Iraqi Journal of Statistical Sciences, Vol. 20, No. 2, 2023, Pp (134-154)

Iraqi Journal of Statistical Sciences

www.stats.mosuljournals.com

Exponential Power-Chen Distribution and Its Some Properties

Noorsl Ahmed ZEENALABIDEN[1] and Buğra SARAÇOĞLU[2]

[1] Selcuk University, Graduate School of Natural and Applied Science, Department of Statistics, Phd Student, Konya, Turkey
[2] Selçuk University, Science Faculty, Department of Statistics, Konya, TURKEY

Article information

Article history:
Received September 28, 2023
Accepted November 28, 2023
Available online December 1, 2023

Keywords:
Exponential Power - X family of distributions, Exponential Power Chen distribution, maximum likelihood estimation (MLE), Monte Carlo simulation

Correspondence:
Noorsl Ahmed ZEENALABIDEN[1]
zeennorsal@yahoo.com

Abstract

In this study, it has been aimed to introduce a new statistical distribution called Exponential Power - Chen by using the method suggested by Alzaatreh et al. (2013). Some statistical properties such as moments, coefficies of skewness and kurtosis, random number generator for Exponential Power Chen (EP-CH) distribution are obtained. Moreover, the maximum likelihood estimators (MLEs) for unknown parameters of EP-CH distribution have been derived and a Monte Carlo simulation study based on mean square errors and biases of this estimators for various sample sizes have been performed. Finally, an application using real data set has been presented for this new distribution.

DOI10.33899/IQJOSS.2023.0181219 ,

1. INTRODUCTION

A data set can show fitting to many distributions. It is important for statistical inference to determine statistical distribution that best fits to a data set. In recent years, many new statistical distributions have been suggested to modeling real data sets. These new distributions have better fit than current distributions for some real data sets. In literature, many methods have been developed to obtain new continuous distributions. Eugene et al. [9] introduced a family of distributions generated by beta distributions. Cordeiro and Castro [6] introduced the family of distributions generated by the Kumaraswamy distribution. Nadarajah and Kotz [15,16] suggested The beta gumbel and the beta exponential distributions. Akinsete et.al. [1] introduced The beta-pareto distribution. Alzaatreh et al. [2] introduced the new class of distributions by extend method of Eugene et al. [8]. The motivation of this study is method suggested by Alzaatreh et al. [2]. This method can be defined as follows. Suppose $k(t)$ and $K(t)$ is probability density function (pdf) and cumulative distribution function (cdf) of a continuous $T \in [a,b]$, $-\infty < a < b < \infty$ random variable, respectively. Let G(x) is cumulative distribution function (cdf) of any random variable X and $W(G(x))$ is a function that has the following properties.

i. $W(G(x)) \in [a,b]$

ii. $W(G(x))$ is a differantiable and monotone non-decreasing function.

iii. When $x \to -\infty$, $W(G(x)) \to a$ and while $x \to \infty$, $W(G(x)) \to b$.

In this case ,the family of new distributions is defined as follows

$$F(x) = \int_a^{W(G(x))} k(t)dt = K\left(W\left(G(x)\right)\right) \quad (1)$$

New distributions obtained by using this method are called as $T-X$ distributions family. (Alzaatreh et al. [2]). Recently, many researchers have found new statistical distributions using this method. Some of these studies can be included as follows: Alzaatreh et al. [2,3] introduced Beta-Exponential-X distribution, Weibull-Pareto distribution and Weibull-X families of distributions. Alzaghal et al. [4] suggested Exponentiated T-X Family of distributions, Tahir et al. [22,23,24] introduced the odd generalized exponential family of distributions, The logistic-X family of distributions and A new Weibull family of distributions. Çelik and Guloksuz [8] suggested a new lifetime distribution called as Uniform-Exponential Distribution.

The main purpose of this study is to introduce a new statiscal distribution with four parameters by using method of Alzaatreh et.al. [2] and this new distribution is called as Exponential Power Chen (EPCh) distribution. The rest of this paper is organized as follows. In section 2, information is given about Exponential power and Chen distributions. In section 3, EPCh distribution with parameters $(\lambda, \alpha, \beta, \theta)$ have been introduced. In section 4, the some statistical properties such as hazard function, random number generator, moment generating function, moments, variance, skewness and kurtosis coefficients, renyi and shannon enropies for this new distribution are presented. In section 5, maximum likelihood (ML) estimators for parameters of EPCh distribution are obtained. In section 6, a simulation study to see the performances of this estimators in terms of mean square errors (MSEs) and biases is performed. In section 7, a real data analysis is presented. Finally, conclusion is given in section 8.

