y, \mu^*)$ is called strongly supra $\alpha-$ continuous if the inverse image of every supra $\alpha-$ open subset of Y is supra open in X .

Definition 2.11 [3] A function $f: (X, \mu) \to (Y, \mu^*)$ is called perfectly supra $\alpha-$ continuous if the inverse image of every supra $\alpha-$ open subset of Y is both supra open and supra closed in X .

Definition 2.12 [3] A function $f: (X, \mu) \to (Y, \mu^*)$ is called totally supra $\alpha-$ continuous if the inverse image of every supra open set in Y is both supra $\alpha-$ closed and supra $\alpha-$ open in X .

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3. Supra α– Connected:

Definition 3.1.[4]. A supra topological space (X,μ) is said to be supra connected if X cannot be written as a disjoint union of two non- empty supra open subsets of X. A subset of (X,μ) is supra connected if it supra connected a subspace.

Definition 3.2 A supra topological space (X,μ) is said to be supra $\alpha-$ connected if X cannot be written as disjoint union of two non-empty supra $\alpha-$ open sets. A subset of (X,μ) is supra $\alpha-$ connected if it is supra $\alpha-$ connected as subspace.

Theorem 3.3

Every supra connected is supra $\alpha-$ connected space .

Proof: Let X be supra connected. To show that X is supra $\alpha-$ connected. Since X is supra connected. Then $X \neq A \cup B$ where A and B are disjoint non-empty supra open sets. Then by Theorem (2.6), we have A and B are disjoint non-empty supra $\alpha-$ open sets. Thus X is supra $\alpha-$ connected.

Theorem 3.4

If $f:(X,\mu) \rightarrow (Y,\mu^*)$ be a surjective and supra $\alpha-$ continuous mapping. Let X be supra $\alpha-$ connected , then Y is supra connected .

Proof:

Suppose Y is not supra connected .

Let $Y = A \cup B$, where A and B are disjoint non- empty supra open sets in Y. Since f is supra $\alpha-$ continuous surjective map , then $X = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non- empty supra $\alpha-$ open sets in X.This contradicts the fact that X is supra $\alpha-$ connected. Hence Y is supra connected.

Theorem 3.5

Let $f:(X,\mu) \longrightarrow (Y,\mu^*)$ be a surjective and strongly supra $\alpha-$ continuous mapping. Let X be supra connected space. Then Y is supra $\alpha-$ connected space.

Proof: Suppose Y is not supra $\alpha-$ connected. Let $Y = A \cup B$, where A and B are disjoint non- empty supra $\alpha-$ open sets in Y.

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Since f is strongly supra $\alpha-$ continuous surjective map, then $X=f^{-1}(A)\cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non- empty supra open sets in X. This contradicts the fact that X is supra connected. Hence Y is supra $\alpha-$ connected.

Theorem 3.6

If $f:(X,\mu)\longrightarrow(Y,\mu^*)$ be a surjective and perfectly supra $\alpha-$ continuous map and X is supra connected, then Y is supra $\alpha-$ connected .

Proof: Suppose Y is not supra $\alpha-$ connected. Let $Y=A\cup B$, where A and B are disjoint non- empty supra $\alpha-$ open sets in Y. Since f is perfectly supra $\alpha-$ continuous surjective map, then $X=f^{-1}(A)\cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non- empty supra open sets and supra closed set in X. This contradicts the fact that X is supra connected. Hence Y is supra $\alpha-$ connected .

Theorem 3.7

If $f:(X,\mu)\longrightarrow(Y,\mu^*)$ be a surjective and $i-$ supra $\alpha-$ continuous map and X is supra $\alpha-$ connected , then Y is supra $\alpha-$ connected .

Proof: Suppose Y is not supra $\alpha-$ connected. Let $Y=A\cup B$, where A and B are disjoint non- empty supra $\alpha-$ open sets in Y. Since f is $i-$ supra $\alpha-$ continuous surjective map, then $X=f^{-1}(A)\cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty supra $\alpha-$ open sets in X. This contradicts the fact that X is supra $\alpha-$ connected. Hence Y is supra $\alpha-$ connected.

Theorem 3.8

If $f:(X,\mu)\longrightarrow(Y,\mu^*)$ be a surjective and totally supra $\alpha-$ continuous map and X is supra $\alpha-$ connected, then Y is supra connected.

Proof: Suppose Y is not supra connected. Let $Y=A\cup B$, where A and B are disjoint non- empty supra open sets in Y. Since every supra open set is supra $\alpha-$ open set, A and B are disjoint non- empty supra $\alpha-$ open set in Y. Since f is totally supra $\alpha-$ continuous surjective map, $X=f^{-1}(A)\cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-

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empty is both supra $\alpha-$ open and supra $\alpha-$ closed in X. This contradicts the fact that X is supra $\alpha-$ connected. Hence Y is supra connected .

Theorem 3.9

Let C and D be subsets of a supra topological space X. Assume that C is supra $\alpha-$ connected and $C \subset D$. Further assume that U and V form a separation of D in X. Then either $C \subset U$ or $C \subset V$.

Proof: Suppose that neither $C \subset U$ nor $C \subset V$. Then $U \cap C \neq \emptyset$ and $V \cap C \neq \emptyset$. It follows that U and V form a separation of C in X, contradicting that C is supra $\alpha-$ connected. Therefore either $C \subset U$ or $C \subset V$.

Theorem 3.10

Let C be a supra $\alpha-$ connected subspace in X , and assume that $C \subseteq A \subseteq$ supra $\alpha-$ $cl(C)$, then A is also supra $\alpha-$ connected .

Proof: Suppose that A is not supra $\alpha-$ connected in X, and let U and V form a separation of A in X. Then by 3.9, either $C \subset U$ or $C \subset V$. We may assume, without loss of generality, that $C \subset U$. Hence $C \cap V = \varphi$. But, since U and V form a separation of A in X, it follows that $A \cap V \neq \varphi$. Pick $x \in A \cap V$. Now, $x \in A$ and $A \subset$ supra $\alpha-$ $cl(C)$ imply $x \in$ supra $\alpha-$ $cl(C)$. But $x \in V$, an open set in X which is disjoint from C. So x cannot be in the supra $\alpha-$ closure of C, yielding a contradiction. Thus, it follows that A is supra $\alpha-$ connected.

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