

## Semi-Analytical Solutions for Time-Fractional Fisher Equations via New Iterative Method

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### Abstract

An effective method for resolving non-linear partial differential equations with fractional derivatives is the New Sumudu Transform Iterative Method (NSTIM). It excels at solving difficult mathematical puzzles and offers insightful information about the behaviour of time-fractional Fisher equations. The method, which makes use of Caputo's sense derivatives and Wolfram in Mathematica, is reliable, simple to use, and gives a visual depiction of the solution. The analytical findings demonstrate that the proposed approach is effective and simple in generating precise solutions for the time-fractional Fisher equations. The results are made more reliable and applicable by including Caputo's sense derivatives. Mathematical modelling relies on the effectiveness and simplicity of the NSTIM approach to solve time-fractional Fisher equations since it enables precise solutions without the use of a lot of processing power. The NSTIM approach is a useful tool for researchers in a variety of domains since it also offers a flexible framework that is easily adaptable to other fractional differential equations. It now becomes possible to examine the dynamics and behaviour of complex systems governed by time-fractional Fisher equations with efficiency and reliability, opening up new research avenues. The ability to solve time-fractional Fisher equations efficiently and reliably using the NSTIM approach has significant implications for various fields such as population dynamics, mathematical biology, and epidemiology. Researchers can now analyze the spread of diseases or study the population dynamics of species with higher accuracy and less computational effort. This advancement in solving fractional differential equations paves the way for deeper insights into the behavior and patterns of complex systems, ultimately advancing scientific understanding and offering new possibilities for practical applications.

**Keywords:** Caputo fractional derivative, Fisher equations, Fractional Calculus, Iterative method, Sumudu Transform.

### Introduction

In recent decades, fractional differential equations have fascinated mathematicians, physicists and engineering researchers<sup>1-3</sup>. A fractional theory, which includes fractional derivatives and fractional

integration, can be used to model a wide range of problems<sup>4-7</sup>. Various methods have been developed to solve both linear and nonlinear fractional differential equations<sup>8-10</sup>, including the Cauchy

reaction-diffusion method, the Adomian decomposition method (A.D.M.)<sup>11,12</sup>, the homotopy method (H.A.M.)<sup>13</sup>, the variational method of iteration (V.I.M.)<sup>14,15</sup>, and the perturbation method of homotopy (H.P.M.)<sup>16</sup>. These methods have been applied to the Cauchy-diffusion of time-fractional equations, which are used to model nonlinear and linear systems in fields such as engineering, biology, ecology, chemistry, and physics<sup>17-19</sup>.

The study of investigates solutions for nonlinear generalized proportional, fractional functional integro-differential Langevin equations using fixed point theorems and Ulam-Hyers stability. It creates a mathematical model to analyse Wolbachia dispersal among *Aedes aegypti* mosquitoes, analysing symmetrical characteristics<sup>20-22</sup>. The model is physically meaningful and assessed for equilibrium points in the presence and absence of disease. Eight equilibrium points are determined, and the basic reproduction number is calculated using the next-generation matrix method. Numerical simulations are conducted to evaluate the basic reproduction number and identify the optimal CI value for reducing disease spread. The study also examines the interaction between prey and predator populations, focusing on the additive Allee effect and intraspecific competition. The study highlights the importance of considering precautionary measures in controlling disease spread, with the rate of precautionary measures playing a crucial role in reducing the chance of infection by the Chickenpox virus<sup>23-25</sup>.

The authors of an article obtained both numerical and analytical solutions to the time-fractional Fisher equations using the New Sumudu transform iterative method (NSTIM). The benefit of this new method is that it makes the calculations easy and gives the most accurate estimate of the exact answer<sup>26-28</sup>. There are many problems in fractional derivatives<sup>29,30</sup>, hydrodynamics<sup>31</sup>, chemical diffusion<sup>32</sup>, and option pricing<sup>33</sup>. Partial differential equations<sup>34</sup> can be used to model computational fluid dynamics<sup>35-38</sup> and control theory<sup>39-42</sup>. Nonlinear P.D.E.s and processes for finding numerical solutions to nonlinear problems have gotten much attention recently (P.D.E.s)<sup>43-45</sup>. The main theme of this research is focused on the solution and analysis of a nonlinear time fractional Fisher's equation with specific boundary conditions.

The analytical method focuses on finding an exact solution to PDEs, but solving the time fractional

Fisher equation is challenging due to the fractional derivative. The iterative method, using techniques like the New Sumudu transform iterative method, discretizes the equation in space and time, and updates it iteratively until it converges to the desired accuracy. The analytical method approximates the fractional derivative term, while the iterative method transforms the equation into linear or nonlinear algebraic equations that can be solved iteratively.

The software Mathematica provides powerful tools for creating visualizations and graphical representations of nonlinear time fractional Fisher's equations. It supports 2D and 3D plotting, graphing, and interactive visualizations, which aids in better understanding solutions of Fisher's equations.

The new Sumudu transform iterative method offers several advantages, including faster convergence, improved accuracy and precision, wide applicability, robustness, memory and computational efficiency, parallelization, adaptability to problem structure, trade-offs, comparative analysis, and experimental results.