2. Exponential Power (EP) and Chen Distributions

EP distribution introduced by Smith and Bain [21] is used to modeling lifetime data. The cdf, pdf and hazard function (hf) of a random variable X having EP distribution with α and β parameters can be written in order as follows :

$$F\left(x\right) = 1 - \exp\left(1 - \exp\left(\frac{x}{\alpha}\right)^{\beta}\right) \quad (2)$$

$$f\left(x\right) = x^{\beta-1}\alpha^{-\beta}\beta\exp\left(\left(\frac{x}{\alpha}\right)^{\beta}\right)\exp\left(1 - \exp\left(\frac{x}{\alpha}\right)^{\beta}\right) \quad (3)$$

$$h\left(x\right) = x^{\beta-1}\alpha^{-\beta}\beta\exp\left(\left(\frac{x}{\alpha}\right)^{\beta}\right) \quad (4)$$

Another distribution used to model lifetime data is Chen distribution suggested by Chen [7]. The cdf, pdf and hf of a random variable X having Chen distribution with λ and θ parameters are given, respectively, by

$$G\left(x\right) = 1 - \exp\left(\lambda\left(1 - \exp(x^{\theta})\right)\right) \quad (5)$$

$$g(x)=\lambda x^{\theta-1}\theta\exp(x^{\theta})\exp\left(\lambda\left(1-\exp(x^{\theta})\right)\right) \tag{6}$$

$$h(x)=\lambda x^{\theta-1}\theta\exp(x^{\theta}) \tag{7}$$

3. Exponential Power-Chen Distribution

This new distribution is obtained by using method of Alzaatreh et al. [2]. Suppose that $W\left(G(x)\right)$ in Eq. (1.1) is defined as follows:

$$\begin{aligned} W\left(G(x)\right) &= -\log\left(1-G(x)\right) \\ &= \lambda\left(\mathrm{e}^{x^{\theta}}-1\right), \end{aligned} \tag{8}$$

where $G(x)$ is defined in Eq. (5). If it is used pdf of EP distribution defined in Eq. (3) instead of $k(t)$ and $a=0$, a new distribution called as EP-Ch distribution with λ,α,β and θ parameters is obtained. Cdf, pdf, hf, inverse hazard functions (ihf) and rf of $EPCh(\lambda,\alpha,\beta,\theta)$ distribution with are given as follows.

$$\begin{aligned} F(x;\alpha,\beta,\lambda,\theta) &= \int_{0}^{\lambda\left(\exp(x^{\theta})-1\right)} t^{\beta-1}\alpha^{-\beta}\beta\exp\left(\left(\frac{t}{\alpha}\right)^{\beta}\right)\exp\left(1-\exp\left(\frac{t}{\alpha}\right)^{\beta}\right)dt \\ &= 1-\exp\left(1-\exp\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^{\theta}}-1\right)^{\beta}\right)\right) \end{aligned} \tag{9}$$

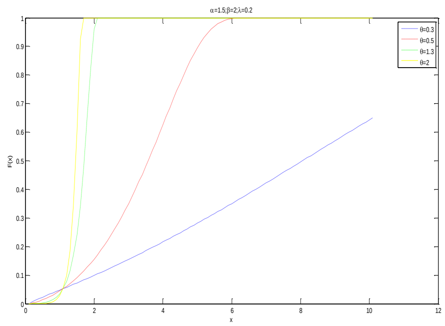
$$\begin{aligned} f(x;\alpha,\beta,\lambda,\theta) &= \left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^{\theta}}-1\right)^{\beta-1}x^{\theta-1}\theta\beta\exp(x^{\theta})\exp\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^{\theta}}-1\right)^{\beta}\right)\times \\ &\times\exp\left(1-\exp\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^{\theta}}-1\right)^{\beta}\right)\right) \end{aligned} \tag{10}$$

