المعركة العشوائية المتجانسة

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المستخلص

نموذج المعركة العشوائية بتوزيع عشوائي للفترة مابين حادثتين (قتل) هو اشمل من نماذج لانكستر المحددة.

ولكن لسوء الحظ الحل التحليلي للمعركة يتطلب حسابات كبيرة مما يجعله من المستحيل استخدامه في المعركة.

الحل التقريبي لنموذج معركة عشوائية تم تناوله في هذه الدراسة.

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4. Conclusions

A fast and reliable approximation algorithm to solve the homogeneous stochastic combat model has been developed *in this study. The algorithm* is based *on solving* a set of approximating the kill rate of one combatant conditioned on the state of the system.

A simulation may be used to generate randomly battles to evaluate the approximation algorithm for accuracy and computation time.

References

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resulting equations set will give us an approximate solution of the stochastic combat.

3. Results

A Q-Basic program is written to solve the Kolmogorov Equations using numerical method. The program computes the state probabilities Pa,b(t)'s as a function of time. The state probabilities are used to compute the important combat statistics.

For example suppose ra= 0.02/ unit time,

r_b =0.04/ unit time, $a_o = 4$ and $b_o = 8$ with $a_f = b_f = 0$. Then the results will be

The mean of the battle duration = 13.35

The standard deviation of the battle duration = 6.56

The mean number of survivors at A side = 7.86E -04

The SD of the number of survivors at A side = 2.5E-03

The mean number of survivors at B side = 7.322

The SD of the number of survivors at A side = 0.757

The probability A wins is = 2.66E-04

The probability B wins is = 0.99262

If we use deterministic Lanchestor model the results will be (3).

The battle duration = 13.06376

The number of survivors at A side = 0

The number of survivors t B side = 7.4833

$$\frac{dp\ (t)}{dt} = -\ P_{a,b}\ (t)\ [ar_A(t|\ a,b)\ +\ br_B(t|\ a,b)]\ +$$
$$P_{a,b+1}(t)ar_A\ (t|\ a,b+1) + P_{a+1,}\ b(t|\ a+1,b)\ . \qquad (4)$$

$$p_{a,b}(t) = e^{-(ar_A+br_B)t}\int_0^t (p_{a,b+1}(x)ar_A + p_{a+1,b}(x)br_B)e^{(ar_A+br_B)x}dx$$

5- For states (a , b_f) , Where a_f < a $\leq$ a_o ,

$$\frac{dp_{a,b_f}\ (t)}{dt} = P_{a,bf+1}\ (t)ar_A\ (t\ |\ a\ ,b_f + 1)\ . \qquad (5)$$

$$p_{a,b_f}(t) = \int_0^t p_{a,b_{f+1}}(x)ar_A dx$$

6- For states (a_f,b) , Where b_f < b $\leq$ b_o ,

$$\frac{dp_{a_f,b}\ (t)}{dt} = P_{af+1b}\ (t)\ br_B\ (t\ |\ a_{f+1}\ ,b). \qquad (6)$$

$$p_{a_f,b}(t) = \int_0^t p_{a_{f+1},b}(x)br_B dx$$

And finally the initial conditions are

$P_{ao,bo}\ (0) = 1$,

$P_{a,b}\ (0) = 0$, for all other states . (7)

If the conditional kill rates r_G (t/a,b)(G=A,B) are available for all states, solving Eqs. (1)(6) together with the initial condition (7) would give the exact solution of the stochastic combat. But unfortunately, the functions r_G (t| a,b) are usually very difficult to obtain except for the initial state (a ,b) and some boundary states. On the other hand, if we can approximate functions r_G (t| a, b) and use the approximations in Eqs. (1)-(6), then solving the

2-For states (a_0,b) . Where $b_f < b < b_0$.