Fisher's equation is a mathematical model that describes the spread of a mutant gene through a population. It is a partial differential equation with constant coefficients.

$$y_{\omega}(\xi, \omega) = y_{\xi, \xi}(\xi, \omega) + y(\xi, \omega)(1 - y(\xi, \omega)). \quad 1$$

This model shows the population density by  $y(\xi, \omega)$ , and the logistic form is indicated by  $y(y - 1)$ . In chemical kinetics and population dynamics, this equation solves problems like the nonlinear growth of a population in a habitat of 1-dimensional and the number of neutrons in a nuclear reaction. Also, one of the same equations is used in models growth of a logistic population<sup>46-48</sup>, the spread of a flame, neurophysiology, chemical reactions that happen on their own and branching Brownian motion. In this article, Fisher's equation of time-fractional can be written as follows:

$$D_{\omega}^{\beta} y(\xi, \omega) = y_{\xi, \xi}(\xi, \omega) + \lambda y(\xi, \omega)(1 - y(\xi, \omega)), \quad 2$$
$$0 < \beta \leq 1,$$

where  $D_{\omega}^{\beta} y(\xi, \omega)$  denotes the caputo fractional derivative (C.F.D.) of order  $\beta$  and  $\lambda$  is a parameter (accurate)<sup>49-51</sup>.

**Background:**

**Definition 1:** The  $\beta \in \mathbb{C}, \text{Re}(\beta) > 0$  R-L fractional integral  $I_{p+}^\beta f$  of order is defined by <sup>50</sup>,

$$\begin{aligned} ({}_p D_q^{-\beta} f)(q) &= (I_{p+}^\beta f)(q) \\ &= \frac{1}{\Gamma(\beta)} \int_p^q \frac{f(c)}{(q-c)^{1-\beta}} dc, (q > p, \text{Re}(\beta) > 0). \end{aligned} \quad 3$$

**Definition 2:** The R-L fractional derivatives  $({}_p D_q^\beta z)(x)$  of order  $\beta \in \mathbb{C}, \text{Re}(\beta) > 0$  is defined by <sup>50</sup>,

$$\begin{aligned} ({}_p D_\xi^\beta z)(\xi) &= \frac{d^j}{d\xi^j} (J_{p+}^{n-\beta} z)(\xi) \\ &= \frac{1}{\Gamma(j-\beta)} \frac{d^j}{d\xi^j} \int_p^\xi (\xi-c)^{n-\beta-1} z(c) dc, \\ c > p, j &= \text{Re}(\beta) + 1. \end{aligned} \quad 4$$

**Definition 3:** Function of Mittag-Leffler and generalation<sup>50</sup>

$$E_\delta(z) = \sum_{m=0}^\infty \frac{z^m}{\Gamma(\delta^{m+1})} (\delta \in \mathbb{C}, \text{Re}(\delta) > 0), \quad 5$$

$E_{\delta,\omega}$  is Mittag-Leffler function in two parameters.

$$\begin{aligned} E_{\delta,\omega}(z) &= \sum_{m=0}^\infty \frac{y^m}{\Gamma(\delta^m + \omega)} \quad \omega, \delta \in \mathbb{C}, \text{Re}(\delta) > 0, \\ \text{Re}(\omega) &> 0. \end{aligned} \quad 6$$

**Definition 4:** A caputo fractional derivative of function  $y(\xi, t)$  is defined as <sup>49</sup>,

$$\begin{aligned} D_{\xi}^\beta y(\xi, t) &= \frac{1}{\Gamma(j-\beta)} \int_0^\xi (\xi-c)^{j-\beta-1} \frac{\partial^j y(c, t)}{\partial c^j} dc, \\ j-1 < \beta &\leq j, j \in \mathbb{N} \end{aligned} \quad 7$$

$d^j \equiv \frac{d^j}{dx^j}$  and  $j_x^\beta$  denote the R-L fractional integral operator of order  $\beta > 0$  defined as  $d^j \equiv \frac{d^j}{dx^j}$  and  $j_x^\beta$  respectively.

$$J_{\xi}^\beta y(\xi, t) = \frac{1}{\Gamma(\beta)} \int_0^\xi (\xi-c)^{(\beta-1)} y(c, t) dc, c > 0, z - 1 < \beta \leq z, z \in \mathbb{N}. \quad 8$$

**Definition 5:** The Sumudu transform of a function  $f(p), p > 0$  is defined as <sup>51</sup>

$$S[f(p)] = \int_0^\infty e^{-pv} f(vp) dp, v \in (-P_1, P_2) \quad 9$$

and  $f(p) \in W$ ,

where  $W =$

$$\left\{ f(p), \mid \exists M, P_1, P_2 > 0, |f(p)| \leq M e^{\frac{|p|}{P_j}}, \right. \\ \left. \text{if } p \in (-1)^j \times [0, \infty) \right\}. \quad 10$$

**Definition 6:** The Sumudu transform of the Caputo fractional derivative is defined as<sup>51</sup>,

$$\begin{aligned} S[D_Y^{n\beta} y(Y, \omega)] &= v^{-n\beta} S[y(Y, \omega)] - \\ \sum_{i=0}^{j-1} v^{-n\beta+i} y^{(i)}(0, \omega), & j-1 < n\beta < j. \end{aligned} \quad 11$$

**The New Sumudu transform Iterative Method (NSTIM):**

To illustrate this New Iterative Transform of Sumudu Method <sup>51-53</sup> take into account a fractional partial differential equation with the initial conditions, which is both non-homogeneous and nonlinear:

$$\begin{aligned} D_\omega^{n\beta} z(Y, \omega) + Lz(Y, \omega) + R(z(Y, \omega)) &= g(Y, \omega), \\ n-1 < n\beta \leq n, z(Y, 0) &= h(Y) \end{aligned} \quad 12$$

where  $D_\omega^{n\beta}$  is the fractional Caputo derivative operator,  $D_\omega^{n\beta} = \frac{\partial^{n\beta}}{\partial \omega^{n\beta}}$ , L-operator(linear), R-operator(non-linear),  $g(Y, \omega)$  is continuous function.