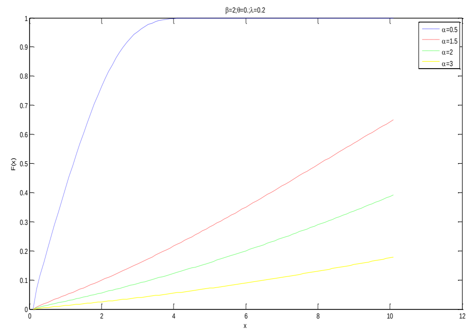
$$\begin{aligned} h(x;\alpha,\beta,\lambda,\theta) &= \frac{f(x)}{1-F(x)}=\frac{f(x)}{R(x)} \\ &= \left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^{\theta}}-1\right)^{\beta-1}x^{\theta-1}\theta\beta e^{x^{\theta}}\exp\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^{\theta}}-1\right)^{\beta}\right) \end{aligned} \tag{11}$$

$$\begin{aligned} r(x) &= \frac{f(x)}{F(x)} \\ &= \frac{\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^{\theta}}-1\right)^{\beta-1}x^{\theta-1}\theta\beta e^{x^{\theta}}}{1-\exp\left(1-\exp\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^{\theta}}-1\right)\right)\right)}\times \\ &\times\exp\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^{\theta}}-1\right)^{\beta}\right)\exp\left(1-\exp\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^{\theta}}-1\right)^{\beta}\right)\right) \end{aligned} \tag{12}$$

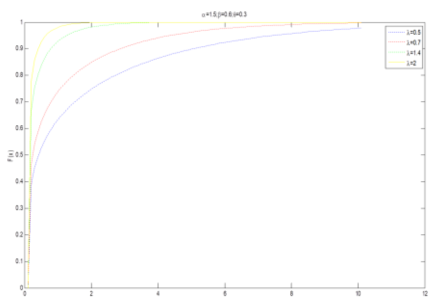
$$R(x;\alpha,\beta,\lambda,\theta)=e\,\mathrm{xp}\left(1-\exp\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^{\theta}}-1\right)^{\beta}\right)\right) \tag{13}$$

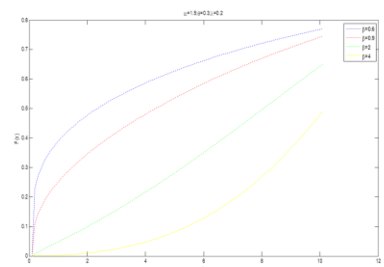
The plots of df, pdf, hf and ihf for the various parameter values of the $EPCh(\lambda,\alpha,\beta,\theta)$ distribution are given in the following order: Figure 1, Figure 2, Figure 3 and Figure 4.





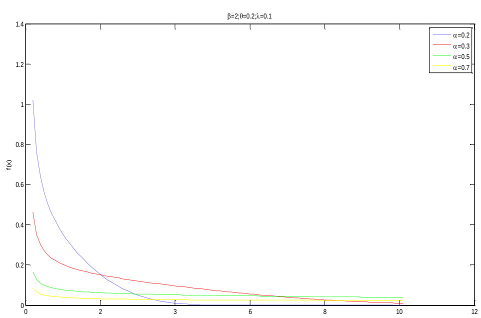
$\alpha=1.5,\beta=2,\lambda=0.2$ and $\theta=0.3,0.5,1.3,2$ $\quad$ $\beta=2,\theta=0.3,\lambda=0.2$ and $\alpha=0.5,1.5,2,3$

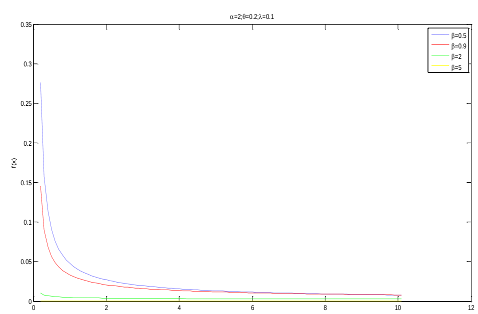




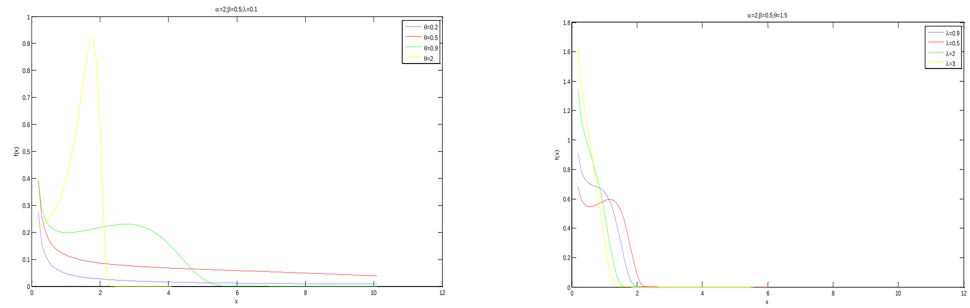
$\alpha=1.5,\theta=0.3,\lambda=0.2$ and $\beta=0.6,0.9,2,4$ $\quad$ $\alpha=1.5,\beta=0.6,\theta=0.3$ and $\lambda=0.5,0.7,1.4,2$