$$\frac{dp_{a_0,b}(t)}{dt} = -P_{a_0,b}(t)\,[a_0 r_A(t|\,a_0,b) + b r_B(t|\,a_0,b)] + P_{a_0,b+1}(t)\,a_0 r_A(t\,|\,a_0,b+1)\,. \qquad (2)$$

$$p_{a_0,b}(t) = e^{-\int_0^t R(x)dx}\left[\int_0^t Q(x)e^{\int_0^x R(u)du}dx\right]$$

where

$R(x)=a_0 r_A + b r_B$

$Q(x)= p_{a_0,b+1}(t)a_0 r_A$

$$p_{a_0,b}(t) = e^{-\int_0^t (a_0 r_A + b r_B)dx}\left[\int_0^t p_{a_0,b+1}(x)a_0 r_A e^{\int_0^x (a_0 r_A + b r_B)du}dx\right]$$

$$= e^{-(a_0 r_A + b r_B)t}\int_0^t p_{a_0,b+1}(x)a_0 r_A e^{(a_0 r_A + b r_B)x}dx$$

3-For states $(a\,,\,b_0)$. where $a_f < a < a_0$,

$$\frac{dp_{a,b_0}(t)}{dt} = -P_{a,b_0}(t)\,[a r_A(t|\,a,b_0) + b_0 r_B(t|\,a,b_0)] + P_{a+1,b_0}(t)\,b_0 r_B(t|\,a+1,b_0). \qquad (3)$$

$$p_{a,b_0}(t) = e^{-(a r_A + b_0 r_B)t}\int_0^t p_{a+1,b_0}(x)b_0 r_B e^{(a r_A + b_0 r_B)x}dx$$

4-For states (a,b_0) . Where $a_f < a < a_0$ and $b_f < b < b_0$,

1. The mean and standard deviation of the battle duration time T_D.
2. The means and standard deviations of the number of survivors, A (x) and B(X), of each side.
3. The winning probabilities for each side, P [A] and P [B].

Therefore, solving the stochastic combat essentially means obtaining the state probabilities.

1.3Kill Rate

Kill rate and kill probability are the key concepts in developing our approximation. In particular, being able to express the probability that one side is going to make a kill during time interval $(t, t+\Delta t)$ given that the combat is in state (a,b) at time t enables us to establish exact differential equations of the state probabilities on which our approximation is based.

2-The Exact Kolmogorov Equations

Using the standard techniques from probability theory to derive differential-difference equations, together with the fact that each combatant is firing independently, it can be shown , in terms of the conditional state-dependently, kill rates , that the exact Kolmogorov equations for this combat are as follows (1).

1- For the initial state (a_o, b_o) .

$$\frac{dp_{a_0,b_0}(t)}{dt} = -P_{a_0,b_0}(t)\ [\ a_o\, r_A(t \mid a_o, b_o) + b_o r_B(t \mid a_o, b_o)].$$

(1)

$$p_{a_0,b_0}(t) = \exp[-\int_0^t (a_0 r_A + b_0 r_B)dx]$$
$$= \exp[-(a_0 r_A + b_0 r_B)t]$$