Employing the Sumudu transform to Eq 12 obtain,

$$S[D_\omega^{n\beta} z(Y, \omega)] + S[L(z(Y, \omega)) + R(z(Y, \omega))] = S[g(Y, \omega)], \quad 13$$

employing the property of sumudu transformation, obtain,

$$S[z(Y, \omega)] - v^{n\beta} \sum_{k=0}^{j-1} v^{-n\beta+k} z^{(k)}(Y, 0) + v^{n\beta} S[L(z(Y, \omega)) + R(z(Y, \omega)) - g(Y, \omega)] = 0. \quad 14$$

employing the Sumudu transform of inverse to Eq 14,

$$z(Y, \omega) = S^{-1} \left[ v^{n\beta} \sum_{k=0}^{j-1} v^{-n\beta+k} z^{(k)}(Y, 0) - v^{n\beta} S[L(z(Y, \omega)) + R(z(Y, \omega)) - g(Y, \omega)] \right]. \quad 15$$

Next, assume that,

$$f(Y, \omega) = S^{-1} \left[ v^{n\beta} \sum_{k=0}^{j-1} v^{-n\beta+k} z^{(k)}(Y, 0) + v^{n\beta} [g(Y, \omega)] \right]; \quad 16$$

$$N(z(Y, \omega)) = -S^{-1} [v^{n\beta} S[R(z(Y, \omega))]]; \quad 17$$

$$K(z(Y, \omega)) = -S^{-1} [v^{n\beta} S[L(z(Y, \omega))]]; \quad 18$$

Thus, Eq 15 will be reduced in the following form,

$$z(Y, \omega) = f(Y, \omega) + K(z(Y, \omega)) + N(z(Y, \omega)). \quad 19$$

The solution of the equation is given in the series form,

$$z(Y, \omega) = \left( \sum_{m=0}^{\infty} z_m(Y, \omega) \right),$$

Obtaining

$$K \left( \sum_{m=0}^{\infty} z_m(Y, \omega) \right) = \sum_{m=0}^{\infty} K(z_m(Y, \omega)). \quad 20$$

Operator N (nonlinear) is decomposed as

$$N(\sum_{m=0}^{\infty} z_m) = N(z_0) + \{N(\sum_{j=0}^m z_j) - N(\sum_{j=0}^{m-1} z_j)\}. \quad 21$$

Therefore, Eq 12 can be represented in the following form, Defining the recursive relation

$$\begin{aligned} z_0(Y, \omega) &= f(Y, \omega), \\ z_1(Y, \omega) &= K(z_0(Y, \omega)) + N(z_0(Y, \omega)), \\ z_{r+1}(Y, \omega) &= K(z_r(Y, \omega)) + \\ &\quad \left\{ N \left( \sum_{j=0}^r z_j(Y, \omega) \right) - N \left( \sum_{j=0}^{r-1} z_j(Y, \omega) \right) \right\}, \\ &\quad \text{for all } r \geq 1 \end{aligned} \quad 22$$

$$\text{Thus, } (z_1 + z_2 + \dots + z_{m+1}) = K(z_0 + \dots + z_m) + N(z_0 + \dots + z_m) \quad 23$$

namely,

$$\begin{aligned} z(Y, \omega) &= \sum_{m=0}^{\infty} z_m(Y, \omega) \\ &= f + K(\sum_{m=0}^{\infty} z_m(Y, \omega)) + \\ &\quad N(\sum_{m=0}^{\infty} z_m(Y, \omega)) \quad 24 \end{aligned}$$

The m-term approximate solution of. Eq 12 is given by

$$z_m(Y, \omega) = z_0(Y, \omega) + z_1(Y, \omega) + \dots + z_{m-1}(Y, \omega). \quad 25$$

### Convergence and Error Analysis:

**Theorem 1:** Let  $z_p(Y, \omega)$  and  $z_n(Y, \omega)$  be the members of Banach space H and the exact solution of Eq 1 be  $z(Y, \omega)$ . The Series solution  $\sum_{p=0}^{\infty} z_p(Y, \omega)$  converges to  $z(Y, \omega)$ , if  $z_p(Y, \omega) \leq \lambda z_{p-1}(Y, \omega)$  for  $\lambda \in (0,1)$ , that is for any  $z > 0, \exists E$  such that  $\|z_{p+n}(Y, \omega)\| \leq z, \forall p, n > E$ .

**Proof:** Let  $u_p(Y, \omega) = z_0(Y, \omega) + z_1(Y, \omega) + z_2(Y, \omega) + \dots + z_p(Y, \omega)$  be the sequence of  $p^{th}$  partial sum of series  $\sum_{p=0}^{\infty} z_p(Y, \omega)$ .