Figure 1. Df plots of EP-Ch distribution for different parameter values



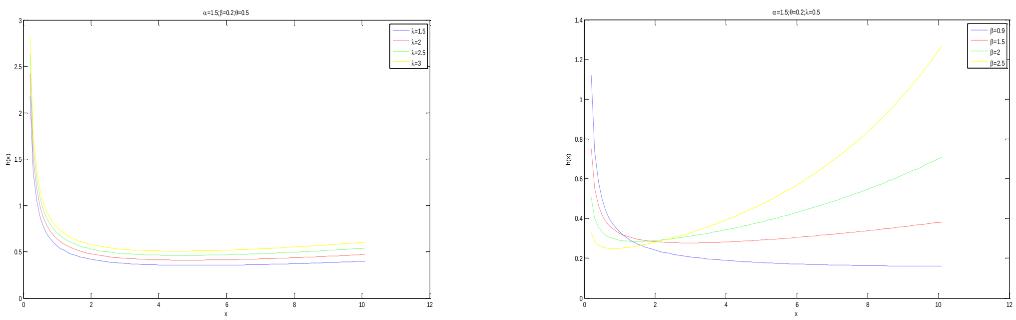


$\beta=2,\theta=0.2,\lambda=0.1$ and $\alpha=0.2,0.3,0.5,0.7$ $\quad$ $\alpha=2.5,\theta=0.2,\lambda=0.1$ and $\beta=0.5,0.9,2,6$

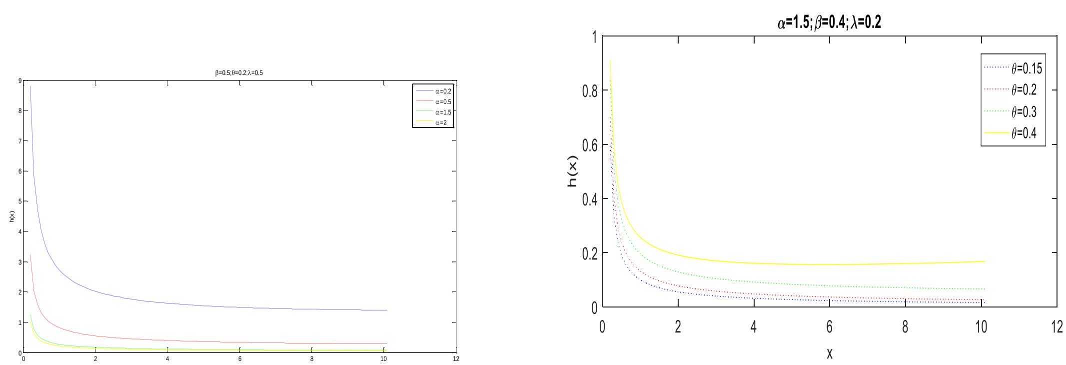


$\alpha = 2, \beta = 0.5, \lambda = 0.1$ and $\theta = 0.2, 0.5, 0.9, 2$ $\quad$ $\alpha = 2, \beta = 0.5, \theta = 1.5$ and $\lambda = 0.9, 0.5, 2, 3$

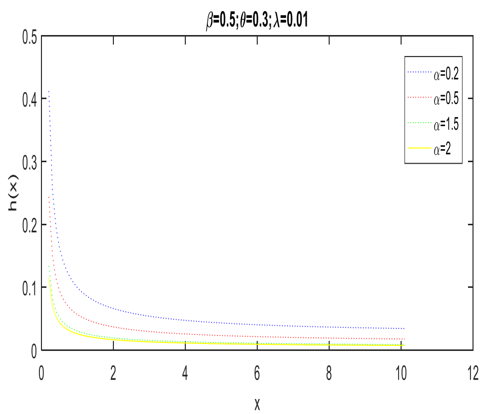
Figure 2. pdf plots of EP-Ch distribution for different parameter values

