

Evaluating Reliability Systems by Using Weibull & New Weibull Extension Distributions

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Abstract :

In this paper we evaluate the reliability systems by using (Weibull & New Weibull Extension) distributions, and by using both of them we determine the Reliability Function and the associated functions such as cumulative distribution function , hazard function ... etc., then we discuss the relationship between them and how effectiveness this to calculate the reliability function and associated functions.

الخلاصة :

في هذا البحث تم تقييم ثقة الأنظمة باستخدام توزيع ويبل وتوزيع ويبل الموسع الجديد ، وباستخدام هذين التوزيعين تم الحصول على دالة الثقة والدوال ذات العلاقة كدالة التجميع الاحتمالية ودالة الخطر ... الخ. ثم ناقشنا العلاقة بين هذين التوزيعين وكيفية تأثير ذلك على إيجاد دالة الثقة والدوال المرتبطة بها.

1.Introduction

Many researchers work to compute the reliability systems by using Weibull distributions as David (2010) , Tang (2004) and Xie (2003).

When manufacturers claim that their products are very reliable they essentially mean that the products can function as required for a long period of time, when used as specified. In order to assess and improve the reliability of an item we need to be able to measure it. Thus a more formal definition is required.

The Weibull distribution (Weibull, 1951), named after the Swedish Professor Waloddi Weibull, is perhaps the most frequently used life time distribution for lifetime data analysis mainly because of not only its flexibility of analyzing diverse types of phenomena, but also its simple and straightforward mathematical forms compared with other distributions.

The Weibull distribution is generalization from exponential distribution. This distribution is appropriate for a system or complex component made up of several parts.

New Weibull extension distribution is subsequently introduced in this paper. This model is regarded as an extension of Weibull distribution which has bathtub shaped failure rate function. It also contains an analysis of the properties of the model.

2. Lifetime Following a Weibull Distribution

When the lifetimes have a Weibull distribution then it's p.d.f. is [Quek S-T. , Ang A. H-S. ,1986] :

$$f(t) = \beta \alpha^{-\beta} t^{\beta-1} \exp \left[- \left(\frac{t}{\alpha} \right)^{\beta} \right] \quad \dots(1)$$

Where α and β are parameters.

The scale parameter, α , reflects the size of the units in which the random variable, t , is measured.

The shape parameter, β , causes the shape of the distribution to vary. By changing the value of β we can generate widely varying set of curves to model real lifetime failure distributions.

The effects of different scale and shape parameters on the Weibull distribution are shown in fig.(1,2):

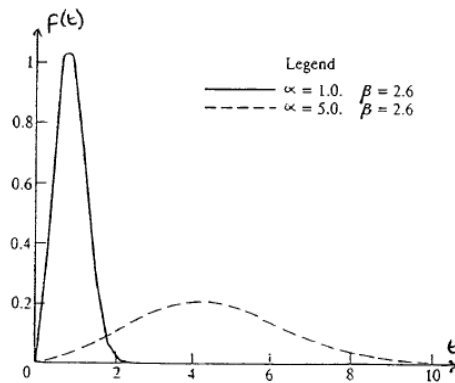


Fig.(1):Effect of different shape parameter , β

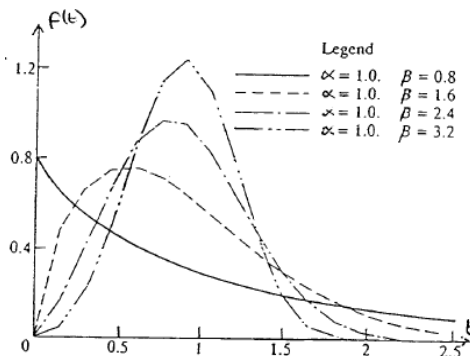


Fig.(2):Effect of different scale parameter , α

Special case:

If $\beta = 1$ then the pdf collapsing is : $f(t) = \alpha' e^{-\alpha' t}$ where $\alpha' = \frac{1}{\alpha}$
 which is an exponential distribution with rate α' (or mean $\frac{1}{\alpha'}$)

2.1 Associated functions

By depending equation (1) , we can find the following equations :

(i) Cumulative Distribution Function is given by :

$$F(t) = 1 - e^{-\left(\frac{t}{\alpha}\right)^\beta} \quad \dots(2)$$

When $R(t) = 1 - F(t)$, we get :

(ii) Reliability Function $R(t) = e^{-\left(\frac{t}{\alpha}\right)^\beta} \quad \dots(3)$

(iii) hazard function

$$h(t) = \frac{f(t)}{R(t)} \Rightarrow h(t) = \frac{\beta \alpha^{-\beta} \left[t^{\beta-1} e^{-\left(\frac{t}{\alpha}\right)^\beta} \right]}{e^{-\left(\frac{t}{\alpha}\right)^\beta}}$$

i.e $h(t) = \beta \alpha^{-\beta} t^{\beta-1} \quad \dots(4)$

(iv) Mean tie Between failure MTBF is $MTBF = \alpha \Gamma\left(1 + \frac{1}{\beta}\right) \quad \dots(5)$

where Γ denotes the gamma function. Tables of gamma functions are available to assist in calculating the mean time between failures.