Now, consider

$$\begin{aligned} \|u_{p+1}(Y, \omega) - u_p(Y, \omega)\| &= \|z_{p+1}(Y, \omega)\| \\ &\leq \lambda \|z_p(Y, \omega)\| \\ &\leq \lambda^2 \|z_{p-1}(Y, \omega)\| \quad 26 \\ &\leq \lambda^3 \|z_{p-2}(Y, \omega)\| \\ &\vdots \\ &\leq \lambda^{p+1} \|z_0(Y, \omega)\|. \end{aligned}$$

for  $\forall n, p \in E$

Consider,

$$\|u_p(Y, \omega) - u_n(Y, \omega)\|$$

$$\begin{aligned}
 &= \|z_{p+n}(Y, \omega)\| \\
 &= \|(u_p(Y, \omega) - u_{p-1}(Y, \omega)) \\
 &+ (u_{p-1}(Y, \omega) - u_{p-2}(Y, \omega)) \\
 &+ (u_{p-2}(Y, \omega) - u_{p-3}(Y, \omega)) \\
 &+ \dots + (u_{n+1}(Y, \omega) - u_n(Y, \omega))\| \\
 &\leq \|(u_p(Y, \omega) - u_{p-1}(Y, \omega))\| \\
 &+ \|(u_{p-1}(Y, \omega) - u_{p-2}(Y, \omega))\| \\
 &+ \|(u_{p-2}(Y, \omega) - u_{p-3}(Y, \omega))\| \\
 &+ \dots + \|(u_{n+1}(Y, \omega) - u_n(Y, \omega))\| \\
 &\leq \lambda^p \|z_0(Y, \omega)\| \\
 &+ \lambda^{p-1} \|z_0(Y, \omega)\| \\
 &+ \lambda^{p-2} \|z_0(Y, \omega)\| \\
 &+ \dots + \lambda^{p-1} \|z_0(Y, \omega)\| \\
 &= \|z_0(Y, \omega)\| (\lambda^p + \lambda^{p-1} + \dots + \lambda^{p+1}) \\
 &= \|z_0(Y, \omega)\| \left(\frac{1 - \lambda^{p-n}}{1 - \lambda}\right) \lambda^{n+1}
 \end{aligned} \tag{27}$$

Since  $0 < \lambda < 1$  and  $z_0(Y, \omega)$  is bounded, so assume that,

$$z = \|z_0(Y, \omega)\| \left(\frac{1 - \lambda^{p-n}}{1 - \lambda}\right) \lambda^{n+1},$$

The anticipated outcome is achieved.. Also  $\sum_{p=0}^{\infty} z_p(Y, \omega)$  is a cauchy sequence in H, which

## Results and Discussion

### Numerical Examples:

**Example 1:** Suppose the following nonlinear time fractional Fisher's equation<sup>54</sup>

$$D_{\omega}^{\beta} y(Y, \omega) = \frac{\partial^2 y(Y, \beta)}{\partial Y^2} + 6y(1 - y), \quad 0 < \beta \leq 1 \tag{29}$$

The initial condition

$$y(Y, 0) = \frac{1}{(1+e^Y)^2}, \tag{30}$$

employing Sumudu transform on Eq 29 and using the initial condition of Eq 30 to obtain,

$$S[y(Y, \omega)] = \frac{1}{(1+e^Y)^2} + \frac{1}{u-\beta} S\left[\frac{\partial^2 y}{\partial Y^2} + 6y(1 - y)\right], \tag{31}$$

employing the Sumudu transform of the inverse formula, the following equation is obtained,

implies that there exists  $z_0(Y, \omega) \in H$  such that  $\lim_{p \rightarrow \infty} z_p(Y, \omega) = z(Y, \omega)$ . Hence, the proof has been completed.

**Theorem 2:** Let  $\sum_{p=0}^q z_p(Y, \omega)$  be the finite and approximate solution of  $z(Y, \omega)$ . If  $\|z_{p+1}(Y, \omega)\| \leq \lambda \|z_0(Y, \omega)\|$  for  $\lambda \in (0, 1)$ , then the maximum absolute error is

$$\|z(Y, \omega) - \sum_{p=0}^q z_p(Y, \omega)\| \leq \frac{\lambda^{q+1}}{1 - \lambda} \|z_0(Y, \omega)\|.$$

**Proof:**  $\|z(Y, \omega) - \sum_{p=0}^q z_p(Y, \omega)\|$

$$\begin{aligned}
 &= \|\sum_{p=0}^{\infty} z_p(Y, \omega)\| \\
 &\leq \sum_{p=q+1}^{\infty} \|z_p(Y, \omega)\| \\
 &\leq \sum_{p=q+1}^{\infty} \lambda^q \|z_0(Y, \omega)\| \\
 &\lambda^{q+1} (1 + \lambda + \lambda^2 + \dots) \|z_0(Y, \omega)\| \\
 &\leq \frac{\lambda^{q+1}}{1 - \lambda} \|z_0(Y, \omega)\|
 \end{aligned} \tag{28}$$

Thus, the proof has been completed.

$$y(Y, \omega) = \frac{1}{(1+e^Y)^2} + S^{-1}\left[\frac{1}{u-\beta} S\left[\frac{\partial^2 y}{\partial Y^2} + 6y(1 - y)\right]\right], \tag{32}$$

namely,

$$y(Y, \omega) = \frac{1}{(1+e^Y)^2} + S^{-1}\left[\frac{1}{u-\beta} S\left[\frac{\partial^2 y}{\partial Y^2} + 6y(1 - y)\right]\right]. \tag{32}$$

