

# مقدرات الاختبار الأولي المقلصة ذات المرحلة الواحدة لمعلمة القياس للتوزيع الأسي

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## الخلاصة

استخدم أسلوب الاختبار الأولي المتقلص ذو المرحلة الواحدة في تقدير معلمة القياس ( $\theta$ ) للتوزيع الأسي عند توافر معلومات أولية عن  $\theta$  بشكل قيمة ابتدائية  $\theta_0$ . حيث اقترح مقدر الاختبار الأولي  $\tilde{\theta}$  في تقدير  $\theta$  بالاعتماد على نتيجة اختبار الفرضية  $H_0: \theta = \theta_0$  ضد الفرضية  $H_A: \theta \neq \theta_0$  بمستوى معنوية  $\alpha$ . ففي حالة قبول الفرضية  $H_0$  سيكون المقدر المقترح  $\tilde{\theta}$  هو المقدر المتقلص  $\tilde{\theta} = \psi_1(\hat{\theta})(\hat{\theta} - \theta_0) + \theta_0$  حيث ان  $\psi_1(\hat{\theta})$  هي عامل او دالة تقلص ربما تكون دالة الاختبار عند اختبار  $H_0$  أو ربما تكون ثابتة حيث ان،  $0 \leq \psi_1(\hat{\theta}) \leq 1$  و  $\hat{\theta}$  هو مقدر غير متحيز. وفي حالة رفض الفرضية  $H_0$  فيكون المقدر  $\tilde{\theta}$  هو المقدر المتقلص  $\tilde{\theta} = \psi_2(\hat{\theta})(\hat{\theta} - \theta_0) + \theta_0$ ، حيث ان  $0 \leq \psi_2(\hat{\theta}) \leq 1$  وهي دالة تقلص موزونة أيضاً. تم اختيار دوال تقلص موزونة  $(\psi_i(\hat{\theta}), i = 1, 2)$  بشكل مناسب ومن ثم تم اشتقاق معادلات التحيز ومعدل مربعات الخطأ للمقدرات المقترحة. قورنت المقدرات المقترحة مع المقدر الكلاسيكي ( $\hat{\theta}$ ) وبعض المقدرات المقترحة في الدراسات السابقة لبيان كفاءتها.

## Abstract

A preliminary test single stage Shrinkage (PTSSS) techniques is used for estimation the scale parameter  $\theta$  of an exponential distribution when a prior knowledge about  $\theta$  is a available in the form of initial estimate  $\theta_0$  of  $\theta$ . It is proposed to estimate  $\theta$  by a testimator  $\tilde{\theta}$  that is based upon the result of a test of the hypothesis  $H_0: \theta = \theta_0$  against the hypothesis  $H_A: \theta \neq \theta_0$  with level of significance  $\alpha$ . If  $H_0$  is accepted we take  $\tilde{\theta} = \psi_1(\hat{\theta})(\hat{\theta} - \theta_0) + \theta_0$ , where the weighting factor  $\psi_1(\hat{\theta})$  is a function of the test statistic for testing  $H_0$  or may be constant such that  $0 \leq \psi_1(\hat{\theta}) \leq 1$ . However if  $H_0$  is rejected we take  $\tilde{\theta} = \psi_2(\hat{\theta})(\hat{\theta} - \theta_0) + \theta_0$ , where  $0 \leq \psi_2(\hat{\theta}) \leq 1$  and  $\hat{\theta}$  is the classical estimator of  $\theta$  (MLE). Choosing the weighting factor  $\psi_i(\hat{\theta})$ , ( $i = 1, 2$ ) appropriately, an expression for the mean squared error (MSE) and Bias of  $\tilde{\theta}$  are derived and comparisons are made with classical estimator ( $\hat{\theta}$ ) in the sense of efficiency and last earlier studies.



## 2. Introduction

Exponential distribution is one of the most useful and widely exploited model, Epstein [1] remarks that the exponential distribution plays as important a role in life experiments as the part played by the normal distribution in agricultural experiments.

