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Some Results on the Average Inverse Shadowing Property and Strong Ergodicity

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Abstract:

Let (X, d_1) and (Y,d_2) be compact metric spaces, $f: (X, d_1) \to (X, d_1)$ and $: (Y, d_2) \to (Y, d_2)$ be continuous maps. If f and $\mathcal{G}$ have dense minimal points and the average inverse shadowing property, we have proved $f \times \mathcal{G}$ has an average inverse shadowing property, topological transitive and dense minimal points. Moreover, we have proved f is totally strongly ergodic and weakly mixing.

Keywords: average inverse Shadowing property, topological transitive, topological ergodic, strongly ergodic.

Introduction:

The shadowing property plays an essential role in the general qualitative theory of dynamical systems. It has been developed intensively in recent years to become a significant concept of the dynamical systems that contains a lot of deep connections to the notions of stability and chaotic behavior. Shadowing of a dynamical system often justifies the validity of computer simulations of the system in use (see (1)).

In (2) he studies many important definitions including orbit and topological transitive. In this paper, more general property inverse shadowing is considered, the concept of inverse shadowing is introduced in (3). It is proved that finite dimensional systems under certain assumptions, such as semi-hyperbolicity are inverse shadowing. Inverse shadowing is extended to infinite dimensional systems.

Niu, Y. in (1) shows that if f has the average-shadowing property and the minimal points of f are dense in X, then f is totally strongly ergodic and weakly mixing. In(4) Ajam , M. H. shows some results on Strong Ergodicity and the Average Bi-Shadowing Property.

In this paper posits some needed definitions, also we have proved some basic properties and main theorems about ergodicity.

Preliminaries

Let $f: (X, d) \longrightarrow (X, d)$ be a map on a metric space (X, d) and consider the dynamical system on X is generated via the iterations of f, that is $f^0 = id_X$ and $f^{\eta+1} = f^{\eta} \circ f$, for all $\eta \in \mathbb{Z}$

Definition 2.1 (4)

In a metric space (X, d) and let $f: (X, d) \longrightarrow (X, d)$ be a map. A sequence $\{x_\iota\}_{\iota=0}^{\infty} \subset X$ is called a (true) orbit of f if $x_{\iota+1} = f(x_\iota)$, $\forall\, \iota \in \mathbb{Z}$.

Definition 2.2 (4)

In a metric space (X, d) and a map $f: (X, d) \longrightarrow (X, d)$. A sequence $\{x_\iota\}_{\iota=0}^{\infty} \subset X$ satisfying $d(f(x_\iota), x_{\iota+1}) \le \delta$, $\forall\, \iota \in \mathbb{Z}$, and $\forall\, \delta > 0$, is called δ-pseudo orbit of f.

Definition 2.3 (4)

We say that a point $x \in X$ ϵ-shadows a δ-pseudo orbit $\{x_\iota\}_{\iota=0}^{\infty}$ if the inequalities $d(f^{\iota}(x_\iota), x_\iota) \le \epsilon$, fix ϵ , $\delta > 0$ and $\iota \in \mathbb{Z}$, holds.

Definition 2.4 (5)

A compact metric space (X, d) and a continuous map $f: (X, d) \longrightarrow (X, d)$ and let f be said to be "inverse shadowing property denoted" by ISP (resp., positive inverse shadowing ISP^+) if $\forall\, \epsilon > 0$ $\exists$ $\delta > 0$ where $\forall\, x \in X$ and any δ-method $\varphi: X \to X^{\mathbb{Z}}$, there is $s \in X$ where $d(f^{k}(x), \varphi_k(s)) < \epsilon$, For all $k \in \mathbb{Z}$.

Definition 2.5 (4)

A metric space (X, d) and a continuous map $f: (X, d) \longrightarrow (X, d)$ be $\forall\, \delta > 0$, we say a sequence $\{x_\iota\}_{\iota=0}^{\infty}$ δ- average is pseudo orbit of f if $\exists$ is a positive integer $\eta_0 > 0$ where $\forall$ integers $\eta \ge \eta_0$, and $\forall$ non-negative integer k, $\frac{1}{\eta}\sum_{\iota=0}^{\eta-1} d\left(f(x_{\iota+k}), x_{\iota+k+1}\right) \le \delta$.