According to NSTIM, result obtain;

$$y_0 = \frac{1}{(1+e^Y)^2}, \tag{33}$$

$$K[y(Y, \omega)] = S^{-1}\left[\frac{1}{u-\beta} S\left[\frac{\partial^2 y}{\partial Y^2} + 6y(1 - y)\right]\right].$$

By iteration, the following results are obtained

$$y_0(Y, \omega) = \frac{1}{(1+e^Y)^2},$$

$$y_1(Y, \omega) = S^{-1}\left[\frac{1}{u^{-\beta}} S\left[\frac{\partial^2 y_0}{\partial Y^2} + 6y_0(1-y_0)\right]\right], \quad 34$$

$$= 10 \frac{e^Y}{(1+e^Y)^3} \frac{(\omega^\beta)}{\Gamma(\beta+1)}.$$

$$y_2(Y, \omega) = S^{-1}\left[\frac{1}{u^{-\beta}} S\left[\frac{\partial^2 (y_0+y_1)}{\partial Y^2}\right]\right]$$

$$- S^{-1}\left[\frac{1}{u^\beta} S\left[\frac{\partial^2 y_0}{\partial Y^2}\right]\right] \quad 35$$

$$= 50 \frac{e^Y(-1+2e^Y)}{(1+e^Y)^4} \frac{(\omega^{2\beta})}{\Gamma(2\beta+1)}$$

$$y_3(Y, \omega) = 50 \frac{e^Y(-1+2e^Y)}{(1+e^Y)^4} \frac{(\omega^{2\beta})}{\Gamma(2\beta+1)} \quad 36$$

Therefore, the Operating analytical solution of the problem in the series form can be obtained as,

$$y(Y, \omega) = y_0(Y, \omega) + y_1(Y, \omega) + \dots$$

$$y(Y, \omega) = \frac{1}{(1+e^Y)^2} + 10 \frac{e^Y}{(1+e^Y)^3} \frac{(\omega^\alpha)}{\Gamma(\beta+1)}$$

$$+ 50 \frac{e^Y(-1+2e^Y)}{(1+e^Y)^4} \frac{(\omega^{2\beta})}{\Gamma(2\beta+1)} + \dots$$

$$250e^Y \left[ 5 - 6e^Y - 15e^{2Y} + 20e^{3Y} - \dots \right]$$

$$12e^Y \frac{\Gamma(2\beta+1)}{\Gamma(\beta+1)^2} \frac{\omega^{3\beta}}{(1+e^Y)^6 \Gamma(3\beta+1)} + \dots \quad 37$$

Where  $E_\beta(\omega^\beta)$  is mittage leffer function defined by Eq 7.

putting  $\beta = 1$ , Eq 29 becomes the following equation,

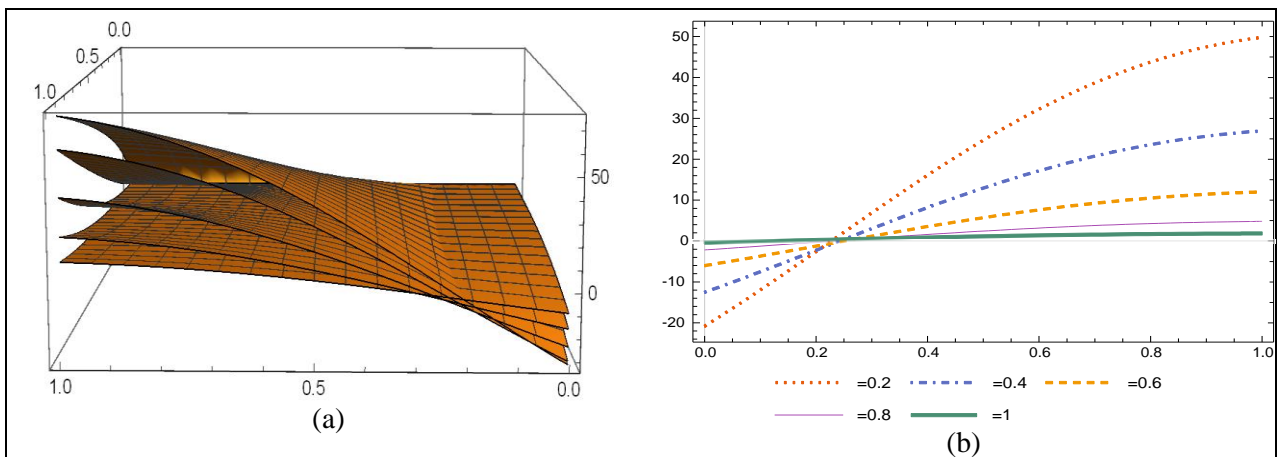
$$y(Y, \omega) = \frac{\partial^2 y}{\partial Y^2} - 6y - y$$

With accurate solution

$$y(Y, \omega) = \frac{1}{(1+e^{Y-5\omega})^2}. \quad 38$$

( See Fig. 1 and Table 1 )

**Remark 1:** The linear time fractional Fisher equations are shown above. The estimated results of fractional Fisher equations of the time linear at values of  $\beta=0.2,0.4,0.6,0.8$  and the accurate solution for  $\beta=1$  are shown below. Fig 1 (a), in 3-dimension view and in Fig 1 (b), in 2-dimension forms, respectively. The answer is so simple to discover that it is constantly dependent on the values of time-fractional derivatives.