The most widely model is one parameter exponential distribution with probability density function

$$f(t) = \frac{1}{\theta} \exp\left(\frac{-t}{\theta}\right),$$

where  $t \geq 0$ ,  $\theta > 0$  is the average or the mean life or mean time to failure (MTTF) and it is also acts as scale parameter.

In this paper we consider Shrinkage technique which introduced by Thompson [2] as follows

$$\tilde{\theta} = \varphi(\hat{\theta})(\hat{\theta} - \theta_0) + \theta_0 \quad \dots(1)$$

where  $\theta_0$  is a prior information about  $(\theta)$  and  $0 \leq \varphi(\hat{\theta}) \leq 1$ , represents a Shrinkage weight factor specifying the degree of belief in  $\theta_0$ . We used the form (1) above to estimate the scale parameter  $\theta$  of an exponential distribution.

In case  $\varphi(\hat{\theta})$  is chosen as follows

$$\varphi(\hat{\theta}) = \begin{cases} \psi_1(\hat{\theta}) & , \text{if } \hat{\theta} \in R \\ \psi_2(\hat{\theta}) & , \text{if } \hat{\theta} \notin R \end{cases} \quad \dots(2)$$

where  $R$  is the preliminary test region of acceptance of size  $\alpha$  for testing the hypothesis  $H_0 : \theta = \theta_0$  against the hypothesis  $H_A : \theta \neq \theta_0$  using the test statistic

$T(\hat{\theta}/\theta) = \frac{2n\hat{\theta}}{\theta_0}$  and  $\hat{\theta}$  is the classical estimator of  $\theta$  (MLE), then the estimator

which is defined in (1) will be written as below

$$\tilde{\theta} = \begin{cases} \psi_1(\hat{\theta} - \theta_0) + \theta_0 & , \text{if } \hat{\theta} \in R \\ \psi_2(\hat{\theta} - \theta_0) + \theta_0 & , \text{if } \hat{\theta} \notin R \end{cases}$$

...(3)

where  $\psi_i(\hat{\theta})$ ,  $i=1,2$ ,  $0 \leq \psi_i(\hat{\theta}) \leq 1$  represents as Shrinkage weight factors which may be a functions of  $\hat{\theta}$  or may be constants.

The resulting estimator (3) is known as preliminary test single stage Shrinkage estimator (PTSSSE).

Several authors had studied the estimator defined in (3) for special distribution and for different parameters also for complete and censored samples as well as for estimate the parameters of linear regression model. For example see [3], [4], [5], [6] and [7].



### 3. Preliminary Test Single Stage Shrinkage Estimators (PTSSSE)

Using the form (3), we proposed two preliminary single stage Shrinkage estimators  $\tilde{\theta}_1$  and  $\tilde{\theta}_2$  for estimate the scale parameter  $\theta$  of an exponent distribution when a prior information  $(\theta_0)$  available about the actual value  $(\theta)$ .

$$\tilde{\theta}_1 = \begin{cases} \theta_0 & , \text{if } \hat{\theta} \in R \\ \hat{\theta} & , \text{if } \hat{\theta} \notin R \end{cases} \quad \dots(4)$$

i.e.  $\psi_1(\hat{\theta}) = 0$  and  $\psi_2(\hat{\theta}) = 1$  in (3).

and

$$\tilde{\theta}_2 = \begin{cases} \theta_0 & , \text{if } \hat{\theta} \in R \\ k(\hat{\theta} - \theta_0) + \theta_0 & , \text{if } \hat{\theta} \notin R, \end{cases}$$

... (5) i.e.  $\psi_1(\hat{\theta}) = 0$  and  $\psi_2(\hat{\theta}) = k$  in (3)

where  $R$  defined in (2) such that

$$R = \left[ \frac{\theta_0}{2n} X_{1-\alpha/2, 2n}^2, \frac{\theta_0}{2n} X_{\alpha/2, 2n}^2 \right]$$

... (6) Assume that  $R = [a, b]$ ,  $a < b$

$$\text{i.e. } a = \frac{\theta_0}{2n} X_{1-\alpha/2, 2n}^2, b = \frac{\theta_0}{2n} X_{\alpha/2, 2n}^2 \quad \dots(7)$$

where  $X_{1-\alpha/2, 2n}^2$  and  $X_{\alpha/2, 2n}^2$  are respectively the lower and upper  $100(\alpha/2)$  percentile point of chi-square distribution with  $(2n)$  degree of freedom.