If $x > 2$, $\Gamma(x) = (x-1) \Gamma(x-1)$

(v) cumulative hazard function is

$$H(t) = -\ln R(t) = -\ln \left[e^{-\left(\frac{t}{\alpha}\right)^\beta} \right] = \left(\frac{t}{\alpha}\right)^\beta \quad \dots(6)$$

2.2 Time-dependant hazard function

For a Weibull lifetime distribution, the hazard function is given by

$$h(t) = \beta \alpha^{-\beta} t^{\beta-1}$$

This will give various forms for the hazard function depending on the value of β , for more details see [Tang Yong, 2004] ,

Example (1):-

The lifetime of a component (in thousands of hours) has a Weibull distribution with $\alpha = 0.5$ and $\beta = 2$, Find the following :

- (i) the probability that the component will fail before 1000 hours of the operation.
- (ii) MTBF.

Solution :

(i) Here $R(t) = e^{-\left(\frac{t}{\alpha}\right)^\beta} = e^{-\left(\frac{t}{0.5}\right)^2}$
 Then $R(1) = e^{-4}$
 $= 0.01$

Required probability = $1 - 0.01$
 $= 0.99$

(ii) $MTBF = \alpha \Gamma\left(1 + \frac{1}{\beta}\right) = 0.5 \Gamma(1+1/2)$
 $= 0.5 \cdot (0.886227)$
 $= 0.443$ (thousands of hours)

i.e. 443 hours.

3. A New Weibull Extension Distribution

The reliability function of new Weibull extension is given by [Casteren J.V., 2001] , [Paul Barringer, P.E. , 2000]

$$R(t) = \exp\left\{ \alpha \lambda \left(1 - \exp\left(\frac{t}{\alpha}\right)^\beta \right) \right\} \quad \dots(7)$$

for any $\lambda, \alpha, \beta > 0$, $t \geq 0$. The new Weibull extension is derived from Chen’s model (for more details see [Paul Barringer, P.E.,2000]), this extension model has the Weibull distribution as a special and asymptotic case , and hence it can be considered as a Weibull extension.

The corresponding failure rate function of Weibull extension model has the following form:

$$h(t) = \lambda \beta \left(\frac{t}{\alpha}\right)^{\beta-1} \exp\left[\left(\frac{t}{\alpha}\right)^\beta\right] \quad \dots(8)$$

The shapes of the failure rate function, which can be of bathtub shape, will be demonstrated.

For the new Weibull extension distribution, the cumulative distribution function is given by:

$$F(t) = 1 - R(t) = 1 - \exp\left\{\alpha\lambda\left(1 - \exp\left(\frac{t}{\alpha}\right)^\beta\right)\right\} \quad \dots(9)$$

and the pdf is given by:

$$f(t) = \lambda\beta\left(\frac{t}{\alpha}\right)^{\beta-1} \exp\left[\left(\frac{t}{\alpha}\right)^\beta + \lambda\alpha\left(1 - \exp\left(\frac{t}{\alpha}\right)^\beta\right)\right] \quad \dots(10)$$

3.1 Characteristic of failure rate function

To study the shape of the failure rate function, We firstly take the derivative of the failure rate function and we get [Lai C. D., Min Xie and Murthy D. N. P. ,2003] , [Tang Yong ,2004]

$$h'(t) = \frac{\lambda\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-2} \exp\left[\left(\frac{t}{\alpha}\right)^\beta\right] \cdot \left[\beta\left(\frac{t}{\alpha}\right)^\beta + (\beta - 1)\right] \quad \dots(11)$$

The shape of the failure rate function depends only on the shape parameter β . Hence, the following two cases will be considered.

Case 1: $\beta \geq 1$

- i). In this case, for any $t > 0$, $h'(t) > 0$, therefore $h(t)$ is a monotonically increasing function .
- ii). $h(0) = 0$ if $\beta > 1$ and $h(0) = \lambda$, if $\beta = 1$.
- iii). $h(t) \rightarrow +\infty$ as $t \rightarrow +\infty$.

Case 2: $0 < \beta < 1$

- i). Let $h'(b_3) = 0$, then we have the equation :

$$\beta\left(\frac{b^*}{\alpha}\right)^\beta + \beta - 1 \quad \dots(12)$$

and by solving this equation, a change point of the failure rate can be obtained as

$$b^* = \alpha\left(\frac{1-\beta}{\beta}\right)^{\frac{1}{\beta}} \quad \dots(13)$$

We can see that when $0 < \beta < 1$, b_3 exist and it is finite.

When $t < b_3$, $h'(t) < 0$, the failure rate function is monotonically decreasing; when $t > b_3$, $h'(t) > 0$, the failure rate function is monotonically increasing.