**Figure 1. The estimated results of fractional Fisher equations of the time linear at values of  $\beta=0.2,0.4,0.6,0.8$  and 1.**

The numerical solution obtained using the NSTIM of 5th order approximation of Example 1 is compared with the accurate solution for  $\beta = 1$  in Table 1,

which shows the efficiency and effectiveness of the method.



**Table 1. Error Analysis of Example 1.**

$\beta = 1$				
Y	$\omega$	y(NSTIM)	y(accurate)	$\ y_{NSTIM} - y_{accurate}\ $
0.2	0.3	0.0539935	0.0539944	$9 \times 10^{-07}$
0.4	0.5	0.26375	0.263795	$5 \times 10^{-05}$
0.6	0.7	0.724381	0.724951	$6 \times 10^{-04}$
0.8	0.9	1.57046	1.57415	$4 \times 10^{-03}$

**Example 2:** Suppose the nonlinear time fractional following Fisher's eq<sup>54</sup>

$$D_{\omega}^{\beta} y(Y, \omega) = \frac{\partial^2 y(Y, \omega)}{\partial Y^2} + y(1 - y), 0 < \beta \leq 1 \quad 39$$

The initial condition

$$y(Y, 0) = \alpha, \quad 40$$

employing Sumudu transform on the Eq 39 and using the initial condition of Eq 40 obtain,

$$S[y(Y, \omega)] = \alpha + \frac{1}{u-\beta} S\left[\frac{\partial^2 y}{\partial Y^2} + y(1 - y)\right], \quad 41$$

employing the Sumudu transform of the inverse formula,

$$y(Y, \omega) = \alpha + S^{-1}\left[\frac{1}{u-\beta} S\left[\frac{\partial^2 y}{\partial Y^2} + y(1 - y)\right]\right],$$

namely,

$$y(Y, \omega) = \alpha + S^{-1}\left[\frac{1}{u-\beta} S\left[\frac{\partial^2 y}{\partial Y^2} + y(1 - y)\right]\right]. \quad 42$$

According to the NSTIM, result obtain

$$y_0 = \alpha, \quad 43$$

$$K[y(Y, \omega)] = S^{-1}\left[\frac{1}{u-\beta} S\left[\frac{\partial^2 y}{\partial Y^2} + y(1 - y)\right]\right].$$

By iteration, the following results are obtained

$$y_0(Y, \omega) = \alpha,$$

$$y_1(Y, \omega) = S^{-1}\left[\frac{1}{u-\beta} S\left[\frac{\partial^2 y_0}{\partial Y^2} + y_0(1 - y_0)\right]\right],$$

$$= \alpha(1 - \alpha) \frac{(\omega^{\beta})}{\Gamma(\beta+1)}. \quad 44$$

$$y_2(Y, \omega) = S^{-1}\left[\frac{1}{u-\beta} S\left[\frac{\partial^2 (y_0 + y_1)}{\partial Y^2}\right]\right]$$

$$- S^{-1}\left[\frac{1}{u-\beta} S\left[\frac{\partial^2 y_0}{\partial Y^2}\right]\right], \quad 45$$

$$= \alpha(1 - \alpha)(1 - 2\alpha) \frac{(\omega^{2\beta})}{\Gamma(2\beta+1)}.$$

$$y_3(Y, \omega) = (\alpha - 5\alpha^2 + 8\alpha^3 - 4\alpha^4) \frac{(\omega^{3\beta})}{\Gamma(3\beta+1)}$$

$$- (\alpha^2 - 2\alpha^3 + \alpha^4) \frac{\Gamma(2\beta+1)}{[\Gamma(\beta+1)]^2} \frac{(\omega^{3\beta})}{\Gamma(3\beta+1)}. \quad 46$$

$$y_4(Y, \omega) = (1 - 2\alpha)(\alpha - 5\alpha^2 + 8\alpha^3 - 4\alpha^4)$$

$$- (\alpha^2 - 2\alpha^3 + \alpha^4) \frac{\Gamma(2\beta+1)}{[\Gamma(\beta+1)]^2} \frac{(\omega^{4\beta})}{\Gamma(4\beta+1)}$$

$$- 2(\alpha - \alpha^2)(\alpha - 3\alpha^3 + 2\beta^3) \frac{\Gamma(3\alpha+1)}{\Gamma(\alpha+1)\Gamma(2\alpha+1)} \frac{(\omega^{4\beta})}{\Gamma(4\beta+1)}. \quad 47$$

Therefore, the analytical estimated results of the problem in the series form can be found as,

$$y(Y, \omega) = y_0(Y, \omega) + y_1(Y, \omega) + \dots \quad 48$$

$$y(Y, \omega) = \left[ \begin{aligned} &\alpha + \alpha(1 - \alpha) \frac{(\omega^{\beta})}{\Gamma(\beta+1)} + \alpha(1 - \alpha)(1 - 2\alpha) \frac{(\omega^{2\beta})}{\Gamma(2\beta+1)} \\ &+ (\alpha - 5\alpha^2 + 8\alpha^3 - 4\alpha^4) \frac{(\omega^{3\beta})}{\Gamma(3\beta+1)} \\ &- (\alpha^2 - 2\alpha^3 + \alpha^4) \frac{\Gamma(2\beta+1)}{[\Gamma(\beta+1)]^2} \frac{(\omega^{3\beta})}{\Gamma(3\beta+1)} \\ &+ (1 - 2\alpha)(\alpha - 5\alpha^2 + 8\alpha^3 - 4\alpha^4) \\ &- 7(\alpha^2 - 2\alpha^3 + \alpha^4) \frac{\Gamma(2\beta+1)}{[\Gamma(\beta+1)]^2} \frac{(\omega^{4\beta})}{\Gamma(4\beta+1)} \\ &- 2(\alpha - \alpha^2)(\alpha - 3\alpha^3 + 2\alpha^3) \frac{\Gamma(3\beta+1)}{\Gamma(\beta+1)\Gamma(2\beta+1)} \frac{(\omega^{4\beta})}{\Gamma(4\beta+1)} + \dots \end{aligned} \right] \quad 49$$