The expressions for Bias  $[B(\cdot)]$  of the estimators  $\tilde{\theta}_1$  and  $\tilde{\theta}_2$  are respectively as follows

$$B(\tilde{\theta}_1 | \theta, R) = E(\tilde{\theta}_1) - \theta \\ = \int_R [\theta_0 - \theta] f(\hat{\theta}) d\hat{\theta} + \int_{\bar{R}} [\hat{\theta} - \theta] f(\hat{\theta}) d\hat{\theta}$$

where  $\bar{R}$  is the complement region of  $R$  in real space and  $f(\hat{\theta})$  is a p.d.f. of  $\hat{\theta}$  which has the following forms

$$f(\hat{\theta}) = \frac{[\hat{\theta}]^{n-1} \exp[-n\hat{\theta}/\theta]}{\Gamma(n)(\theta/n)^n} \quad \text{for} \quad 0 < \hat{\theta} < \infty$$

... (8) we conclude,

$$B(\tilde{\theta}_1 | \theta, R) = \theta \{ \lambda j_0(a^*, b^*) - j_1(a^*, b^*) \}$$

$$\dots(9) \quad \text{where} \quad j_\ell(a^*, b^*) = \frac{1}{n^\ell \Gamma(n)} \int_{a^*}^{b^*} y^\ell y^{n-1} e^{-y} dy, \quad \text{for} \quad \ell = 0, 1, 2, \dots$$

$$\dots(10) \quad \lambda = \theta_0 / \theta, a^* = \lambda X_{1-\alpha/2, 2n}^2 \quad \text{and} \quad b^* = \lambda X_{\alpha/2, 2n}^2$$



...(11) and,

$$B(\tilde{\theta}_2 | \theta, R) = E(\tilde{\theta}_2 - \theta) \\ = \theta \{ (\lambda - 1)(j_0(a^*, b^*) + 1) - k(j_1(a^*, b^*) + j_0(a^*, b^*)\lambda) \} \quad \dots(12)$$

The expression for mean square error (MSE) of  $\tilde{\theta}_1$  and  $\tilde{\theta}_2$  are respectively given as below:

$$MSE(\tilde{\theta}_1 | \theta, R) = E(\tilde{\theta}_1 - \theta)^2 \\ = \theta^2 \left\{ (\lambda - 1)^2 j_0(a^*, b^*) + \frac{1}{n} - [j_2(a^*, b^*) - 2j_1(a^*, b^*) + j_0(a^*, b^*)] \right\}$$

...(13) and

$$MSE(\tilde{\theta}_2 | \theta, R) = \theta^2 \{ k^2 / n + (\lambda - 1)^2 (k^2 + 2) - 2k(\lambda - 1)[\lambda(1 - j_0(a^*, b^*) + j_1(a^*, b^*) - 1) - k^2(j_2(a^*, b^*) - 2j_1(a^*, b^*)\lambda + \lambda^2 j_0(a^*, b^*))] \}$$

...(14) The Relative Efficiency of estimator  $\tilde{\theta}_i$  ( $i=1,2$ ) with respect to the classical estimator ( $\hat{\theta}$ ) is defined as below

$$R.Eff(\tilde{\theta}_i | \theta, R) = \frac{MSE(\hat{\theta})}{MSE(\tilde{\theta}_i)} = \frac{\theta^2/n}{MSE(\tilde{\theta}_i)}$$

...(15)

#### 4. Numerical Results

The computations of relative Efficiency [R.Eff( $\cdot$ )] and Bias ratio [B( $\cdot$ )] were used for the estimators  $\tilde{\theta}_1$  and  $\tilde{\theta}_2$ . These computations were performed for  $\alpha = 0.01, 0.05, 0.1$ ,  $\lambda = 0.25(0.25)2$ ,  $n = 4, 6, 8, 12$  and  $k = 0.1, 0.3, 0.5$ . Some of these computations are given in four annexed tables. The observation mentioned in the tables lead to the following results:

1. R.Eff( $\cdot$ ) of  $\tilde{\theta}_i$  ( $i=1,2$ ) are adversely proportional with small value of  $\alpha$ ,  $k$  and  $n$ .
2. R.Eff( $\cdot$ ) of  $\tilde{\theta}_i$  ( $i=1,2$ ) are maximum when  $\theta = \theta_0(\lambda = 1)$  for all  $\alpha$ ,  $k$  and  $n$ .
3. R.Eff of  $\theta_2$  is better than R.Eff( $\cdot$ ) of  $\tilde{\theta}_1$  specially when  $\theta = \theta_0$ .
4. Bias ratio [B( $\cdot$ )] of  $\tilde{\theta}_i$  ( $i=1,2$ )  $[B(\cdot) = \frac{Bias(\tilde{\theta}_i)}{\theta}, i = 1, 2]$  are reasonably small when  $\theta \sqcup \theta_0$ , otherwise start to be maximum for all  $\alpha$  and  $n$ .
5. Bias ratio [B( $\cdot$ )] of  $\tilde{\theta}_i$  ( $i=1,2$ ) are at most increasing function with  $\alpha$ .
6. Effective Interval [the values of  $\lambda$  that makes R.Eff. greater than one] for  $\tilde{\theta}_i$  ( $i = 1, 2$ ) is [0.5, 1.5].



7.  $\tilde{\theta}_2$  has higher R.Eff. with small value of k.  
i.e. R.Eff. of  $\theta_2$  is decreases function w.r.t. k.
8. Bias ratio of  $\tilde{\theta}_2$  is increasing function with respect to k and sample size n.
9. The suggested estimators  $\tilde{\theta}_i$  (i=1,2) are more efficient than the estimators introduced by [8], [9] and [10] in the sense of mean square error and Relative efficiency.

## 5. Conclusions

1. The proposed estimator  $\tilde{\theta}_i$  (i=1,2) are consistent .  
i.e  $\lim_{n \rightarrow \infty} \text{MSE}(\tilde{\theta}_i | \theta, R) = 0$
2. The proposed estimator  $\tilde{\theta}_i$  (i=1,2) are dominate to  $\hat{\theta}$  .  
i.e  $\lim_{n \rightarrow \infty} [\text{MSE}(\tilde{\theta}_i | \theta, R) - \text{MSE}(\hat{\theta})] \leq 0$
3. The considered estimator  $\tilde{\theta}_i$  (i=1,2) has high efficiency when  $\theta \approx \theta_0 (\lambda = 1)$ .
4. The considered estimator  $\tilde{\theta}_i$  (i=1,2) has lower bias when  $\theta \approx \theta_0 (\lambda = 1)$ .

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Table (1) Shown Bias Ratio [B(.)] and Relative Efficiency [R.Eff.(.)] of  $\tilde{\theta}_1$ 