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Note that the δ- pseudo orbit is δ- average pseudo orbit but the convers is not always true.

Definition 2.6 (4)

A mapping f is said to have the average shadowing property (Abbrev. ASP) if $\forall \epsilon > 0$ $\exists\, \delta > 0$ where $\forall$ $\delta -$ average pseudo orbit $\{x_\iota\}_{\iota=0}^{\infty}$ is $\epsilon -$ shadowed in average by the orbit of some point $x \in X$, is that

$$\lim_{\eta\to\infty} \sup \frac{1}{\eta}\sum_{\iota=0}^{\eta-1} d\left(f^{\iota}(x)\,,\, x_\iota\right) < \epsilon\,.$$

I will give a new definition which is the average inverse shadowing property .

Definition 2.7

A mapping f is said to have the average inverse shadowing property (Abbrev. AISP) if $\forall$ $\epsilon > 0$ $\exists\, \delta > 0$ where every δ- average pseudo orbit $\{x_\iota\}_{\iota=0}^{\infty}$ is ϵ - inverse shadowed in average by the orbit of some point $s \in X$, is that

$d^*\left(f(x_\iota)\,,\,\varphi(s)\right) = max\{\, d(f(x_\iota), x_{\iota+1})\,, d(x_\iota, \varphi(s))$, Where

$$\lim_{\eta\to\infty} \sup \frac{1}{\eta}\sum_{\iota=0}^{\eta-1} d\left((f(x_\iota), x_{\iota+1}\right)$$

$< \epsilon$. And $\lim_{\eta\to\infty} \sup \frac{1}{\eta}\sum_{\iota=0}^{\eta-1} d\,(x_\iota\,, \varphi(s)) < \epsilon$.

Then $$\lim_{\eta\to\infty} \sup \frac{1}{\eta}\sum_{\iota=0}^{\eta-1} d^*\left(f(x_\iota)\,,\,\varphi(s)\right) < \epsilon\,.$$

Note that the maps that have the inverse shadowing property also have the average inverse shadowing property but the converse is not always true.

Definition 2. 8 (6)

In a metric space (X , d) a continuous mapping $f: (X , d) \to (X , d)$ is said to be "topological transitive" if every pair of non-empty is an open subset U and $V$of X , $\exists\, k \in \mathbb{N}$ where $f^{k}(U) \cap V \neq \emptyset$.

Definition 2.9 (6)

Let (X,d) be a metric space and a continuous map $f: (X , d) \to (X , d)$. If $\mathcal{Z}$ and V are two non-empty subsets of X, so let $N(\mathcal{Z}, V) = \{\iota :\, f^{\iota}(\mathcal{Z}) \cap V \neq \varphi\,, 0 \le \iota < \infty\}$.

A mapping f is called "topologically ergodic" if for any pair of nonempty open subsets $\mathcal{Z}$ and V of X, $N(\mathcal{Z}, V)$ has positive upper density , is that ,

$$\overline{D}N(\mathcal{Z}, V) = \lim_{\iota\to\infty} \sup \frac{Card(N(\mathcal{Z}, V) \cap \{0,1,\ldots,\iota-1\})}{\iota} > 0\,.$$

Definition 2.10 (7)

Let a compact metric space (X , d) and $f: (X , d) \to (X , d)$ be a continuous map , a set $k \in \mathbb{N}_0$ is said to be "syndetic" if $\exists\, m \in \mathbb{N}$ where $[j , j + m\,] \cap k \neq \varphi$ $\forall\, j \in \mathbb{N}$.

Definition 2.11 (7)

A map f is said to be "strongly ergodic" if $\forall$ pair of non-empty open subset $\mathcal{Z}, V \subset X$, $N(\mathcal{Z}, V)$ is a syndetic set.

Definition 2.12 (7)

A mapping f is said to be" totally strongly ergodic" if f^{k} is strongly ergodic $\forall\, k \in \mathbb{N}$.

Definition 2.13 (7)

A point $x \in X$ is said to be "minimal point" if $\forall$ neighborhood U of x , $N(x, U)$ is syndetic, denoted by $AP(f)$ the set of all minimal points of f.