Where -  $E_{\beta}(\omega^{\beta})$  is mittage leffer function defined by Eq 7.

putting  $\beta = 1$ , Eq 39 becomes the following equation,

$$y(Y, \omega) = \frac{\partial^2 y}{\partial Y^2} + y(1 - y), \quad 50$$

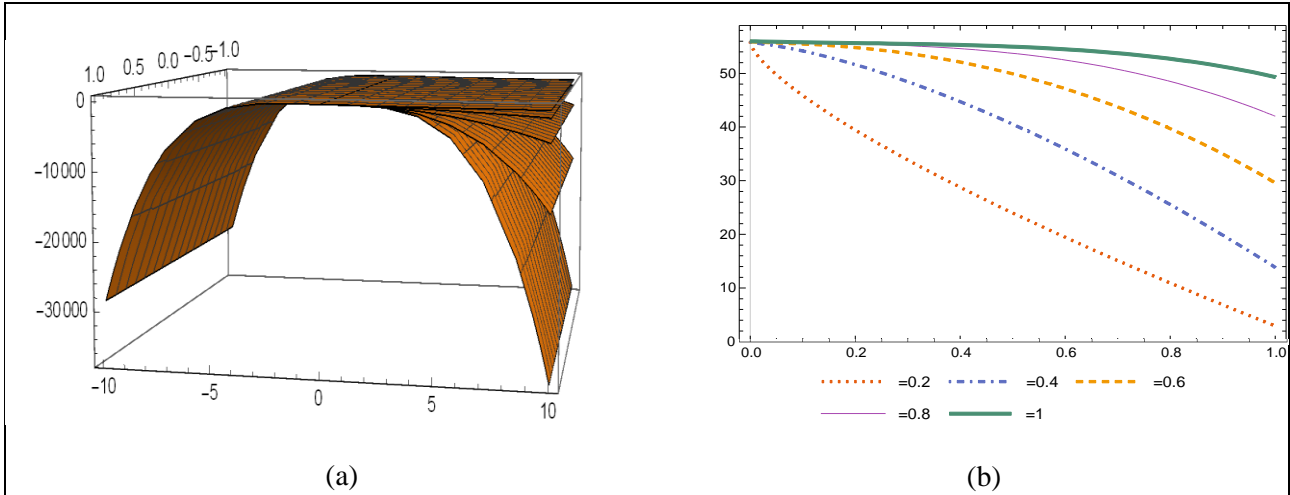
with accurate solution

$$y(Y, \omega) = \frac{ae^Y}{(1 - \alpha + ae^{\omega})}$$

( See Fig. 2 and Table 2 )

**Remark 2:** The linear time fractional Fisher equations are shown above. The estimated results of the linear time fractional Fisher equations at values of  $\beta=0.2,0.4,0.6,0.8$  and the accurate solution for  $\beta=1$  are shown below in Fig. 2 (a), in 3-dimension

view and in Fig. 2 (b), in 2-dimension forms respectively. The solution is so simple to discover that it is constantly dependent on the values of time-fractional derivatives.



**Figure 2. The estimated results of the linear time fractional Fisher equations at values of  $\beta=0.2,0.4,0.6,0.8$  and 1**

The numerical solution obtained using the NSTIM of 5th order approximation of Example 2 is compared with the accurate solution for  $\beta = 1$  in Table 2,

which shows the efficiency and effectiveness of the method

**Table 2. Error Analysis of Example 2**

		$\beta = 1$		
$Y$	$\omega$	$y(\text{NSTIM})$	$y(\text{ accurate})$	$\ y_{\text{NSTIM}} - y_{\text{accurate}}\ $
0.2	0.3	0.109149	0.109032	$1 \times 10^{-04}$
0.4	0.5	0.146032	0.143259	$3 \times 10^{-03}$
0.6	0.7	0.159643	0.139239	$2 \times 10^{-02}$
0.8	0.9	0.204733	0.118578	$9 \times 10^{-02}$

**Remark 3:** Fig 3(a) and Fig 3(b) depict the absolute error between estimated and accurate solutions for  $\beta=1$ . By comparison, it is clear that by computing additional terms, the efficiency and accuracy of this method (NSTIM) can be significantly improved. The authors have used a few iterations in this post.

However, the precision of the estimated solution could be substantially enhanced if they employed additional terms. As a result, the recommended method for solving the linear differential equation is both precise and efficient.