| $\alpha$ | $\lambda$ |                  | 0.25                 | 0.75                | 1                    | 1.25                 | 1.75                  | 2                     |
|----------|-----------|------------------|----------------------|---------------------|----------------------|----------------------|-----------------------|-----------------------|
|          | n         |                  |                      |                     |                      |                      |                       |                       |
| 0.01     | 4         | R.Eff(.)<br>B(.) | 0.41924<br>(0.44121) | 3.3456<br>(0.2606)  | 9.3931<br>(0.035483) | 2.5037<br>(0.16552)  | 0.4844<br>(0.46998)   | 0.31292<br>(0.56854)  |
|          | 6         | R.Eff(.)<br>B(.) | 0.33482<br>(0.38347) | 2.2162<br>(0.26314) | 5.1046<br>(0.054174) | 1.6525<br>(0.1648)   | 0.44508<br>(0.25713)  | 0.34161<br>(0.26193)  |
|          | 8         | R.Eff(.)<br>B(.) | 0.2946<br>(0.33019)  | 1.6589<br>(0.26486) | 3.5676<br>(0.070868) | 1.379<br>(0.054027)  | 0.53456<br>(0.11125)  | 0.49468<br>(0.089647) |
| 0.05     | 4         | R.Eff(.)<br>B(.) | 0.48505<br>(0.2853)  | 2.396<br>(0.28444)  | 3.7688<br>(0.10626)  | 1.9607<br>(0.026597) | 0.69944<br>(0.15839)  | 0.54404<br>(0.17415)  |
|          | 6         | R.Eff(.)<br>B(.) | 0.41925<br>(0.23496) | 1.7427<br>(0.2858)  | 2.8176<br>(0.12389)  | 1.6932<br>(0.025157) | 0.85668<br>(0.033818) | 0.78732<br>(0.030896) |
|          | 8         | R.Eff(.)<br>B(.) | 0.39365<br>(0.19026) | 1.398<br>(0.28361)  | 2.5032<br>(0.12857)  | 1.6863<br>(0.044191) | 1.0242<br>(0.000678)  | 0.97988<br>(0.002065) |

Table (2) Shown Bias Ratio [B(.)] and Relative Efficiency [R.Eff.(.)] of  $\tilde{\theta}_2$ 

when k = 0.1

| $\alpha$ | $\lambda$ |                  | 0.25                 | 0.75                 | 1                    | 1.25                | 1.75                 | 2                    |
|----------|-----------|------------------|----------------------|----------------------|----------------------|---------------------|----------------------|----------------------|
|          | n         |                  |                      |                      |                      |                     |                      |                      |
| 0.01     | 4         | R.Eff(.)<br>B(.) | 0.2307<br>(0.86912)  | 1.9905<br>(0.30106)  | 939.31<br>(0.003548) | 2.0564<br>(0.29155) | 0.23005<br>(0.872)   | 0.13019<br>(1.1569)  |
|          | 6         | R.Eff(.)<br>B(.) | 0.15494<br>(0.86335) | 1.317<br>(0.30131)   | 143.7<br>(0.005417)  | 1.3917<br>(0.28565) | 0.15762<br>(0.85071) | 0.089468<br>(1.1262) |
|          | 8         | R.Eff(.)<br>B(.) | 0.11720<br>(0.85802) | 0.98389<br>(0.30149) | 78.107<br>(0.007087) | 1.0619<br>(0.2804)  | 0.12057<br>(0.8361)  | 0.068309<br>(1.109)  |
| 0.05     | 4         | R.Eff(.)<br>B(.) | 0.23545<br>(0.85353) | 1.969<br>(0.30344)   | 376.88<br>(0.010626) | 2.1627<br>(0.27766) | 0.23967<br>(0.84084) | 0.13543<br>(1.1174)  |
|          | 6         | R.Eff(.)<br>B(.) | 0.15803<br>(0.8484)  | 1.3044<br>(0.30358)  | 116.97<br>(0.012389) | 1.4644<br>(0.27248) | 0.16253<br>(0.82838) | 0.09165<br>(1.1031)  |
|          | 8         | R.Eff(.)<br>B(.) | 0.11925<br>(0.84403) | 0.97632<br>(0.30336) | 71.455<br>(0.012858) | 1.105<br>(0.27058)  | 0.12247<br>(0.82507) | 0.06895<br>(1.1002)  |