3. Basic properties

The goal of this section of the paper in to give the main theorems and give a proof of some properties.

Theorem 3.1

Let (X , d) be a metric space and $f: (X , d) \to (X , d)$ be a map. If f has the average inverse shadowing property, then f^{k} has the average inverse shadowing property $\forall\, k \in \mathbb{N}$.

Proof :

Let $k \in \mathbb{N}$, since f has the average inverse shadowing property, $\forall$ $\epsilon > 0$ $\exists\, \delta > 0$ where every δ-average pseudo orbit is $\frac{\epsilon}{k}$ inverse shadowing in average by some orbit in X .Suppose $\{y_\iota\}_{\iota=0}^{\infty}$ is δ-average pseudo orbit of f^{k} , is that , $\exists\, m_0 > 0$ where

$$\frac{1}{\eta}\sum_{\iota=0}^{\eta-1} d\left(f^{k}(y_{\iota+h})\,,\, y_{\iota+h+1}\right) < \delta\,. \quad \text{for all } \eta \ge m_0 \text{ and } h \in \mathbb{N}_0\,.$$

We write $x_{\eta k + j} = f^{j}(x_\eta)$ for $0 \le j < k$, $\eta \in \mathbb{N}_0$, that is $\{x_\iota\}_{\iota=0}^{\infty} = \{\, y_0 , f(y_0), \ldots , f^{k-1}(y_0), y_1, f(y_1), \ldots , f^{k-1}(y_1), \ldots .\}$.

We have $$\frac{1}{\eta}\sum_{\iota=0}^{\eta-1} d\left(f(x_{\iota+h})\,,\, x_{\iota+h+1}\right) < \delta\,.$$

For all $\eta \ge m_0$ and $h \in \mathbb{N}_0$.

Then $\{x_\iota\}_{\iota=0}^{\infty}$ is δ-average pseudo orbit of f . by definition of map that have the average inverse shadowing property there is f continuous map on X . satisfying :

$$\frac{1}{\eta}\sum_{\iota=0}^{\eta-1} d^*\left(f(x_\iota)\,,\,\varphi(s)\right) < \epsilon\,. \text{ Where}$$

$$\lim_{\eta\to\infty} \sup \frac{1}{\eta}\sum_{\iota=0}^{\eta-1} d^*\left(f(x_\iota)\,,\,\varphi(s)\right) \le \frac{\epsilon}{k}\,. \quad (3.1)$$

Claim that there are infinite $\eta \in \mathbb{N}$ where

$$\frac{1}{\eta}\sum_{\iota=0}^{\eta-1} d^{*}\left(f^{k}(x_{\iota}),\ \varphi(s)\right) < \epsilon\ .$$

To proof the claim , assume on the contrary that $\exists\ \eta_0 \in \mathbb{N}$ where :

$$\frac{1}{\eta}\sum_{\iota=0}^{\eta-1} d^{*}\left(f^{k}(x_{\iota}),\ \varphi(s)\right) \geq \epsilon\ .\ \text{For all}\ \eta \geq \eta_0\ .\ \text{Then}$$

$$\lim_{\eta\to\infty}\sup \frac{1}{\eta}\sum_{\iota=0}^{\eta-1} d^{*}\left(f(x_{\iota}),\ \varphi(s)\right) \geq \frac{\epsilon}{k}\ .$$

Which contracts with (3.1). The proof of the claim is completed.

by the claim ,we have :

$$\lim_{\eta\to\infty}\sup \frac{1}{\eta}\sum_{\iota=0}^{\eta-1} d^{*}\left(f^{k}(x_{\iota}),\ \varphi(s)\right) < \epsilon\ .$$

Since $\varphi(s) = \ \varphi_{k}(s)$

$$\lim_{\eta\to\infty}\sup \frac{1}{\eta}\sum_{i=0}^{\eta-1} d^{*}\left(f^{k}(x_{\iota}),\ \varphi_{k}(s)\right) < \epsilon\ .$$

Hence f^{k} have the average inverse shadowing property . ■

Theorem 3.2

Let (X, d_1) and (Y, d_2) be two metric space , f: $(X, d_1) \to (X, d_1)$ and $\mathcal{G}$: $(Y, d_2) \to (Y, d_2)$ be maps .If f and $\mathcal{G}$ have the average inverse shadowing property, then $f\times\mathcal{G} : (X \times Y, d_1) \to (X \times Y, d_2)$ has the average inverse shadowing property .