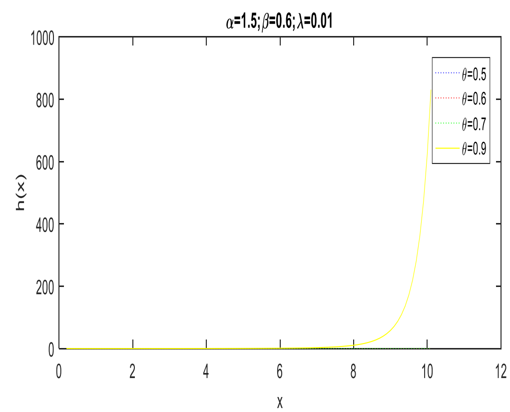


$\alpha = 1.5, \beta = 0.2, \theta = 0.5$ ve $\lambda = 1.5, 2, 2.5, 3$ $\quad$ $\alpha = 1.5, \theta = 0.2, \lambda = 0.5$ *ve* $\beta = 0.9, 1.5, 2, 2.5$

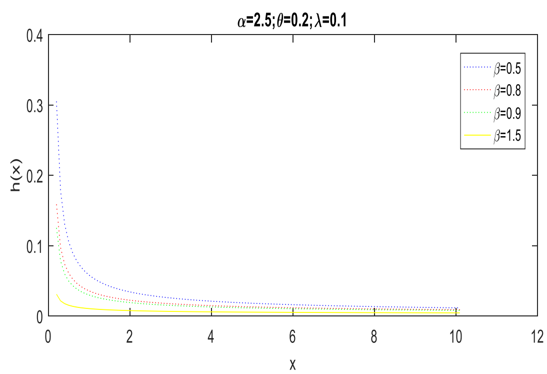


$\beta = 0.5, \theta = 0.2, \lambda = 0.5$ ve $\alpha = 0.2, 0.5, 1.5, 2$ $\quad$ $\alpha = 1.5, \beta = 0.4, \lambda = 0.2$ ve $\theta = 0.15, 0.2, 0.3, 0.4$



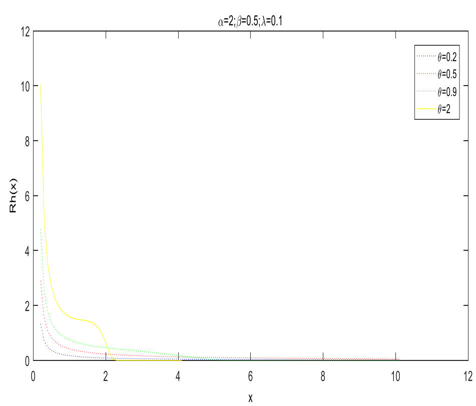


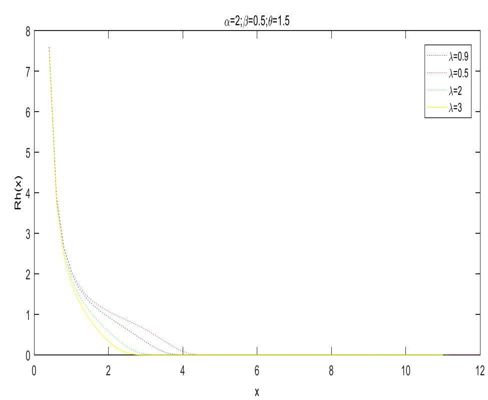
$\beta = 0.5, \theta = 0.3, \lambda = 0.01$ ve $\alpha = 0.2, 0.5, 1.5, 2$ $\alpha = 1.5, \beta = 0.6, \lambda = 0.01$ *ve* $\theta = 0.5, 0.6, 0.7, 0.9$



$\alpha = 2.5, \theta = 0.2, \lambda = 0.1$ *ve* $\beta = 0.5, 0.8, 0.9, 1.5$

Figure 3. hf plots of EP-Ch distribution for different parameter values





$\alpha = 2, \beta = 0.5, \lambda = 0.1$ and $\theta = 0.2, 0.5, 0.9, 2$ $\alpha = 2, \beta = 0.5, \theta = 1.5$ and $\lambda = 0.9, 0.5, 2, 3$

Figure 4. ihf function plots of EP-Ch distribution for different parameter values

4. Some Statistical Properties for EP-Ch distribution

4.1. Random Numer Generator for EP-Ch Distribution

The method of inversion transformation has been used to generate random numbers from $EPCh(\lambda, \alpha, \beta, \theta)$ distribution as following

$$F(x)=u \Rightarrow x=\frac{1}{\theta}\log\left(\exp\left(-\frac{-\log\left(\log\left(1-\log\left(1-u\right)\right)\right)+\beta\log\left(\frac{\lambda}{\alpha}\right)}{\beta}+1\right)\right) \tag{14}$$