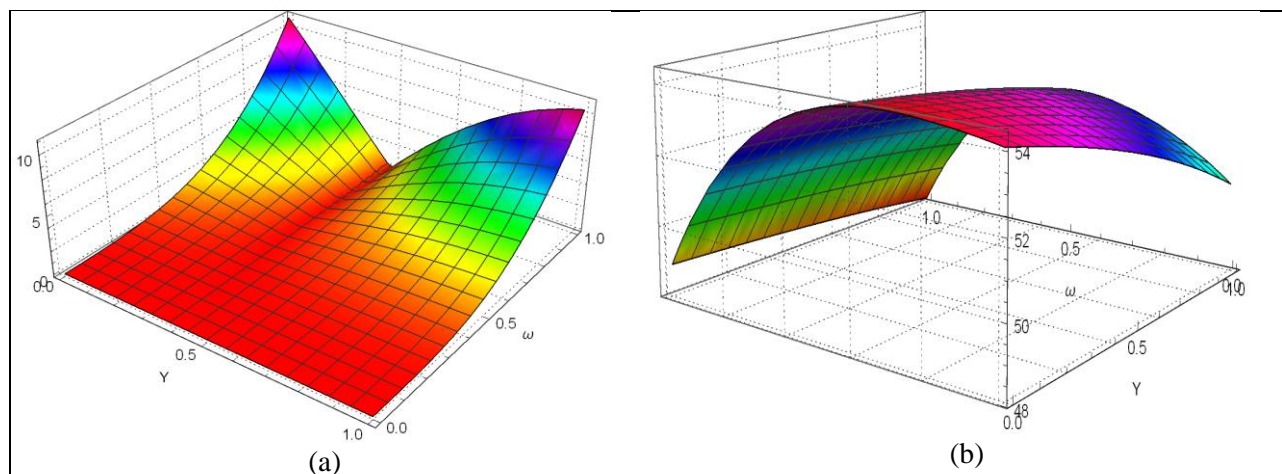


Figure 3. Absolute error between estimated and accurate solutions for  $\beta=1$ .

## Conclusion

This research used the novel Sumudu transform iterative technique to solve linear time fractional Fisher equations. The novel Sumudu transform iterative approach (NSTIM) combines NIM and Sumudu to solve linear time fractional Fisher equations. The iterative Sumudu transform approach is more structured and accurate and requires less numerical calculation, according to the numbers.

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## Authors' Declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for republication, which is attached to the manuscript.

## Authors' Contribution Statement

S. T. proposed the Concepts, ideas, method, and analysis. A. B. designed the manuscript. S. G. read the manuscript and revised it. K. K. Drafting of the

The approach reduces computational effort compared to conventional methods while maintaining good numerical precision. It can aid mathematicians and researchers working in the field of partial differential equations. The key advantage of this method is quick convergence and accuracy. NSTIM is an effective tool for discovering approximation and semi-analytical solutions.

Bhadane, S.B. Gaikwad, and Kishor A. Kshirsagar, among others.

- Ethical Clearance: The project was approved by the local ethical committee at Commerce and Science College, Ahmednagar, Maharashtra, India.

manuscript with interpreted and plotted the graphs of the solution of examples using Mathematica 11.3. All authors read and approved the final manuscript.

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## الحلول شبه التحليلية لمعادلات فيشر الكسرية للزمن عبر الطريقة التكرارية الجديدة

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### الخلاصة

إحدى الطرق الفعالة لحل المعادلات التفاضلية الجزئية غير الخطية ذات المشتقات الكسرية هي الطريقة التكرارية لتحويل سومودو الجديدة (NSTIM). إنه يبرع في حل الألغاز الرياضية الصعبة ويقدم معلومات ثابتة حول سلوك معادلات فيشر ذات الكسر الزمني. الطريقة، التي تستخدم مشتقات كابوتو الحسية و Wolfram في Mathematica، موثوقة وسهلة الاستخدام وتعطي تصويرًا مرئيًا للحل. أظهرت النتائج التحليلية أن الطريقة المقترحة فعالة وبسيطة في توليد حلول دقيقة لمعادلات فيشر الكسرية للزمن. أصبحت النتائج أكثر موثوقية وقابلة للتطبيق من خلال تضمين مشتقات كابوتو الحسية. تعتمد النمذجة الرياضية على فعالية وبساطة منهج NSTIM لحل معادلات فيشر ذات الكسر الزمني لأنها تتيح حلولاً دقيقة دون استخدام الكثير من قوة المعالجة. يعد نهج NSTIM أداة مفيدة للباحثين في مجموعة متنوعة من المجالات لأنه يوفر إطارًا مرئيًا يمكن تعديله بسهولة مع المعادلات التفاضلية الكسرية الأخرى. أصبح من الممكن الآن فحص ديناميكيات وسلوك الأنظمة المعقدة التي تحكمها معادلات فيشر الكسرية الزمنية بكفاءة وموثوقية، مما يفتح طرقًا بحثية جديدة. إن القدرة على حل معادلات فيشر ذات الكسور الزمنية بكفاءة وموثوقية باستخدام نهج NSTIM لها آثار مهمة على مجالات مختلفة مثل الديناميات السكانية والبيولوجيا الرياضية وعلم الأوبئة. يمكن للباحثين الآن تحليل انتشار الأمراض أو دراسة الديناميكيات السكانية لأنواع بدقة أعلى وجهد حسابي أقل. يمهّد هذا التقدم في حل المعادلات التفاضلية الكسرية الطريق لرؤى أعمق حول سلوك وأنماط الأنظمة المعقدة، مما يؤدي في نهاية المطاف إلى تعزيز الفهم العلمي وتقديم إمكانيات جديدة للتطبيقات العملية.

**الكلمات المفتاحية:** مشتقة كابوتو الكسرية، معادلات فيشر، حساب التفاضل والتكامل الكسري، الطريقة التكرارية، تحويل سومودو.