Proof :

Suppose f has the average inverse shadowing property by definition if $\forall\ \epsilon > 0$ there is $\delta_1 > 0$ where $\forall\ \delta$ - average pseudo orbit $\{x_{\iota}\}_{\iota\in\mathbb{Z}}$ is ϵ - inverse shadowed in average by the orbit of some point $s_1 \in X$ is that

$$\frac{1}{\eta}\sum_{\iota=0}^{\eta-1} d^{*}\left(f(x_{\iota}),\ \varphi(s_1)\right) < \epsilon\ .$$

Since $\mathcal{G}$ has the average inverse shadowing property by definition if $\forall\ \ \epsilon_2 > 0\ \exists\ \delta_2 > 0$ where $\forall\ \delta$ - average pseudo orbit $\{y_{\iota}\}_{\iota\in\mathbb{Z}}$ is ϵ - inverse shadowed in average by the orbit of some point $s_2 \in Y$ is that

$$\frac{1}{\eta}\sum_{\iota=0}^{\eta-1} d^{*}\left(\mathcal{G}(y_{\iota}),\ \varphi(s_2)\right) < \epsilon_2\ .$$

We choose $\delta = max\{\delta_1, \delta_2\}$ where $\forall\ \delta$- average pseudo orbit $w = \{(x_{\iota}, y_{\iota})\}_{\iota\in\mathbb{Z}} \in X\times Y$ is ϵ - inverse shadowed in average by the orbit of some point $s = s_1 \times s_2 \in X\times Y$ is that

$$\frac{1}{\eta}\sum_{\iota=0}^{\eta-1} d^{*}\left(f\times\mathcal{G}(w),\ \varphi(s)\right) < \epsilon\ .$$

Hence $f\times\mathcal{G}$ has the average inverse shadowing property .■

Main results:

The goal of this section is to view the main results and theorems.

Theorem 4.1

Let (X, d) be compact metric space and $f : (X, d) \to (X, d)$ be continuous map , if f has the average inverse shadowing property and the set of all minimal *points* of f are dense in X, then f is strongly ergodic .

Proof :

Let $\mathcal{Z}$ and V be any nonempty open subsets of X . Assume that the minimal point of f is dense in X then we select $u \in \mathcal{Z} \cap AP(f)$, $w \in V \cap AP(f)$, and $\epsilon > 0$ so that $B(u,\epsilon) \subset \mathcal{Z}$,$B(w,\epsilon) \subset V$, where $B(\alpha,\epsilon) = \{\beta \in X :\ d(\alpha,\beta) < \epsilon\}$. By $u, w\ \in AP(f)$,we have that $I_{u} = \{\eta \in \mathbb{N}_0, f^{\eta}(u) \in B\left(u, \frac{\epsilon}{2}\right)\}$ and $I_{w} = \{\eta \in \mathbb{N}_0, f^{\eta}(w) \in B\left(w, \frac{\epsilon}{2}\right)\}$ are syndetic , then there are $k', k'' \in \mathbb{N}$ so that $[\kappa, \kappa + k'] \cap I_{\eta} \neq \varphi$ and $[\kappa, \kappa + k''] \cap I_{w} \neq \varphi\ \ \forall\ k \in \mathbb{N}$. Let $k = \max\left\{k', k''\right\}$. $\exists\ \delta > 0$ So that $d(u, w) < \delta$ implies $d(f^{\iota}(u), f^{\iota}(w)) < \frac{\epsilon}{2}$ when $i = 0,1, \dots, k$ by continuity and compactness. Since f has the average inverse shadowing property for $\epsilon > 0$ when $\frac{\delta}{2} = \epsilon$, $\exists\ \delta_1$ where $0 < \delta_1 < \frac{\delta}{2}$ and every δ_1-average pseudo orbit is $\frac{\delta}{2}$ inverse shadowing in average by some orbit in X .

