Generalized Theory for Indeterminate Coefficients Procedure

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Abstract:

As it been mentioned by D.R Merk in 1994 that the method of indeterminate coefficients for constructing a Liapunov function is not suitable for every problem. Anew theory give in [2] and in this paper we generalized this idea in which is applicable for more extended problems. The prove of this idea is given with example.

تعميم نظري لطريقة المعاملات الغير محددة

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الخلاصة:

في هذا البحث تم تعميم نظرية جديدة لتكوين دوال ليبانوف الخاصة باختبار استقرارية الانظمة اللاخطية ذات المعاملات الثابتة. وكذلك عرض برهان لهذه النظرية مع ذكر مثال تطبيقي لها.

Introduction:-

Application of main theorems of the direct method necessitates obtaining proper Liapunov functions that satisfy the given requirments. Unfortunately, no general procedures are available for constructing such functions, although in certain cases they could be constructed (see [1]), If it is difficult to find a Liapanov function for the given equations of perturbed solution, often by transferring to new coordinate system the equation can take for which the corresponding function can be found with relative case (of course one should try a linear transformation with constant coefficients).
Many papers, it had been seek a Liapunov function in a quadratic form with constant coefficients

$$V(x)=\frac{1}{2}\sum_{k=1}^{m}\sum_{j-1}^{m} c_{kj}\, x_k\, x_j \qquad (1)$$

In which the indeterminate coefficients c_{kj} must be satisfies Sylvester's criterion, (see[4]), then the function V is position definite . Since the number of coefficients c_{kj} equal to n(n+1)/2, then we have n(n-1)/2 independent coefficients that can be manipulated (see [3]).

Now,in order to be applicable for any system of ordinary differential equation, let us assume that we need to determine the conditions satisfied by the parameters of the system would result in the stability of unperturbed solution. Then we try to choose the remaining independent coefficients c_{kj} in such a way that the derivative V', obtained by virtue of the equations of the pertubed solution , is either a negative definite function or that it satisfies the conditions of Krasovsky theorem (see [4] , [5]).

If such coefficients c_{kj} could be found , then the perturbed solution is asymptotically stable .

Since this approach is not suitable for every problem , but in some cases it reduces good results .Therefore , in this paper ,we are built a criteria for testing the stability for general extended system based on the following theorem:

Theorem:- The system

$$X'_k = \sum_{j=1}^{m} a_{kj}\ x_j^{n}\ x_k^{n-1} \qquad (2)$$

Where a_{kj}, (k=1,2,…,m) are any constants and n is any real number such that $n \neq m$
is asymptotically stable if the Sylvester's criterion on V' is satisfied.
Proof:- consider the systern (1)

$$V = \frac{1}{2}\sum_{k=1}^{m}\ \sum_{j=1}^{m} c_{kj} x_k x_j \text{ with } C_{ii}=1,\ i=2,3,\ \dots\ ,m$$

$$\text{Let } C = \begin{pmatrix} c_{11} & c_{12} & c_{13} & \cdots & c_{1m} \\ c_{21} & 1 & c_{23} & \cdots & c_{2m} \\ c_{31} & c_{32} & 1 & \cdots & c_{3m} \\ \vdots & & & \ddots & \vdots \\ c_{m1} & c_{m2} & c_{m3} & \cdots & 1 \end{pmatrix}$$

Then Sylvester's criterion for the coefficients matrix C has the from:

$$\Delta 1 = c_{11} > 0,\ \Delta 2 = \begin{vmatrix} c11 & c12 \\ c21 & 1 \end{vmatrix} = c_{11} - c_{21}c_{12} > 0, \dots\ \Delta m = |C| > 0$$

Thus

$$V^{/} = \sum_{k=1}^{m}\sum_{j=1}^{m} c_{kj} x_j (\sum_{j=1}^{m} a_{kj} x_j^{n} x_k^{n-1})$$

Now, if $C_{kj} \neq 0\ \forall k, j$ and $K \neq j$ (k,j=1,2,…,m) then $V^{/}$ is an indefinite function.

Therefor, we assume $C_{kj} = 0$ for $k \neq j$, k, j = 1,2, ... ,m, and given as:

$$V^{/} = \sum_{k-1}^{m}\ \sum_{j=1}^{m} c_{KK}\, a_{kj} x_k^{n} x_j^{n}$$

then system (2) is asymptotically stable if the Sytresters criterion on V' is satisfied. That is if the following inequalities are satisfied:-

$$\Delta_1^{*} < 0,\ \Delta_2^{*} > 0, \dots .$$

i.e the determinants $\Delta_j^{*}, (j = 1,2,\dots,m)$ should alternately change their signs, and the sign of Δ_1^{*} should be nagative.

Example:- consider the system of equations

$$x_1^{/} = -2x_1^{3} + x_1 x_2^{2} + x_1 x_3^{2}$$

$$x_2' = x_1^2 x_2 - 4x_2^3 - x_2 x_3^2 \qquad (3)$$
$$x_3' = x_1^2 x_3 - x_2^2 x_3 + x_3^3$$

let V be a Liapunov function given by

$$V = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2)$$

Sylvester is criterion for the coeficient matrix has the form:

$$\Delta 1 = 1 > 0, \Delta 2 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1, \Delta 3 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 > 0$$

We evaluate the derivative V'

$$v' = x_1 x_1' + x_2 x_2' + x_3 x_3'$$

Substituting for x_1', x_2' and x_3' from cquation (3)we get

$$v' = -2x_1^4 + 2x_1^2 x_2^2 - 4x_2^4 + 2x_1^2 x_3^2 - 2x_2^2 x_3^2 - 2x_2^2 x_3^2 + x_3^4$$

Now, applying Sylvester's criterion on V' we get

$$\Delta_1^* = -2 < 0, \Delta_2^* = \begin{vmatrix} -2 & 2 \\ 2 & -4 \end{vmatrix} = 4 > 0, \Delta_3^* = \begin{vmatrix} -2 & 2 & 2 \\ 2 & -4 & -2 \\ 2 & -2 & 1 \end{vmatrix} = -4 < 0$$

.: system (3) is asymptotically stable

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