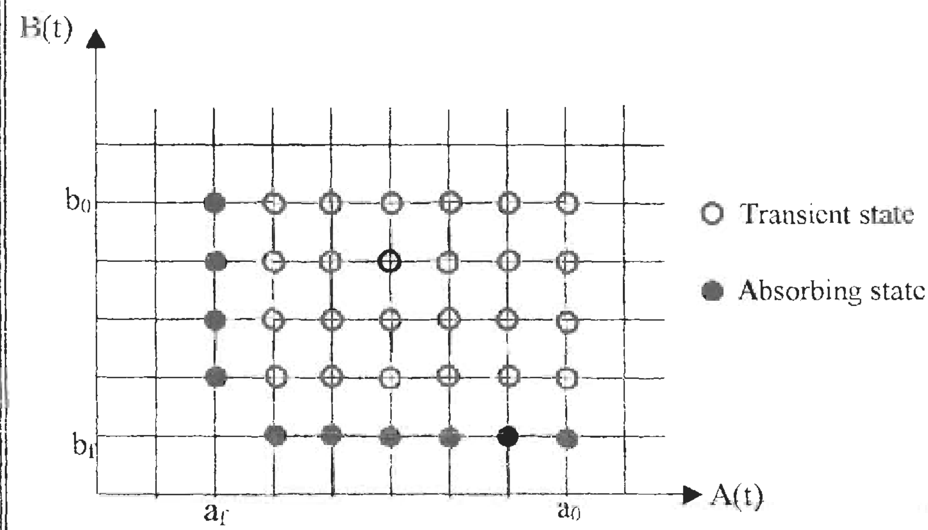

Figure 1. State Space of Homogenous Stochastic Combat

Combat can represented by a finite number of lattice points on a two-dimensional space, as indicated in Figure 1. The combat starts at the initial state (a_0 , b_0) and evolves in a discrete manner, transferring from state to state, and finally terminates in one of the absorbing states. The state probability $P_{a,b}$ (t) is defined as the probability that the combat is in stat (a , b) at time t . the stat probability $P_{a,b}$ (t)'s, as functions of time, contain all the pertienent information about this stochastic combat. In fact, in terms of the state probabilities we can write expressions for the following important combat statistics.

at which time he immediately shifts to a new target picked at random and resumes firing.

3. Each combatant on the B side does the same to its opponent.

4. All the combatants on one side are visible and within the weapon range of all the combatants on the other side.

5. The ammunition supply is unlimited.

6. Every combatant fires independently.

7. When the force level on the A side reaches its break point, a_f or the force level on the B side reaches its break points, b_f whichever comes first, the battle will terminate.

8. All the combatants on the same side are identical; that is, they have the same interkilling time distribution (homogeneous force assumption).

1.2. State Probabilities and Combat Statistics

Let **A** (t) and B (t) be the number of live combatants on the **A** side and on the B side respectively at time **t.** We define the state of the combat at a given time by specifying the number of live combatants on each side at that time; that is, we say that the combat is in state (a , b) at time **t** when **A** (t) = **a** and B (t) =**b.** By this definition, the state space of the small –to moderate-size firefights. These models, which were called general renewal (GR) models, in some sense, are *generalizations of the Lanchester sequre-law model* because they allow the interkilling (or interfering) time of a combatant to be an arbitrary random variable.

Simulation also has been used to produce solutions for battles of the m-on-n GR model .Because relative error of a simulation results is inversely proportional to the square root of the sample size, it is very time consuming to obtain a high precision simulation result.

Solving small-to moderate-size firefight model in a reasonable time is essential to studying large –scale combat because a large-scale battle is viewed as network of many small-scale engagements (2) . Because none of the available solution techniques of the GR models are able to accomplish this, a fast reliable approximation is needed. This article is motivated by this need.

1.1. The Model

In this article we are going to consider a version of the m-on-n homogeneous interkilling stochastic combat. The basic assumptions are:

1. Two sides, A and B, conduct a continuous time engagement with initial force size a_0 on the A side and b_0 on the B side.
2. At the beginning of the battle each combatant on the A side picks a target (a live combatant on the enemy side) at random and fires. The time it takes for the combatant to kill its target, called interkilling time, is an arbitrary nonnegative continuous random variable. Each combatant fires until he is killed or makes a kill,

Homogeneous Stochastic Combat

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Abstract

The stochastic combat model with arbitrary inter killing time distribution is more general than the deterministic Lanchester model. Unfortunately, the exact analytical solution of the more general combat model requires a huge amount of computation time, which makes it practically impossible to use for battles .An approximate solution for a class of homogeneous stochastic combats with arbitrary inter killing time distribution is developed in this study.

1. Introduction

It is believed that the development of realistic combat models involving a large number of combatants will depend on successfully modeling (1) (a) the decomposition of the large battle into the separate small engagements, and (b) the attrition process in the separate small engagements. With respect to (b), a large amount of effort has been devoted to developing more realistic models of

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