**Table (3) Shown Bias Ratio [B(·)] and Relative Efficiency [R.Eff.(·)] of  $\tilde{\theta}_2$  when k = 0.3**

| $\alpha$ | $\lambda$<br>n |                   | 0.25                 | 0.75                 | 1                    | 1.25                | 1.75                 | 2                    |
|----------|----------------|-------------------|----------------------|----------------------|----------------------|---------------------|----------------------|----------------------|
|          |                | R.Eff.(·)<br>B(·) |                      |                      |                      |                     |                      |                      |
| 0.01     | 4              | R.Eff.(·)<br>B(·) | 0.24254<br>(1.1074)  | 1.9652<br>(0.40318)  | 104.37<br>(0.010645) | 2.0922<br>(0.37466) | 0.2421<br>(1.116)    | 0.13884<br>(1.4706)  |
|          | 6              | R.Eff.(·)<br>B(·) | 0.16482<br>(1.09)    | 1.2344<br>(0.40394)  | 15.967<br>(0.016252) | 1.3787<br>(0.35695) | 0.17355<br>(1.0521)  | 0.10092<br>(1.3786)  |
|          | 8              | R.Eff.(·)<br>B(·) | 0.12606<br>(1.0741)  | 0.8995<br>(0.40446)  | 8.6785<br>(0.02126)  | 1.0575<br>(0.34121) | 0.13805<br>(1.0084)  | 0.080044<br>(1.3269) |
| 0.05     | 4              | R.Eff.(·)<br>B(·) | 0.25657<br>(1.0606)  | 1.8885<br>(0.41033)  | 41.876<br>(0.03188)  | 2.3279<br>(0.33298) | 0.27168<br>(1.0225)  | 0.15581<br>(1.3522)  |
|          | 6              | R.Eff.(·)<br>B(·) | 0.17444<br>(1.0455)  | 1.1969<br>(0.41075)  | 12.997<br>(0.037166) | 1.5599<br>(0.31745) | 0.19074<br>(0.98515) | 0.10897<br>(1.3093)  |
|          | 8              | R.Eff.(·)<br>B(·) | 0.13328<br>(0.13328) | 0.87903<br>(0.41008) | 7.9394<br>(0.03857)  | 1.1748<br>(0.31174) | 0.14525<br>(0.9752)  | 0.082585<br>(1.3006) |

**Table (4) Shown Bias Ratio [B(·)] and Relative Efficiency [R.Eff.(·)] of  $\tilde{\theta}_2$  when k = 0.5**

| $\alpha$ | $\lambda$<br>n |                   | 0.25                 | 0.75                 | 1                    | 1.25                 | 1.75                 | 2                      |
|----------|----------------|-------------------|----------------------|----------------------|----------------------|----------------------|----------------------|------------------------|
|          |                | R.Eff.(·)<br>B(·) |                      |                      |                      |                      |                      |                        |
| 0.01     | 4              | R.Eff.(·)<br>B(·) | 0.24566<br>(1.3456)  | 1.9323<br>(0.5053)   | 37.572<br>(0.017742) | 2.0197<br>(0.45776)  | 0.24792<br>(1.36)    | 0.14418<br>(1.7843)    |
|          | 6              | R.Eff.(·)<br>B(·) | 0.16798<br>(1.3167)  | 1.1063<br>(0.50657)  | 5.7479<br>(0.027087) | 1.2123<br>(0.42824)  | 0.18235<br>(1.2536)  | 0.10938<br>(1.631)     |
|          | 8              | R.Eff.(·)<br>B(·) | 0.12963<br>(1.2901)  | 0.77486<br>(0.50743) | 3.1243<br>(0.035434) | 0.90064<br>(0.40201) | 0.14978<br>(1.1806)  | 0.089977<br>(1.5448)   |
| 0.05     | 4              | R.Eff.(·)<br>B(·) | 0.26625<br>(1.2677)  | 1.7887<br>(0.51722)  | 15.075<br>(0.053129) | 2.206<br>(0.3883)    | 0.29347<br>(1.2042)  | 0.17218<br>(1.587)     |
|          | 6              | R.Eff.(·)<br>B(·) | 0.18286<br>(1.2425)  | 1.0509<br>(0.51791)  | 4.6789<br>(0.061944) | 1.384<br>(0.36242)   | 0.21224<br>(1.1419)  | 0.12445<br>(1.5154)    |
|          | 8              | R.Eff.(·)<br>B(·) | 0.14122<br>(0.14122) | 0.7477<br>(0.7477)   | 2.8582<br>(2.8582)   | 1.0214<br>(1.0214)   | 0.16337<br>(0.16337) | 0.095117<br>(0.095117) |