Choose $k_0 \in \mathbb{N}$ where $\frac{3D}{k_0} < \delta_1$. Where $D = daim(x)$, is that $D = sup\{d(u, w): u, w \in X\}$

We define the $2k_0$ periodic sequence $\{x_{\iota}\}_{\iota=0}^{\infty}$ with $x_0 = u, x_1 = f(u), \dots, x_{k_0-1} = f^{k_0-1}(u)$, $x_{k_0} = w, x_{k_0+1} = f(w), \dots, x_{2k_0-1} = f^{k_0-1}(w)$.

$\forall\ \eta \geq k_0$ and $0 \leq h < \infty$,

$$\frac{1}{\eta}\sum_{\iota=0}^{\eta-1} d\left(f(x_{\iota+h}),\ x_{\iota+h+1}\right) < \frac{[\frac{\eta}{k_0}]\times 3D}{\eta} \leq \frac{3D}{k_0} < \delta_1\ .$$

Thus $\{x_{\iota}\}_{\iota=0}^{\infty}$ is a periodic δ_1- average pseudo orbit of f, via definition of map that have the average inverse shadowing property, $\exists\ f$ continuous map on X .

$$\lim_{\eta\to\infty}\sup \frac{1}{\eta}\sum_{\iota=0}^{\eta-1} d^{*}\left(f(x_{\iota}),\ \varphi(s)\right) < \epsilon = \frac{\delta}{2}\ . \quad (4.1)$$

Claim one: There are infinite $\iota \in \mathbb{N}$ where $x_\iota \in \{w, f(w), \dots, f^{k_0-1}(w)\}$ and $d^*(f(x_i), \varphi(s)) < \delta$.

Claim Two: There are infinite $\iota \in \mathbb{N}$ where $x_\iota \in \{w, f(w), \dots, f^{k_0-1}(w)\}$ and $d^*(f(x_\iota), \varphi(s)) < \delta$.

To proof claim one: Assume on the contrary that $\exists\ L \in \mathbb{N}$ where if $x_\iota \in \{w, f(w), \dots, f^{k_0-1}(w)\}$ then $d^*(f(x_\iota), \varphi(s)) \geq \delta$. $\forall\ \iota \geq L$ we have

$$\limsup_{\eta\to\infty} \frac{1}{\eta}\sum_{\iota=0}^{\eta-1} d^*\left(f(x_\iota), \varphi(s)\right) \geq \frac{\delta}{2}.$$

Which contracts (4.1) based on the claim is true.

In the same way we can prove claim two by the claim one, there are $j_0 > k_0$, $0 \leq j_0 \leq k_0 - 1$, $M_0 > j_0 + k$, $0 \leq j_0 \leq k_0 - 1$ where

$x_{j_0} = f^{\iota_0}(w), d^*\left(f(x_{j_0}), \varphi(s)\right) < \delta$ (2.3)

$x_{m_0} = f^{j_0}(w), d^*\left(f(x_{m_0}), \varphi(s)\right) < \delta$ (2.4)

Since $[\iota_0, \iota_0 + k] \cap I_u \neq \varphi$ and $[j_0, j_0 + k] \cap I_w \neq \varphi$ there are $0 \leq \iota, j \leq k$ where $f^{\iota_0+\iota}(w) \in B\left(w, \frac{\epsilon}{2}\right)$, $f^{j_0+j}(w) \in B\left(w, \frac{\epsilon}{2}\right)$ by the formula (2.3), we obtain $d^*\left(f^{\iota_0+\iota}(u), \varphi(s)\right) < \frac{\epsilon}{2}$ and by (2.4) $d^*\left(f^{j_0+j}(w), \varphi(s)\right) < \frac{\epsilon}{2}$. Thus $w_{\iota_0+\iota} \in B(w, \epsilon) \subset \mathcal{Z}, w_{j_0+j} \in B(w, \epsilon) \subset V$

Let $k_0 = m_0 + j - j_0 - I > 0$, it follows that $\mathcal{Z} \cap f^{k_0}(V) \neq \varphi$.

Since $\mathcal{Z}$, Vare arbitrary, f is topological transitive .