Hence, the failure rate function has a bathtub shape property.

- ii). $h(t) \rightarrow +\infty$ for $t \rightarrow 0$ and $t \rightarrow +\infty$;
- iii). The change point b_3 increases as the shape parameter β decreases from 1 to 0.

Figure (3) shows the plots of the failure rate function for Weibull extension model at several different parameter combinations. From Figure (3) , we can observe the failure rate function has an increasing function when $\beta \geq 1$, and $h(t)$ is a bathtub shaped function when $0 < \beta < 1$.

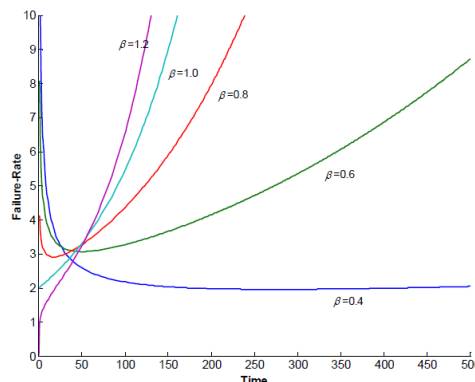


Figure (3): Plots of the failure rate function with $\lambda = 2$, $\alpha = 100$ and β changing from 0.4 to 1.2

3.2 Mean Time Between Failure

The expected time to failure of the Weibull extension distribution, or the mean time between failure (MTBF) can be expressed as [Casteren J. V. , 2001] ,[Constantin Tarcolea, Adrian Paris, Cristian Andreescu, 2008] :

$$\begin{aligned}
 MTBF &= \int_0^{+\infty} tF(t)dt = \int_0^{+\infty} R(t)dt \\
 &= \int_0^{+\infty} \exp\left\{\lambda\alpha\left[1 - \exp\left(\frac{t}{\alpha}\right)^\beta\right]\right\} dt \qquad \dots(14)
 \end{aligned}$$

Example(2):

We can solve previous example by using Weibull extension distribution with $\lambda = 1$ and we will get from eq.(7)

$$\begin{aligned}
 R(1) &= \exp\{0.5(1 - 54.5)\} \\
 &= \exp\{-26.7\} \approx 2.5 \times 10^{-12}
 \end{aligned}$$

4. Relationship Between New Weibull Extension Distribution and Weibull Distribution

The new model is related to Weibull distribution in an interesting way [C. D. Lai, Min Xie and Murthy D. N. P., 2003] ,[Constantin Tarcolea, Adrian Paris, Cristian Andreescu, 2008] , [Polpo A., Coque M. A. , Pereira CAB., 2009]

Weibull distribution can be seen as an asymptotic case of the new distribution.

When α is too large then :

$$1 - \exp\left[\left(\frac{t}{\alpha}\right)^\beta\right] = 1 - \left[1 + \left(\frac{t}{\alpha}\right)^\beta + o(t)^\beta\right] \approx -\left(\frac{t}{\alpha}\right)^\beta \qquad \dots(15)$$

Therefore, the reliability function can be approximated by

$$R(t) = \exp\left\{\lambda\alpha\left[1 - e\left(\frac{t}{\alpha}\right)^\beta\right]\right\} \approx \exp\{-\lambda\alpha^{1-\beta}t^\beta\} \qquad \dots(16)$$

which is a standard two-parameter Weibull distribution with a shape parameter of β , and a scale parameter of $\alpha^{\beta-1} / \lambda$. That is, in the limiting case when α approaches infinity while $\alpha^{\beta-1} / \lambda$ remains constant, the new distribution remains a standard two parameter Weibull distribution. In this limiting case, the Weibull extension model is capable of handling both increasing and decreasing failure rates, which are in fact, special cases of bathtub curve.

A further special case is, when $\beta = 1$, α is large enough, we have

$$R(t) = \exp\left\{\lambda\alpha\left[1 - e\left(\frac{t}{\alpha}\right)\right]\right\} \approx \exp\{-\lambda t\}$$

Hence, the model reduces to the exponential distribution with parameter λ . It is well-known that the exponential distribution has a constant failure rate, which is again, a very special case of bathtub curve [Tang Yong , 2004] .

5. Conclusions

- One of the good properties of Weibull distribution is that it can have different monotonic types of hazard rate shapes so that it can be applied to different kinds of products. From Equation (4), it is clear that the shape of hazard rate function depends solely on the shape parameter.
- There are some relationships between Weibull distribution and other distributions. For example when $\beta = 1$, it is reduced to exponential distribution. When $\beta = 2$, it has the form of Rayleigh distribution. When $\beta > 3.6$, Weibull distribution is very similar to normal distribution and When the scale parameter α becomes very large or approaches infinity Weibull distribution can be seen as an asymptotic case of the new Extension Weibull distribution.

- The Reliability System by Using Weibull distribution is larger than the new Extension Weibull distribution as in examples (1)& (2) respectively.

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