We write $U = \mathcal{Z} \cap f^{k_0}(V) \neq \varphi$, then $\exists\ p \in AP(f) \cap U$.

Let $J = \{\eta \in \mathbb{N}_0, f^{\eta}(P) \in U\}$, then J is syndetic when $s \in J, U \cap f^{-s}(U) \neq \varphi$ since $\varphi \neq \mathcal{Z} \cap f^{-k_0}(V) \cap f^{-s}\left(\mathcal{Z} \cap f^{-k_0}(V)\right) \subset \mathcal{Z} \cap f^{-(k_0+s)}(V)$.

$N(\mathcal{Z}, V) \supset \{k_0 + s : s \in J\}$.Based on $N(\mathcal{Z}, V)$ is syndetic, since $\mathcal{Z}, V$ are arbitrary, f is strongly ergodic .■

Theorem 4.2

Let (X, d) be compact metric space and $f: (X, d) \to (X, d)$ be continuous map. If f has the average inverse shadowing property and minimal point of f are dense in X, then f is totally strongly ergodic .

Proof :

$\forall\ k \in N$,by Theorem(3.1), f^{k} have the average inverse shadowing property .It is well known that $AP(f) = AP(f^{k})$. Enforcement Theorem (4. 1), for f^{k}, we obtain that f^{k} is strongly ergodic .Therefore; f is totally strongly ergodic .■

Proposition 4.3 (1)

Let (X, d_1) and (Y, d_2) be compact metric spaces , $f: (X, d_1) \to (X, d_1)$ and $\mathcal{G}: (Y, d_2) \to (Y, d_2)$ be continuous maps and f and $\mathcal{G}$ are onto. If f and $\mathcal{G}$ each have dense minimal points then so does the product $f \times \mathcal{G} : (X \times Y, d_1) \to (X \times Y, d_2)$.

Lemma 4.4

Let (X, d_1) and (Y, d_2) be compact metric spaces, $f: (X, d_1) \to (X, d_1)$ and $\mathcal{G}: (Y, d_2) \to (Y, d_2)$ be continuous maps. If f and $\mathcal{G}$ have dense minimal points and have the average inverse shadowing property, then:

1. $f \times \mathcal{G}$ has dense minimal points and has the average inverse shadowing property .
2. $f: (X, d_1) \to (X, d_1)$ and $\mathcal{G}: (Y, d_2) \to (Y, d_2)$ are weakly disjoint, that is , $f \times \mathcal{G}$ is topological transitive .
3. f is topologically weakly mixing and totally strongly ergodic .

Proof

(1) Since each of f and $\mathcal{G}$ has dense minimal points and has the average inverse shadowing property , by Theorem(4.1), each of f and $\mathcal{G}$ is strongly ergodic , so that f and $\mathcal{G}$ are (2.8) . As a topological transitive map is onto by Theorem(3.2) and Proposition (3.4) $f \times \mathcal{G}$ have dense minimal points and has the average inverse shadowing property . ■

(2) By Theorem (4.1) and (1) $f \times \mathcal{G}$ is topological ergodic, it is of course topological transitive. ■

(3) By (2), $\mathcal{G}$ is topologically weakly mixing and by Theorem (4.2) , $\mathcal{G}$ is totally strongly ergodic.■

Conflicts of Interest: None.

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بعض النتائج حول خاصية معدل معكوس الظل والاركوديكية قوية

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الخلاصة:

ليكن (X, d_1) و (Y, d_2) فضائيين متريين مرصوصين، و ليكن $f: (X, d_1) \to (X, d_1)$, $\mathcal{G}: (Y, d_2) \to (Y, d_2)$ دوال مستمرة. برهننا على ان إذا f و $\mathcal{G}$ لديهما نقاط الحد الأدنى كثيفة وخاصية معدل معكوس الظل و فان $f \times \mathcal{G}$ لديها خاصية معدل معكوس الظل، ونقاط الحد الأدنى لها كثيفة، متعدية تبولوجيا. كما اثبتنا ان f هو إرجوديك كليا بقوة و خلط تبولوجي ضعيف.

الكلمات المفتاحية: خاصية معدل معكوس الظل، متعدية تبولوجيا، تبولوجي ارجوديك، ارجوديك كليا بقوة .