Where u is defined on the unit interval (0,1). When u = 0.5 in Eq (14) the median for EPCh distribution is obtained. In this case, the median can be written as follows;

$$median=\frac{1}{\theta}\log\left(\exp\left(-\frac{-\log\left(\log\left(1-\log\left(\frac{1}{2}\right)\right)\right)+\beta\log\left(\frac{\lambda}{\alpha}\right)}{\beta}+1\right)\right) \tag{15}$$

4.2.Moments for EP-Ch distribution

The r^{th} moment of a random variable X having EPCh distribution with $(\lambda,\alpha,\beta,\theta)$ parameter is obtained as follows;

$$\begin{aligned} E\left(X^r\right)&=\int_0^\infty x^r f\left(x\right)dx \\ &=\int_0^\infty x^r\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^\theta}-1\right)^{\beta-1}\beta x^{\theta-1}\theta y(x,\alpha,\beta,\lambda,\theta)dx \end{aligned} \tag{16}$$

Where y can be written as follows:

$$y(x,\alpha,\beta,\lambda,\theta)=\exp\left(x^\theta+\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^\theta}-1\right)^{\beta}+1-\exp\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^\theta}-1\right)^{\beta}\right)\right)$$

From the equation (16), as a result of $u=\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{\left(x^\theta\right)}-1\right)^{\beta}\right)$ transformation, r^{th} moment is obtained as follows.

$$E\left(X^r\right)=\int_0^\infty\left(\ln\left(1+\frac{\alpha}{\lambda}u^{\frac{1}{\beta}}\right)\right)^{\frac{r}{\theta}}e^{u}e^{1-e^{u}}du \tag{17}$$

By using the equation (17), the coefficients of skewness (CS) and kurtosis (CK) can be computed using the following formulas;

$$CS=\frac{E\left(X^3\right)-3E\left(X\right)E\left(X^2\right)+2\left(E\left(X\right)\right)^3}{\left(E\left(X^2\right)-\left(E\left(X\right)\right)^2\right)^{\frac{3}{2}}} \tag{18}$$

$$\text{CK}=\frac{E\left(X^4\right)-4E\left(X\right)E\left(X^3\right)+6\left(E\left(X\right)\right)^2E\left(X^2\right)-3\left(E\left(X\right)\right)^4}{\left[E\left(X^2\right)-\left(E\left(X\right)\right)^2\right]^2} \tag{19}$$

For different parameter values of EP-Ch distribution, the r^{th} moment, variance, skewness and kurtosis coefficients are given in Table 1.

Table1. r^{th} moment, variance skewness and kurtosis values for EP-Ch distribution

$(\alpha,\beta,\theta,\lambda)$	$E(X)$	$E(X^2)$	$E(X^3)$	$E(X^4)$	$Var(X)$	Skewness	Kurtosis
$(0.6,0.5,1,0.5)$	0.4065	0.3187	0.3163	0.3591	0.1535	1.0300	3.3540
$(0.6,0.5,1,1)$	0.2414	0.1221	0.0812	0.0632	0.0638	1.2960	4.2560
$(0.6,0.5,1,2)$	0.1357	0.0413	0.0177	0.0086	0.0229	1.5390	5.2950
$(0.6,2,0.5,0.7)$	0.2424	0.0786	0.0295	0.0122	0.0198	0.3006	2.3390
$(0.6,2,1,0.7)$	0.4661	0.2424	0.1349	0.0786	0.0251	-0.4042	2.5590
$(0.6,2,2,0.7)$	0.6704	0.4661	0.3327	0.2424	0.0167	-0.9544	3.8510
$(0.6,0.5,1,0.7)$	0.3180	0.2033	0.1675	0.1596	0.1022	1.1610	3.7670
$(0.6,1,1,0.7)$	0.3859	0.2019	0.1223	0.0813	0.0529	0.2814	2.2520
$(0.6,2,1,0.7)$	0.4661	0.2424	0.1349	0.0786	0.0251	-0.4042	2.5590
$(0.5,0.5,1,0.7)$	0.2768	0.1573	0.1164	0.1003	0.0807	1.2310	4.0110
$(1,0.5,1,0.7)$	0.4591	0.3976	0.4318	0.5337	0.1868	0.9616	3.1610
$(2,0.5,1,0.7)$	0.7158	0.8854	1.3263	2.2171	0.3730	0.6951	2.5440

The plots of coefficients of skewness and kurtosis are given in Figure 5 and Figure 6.

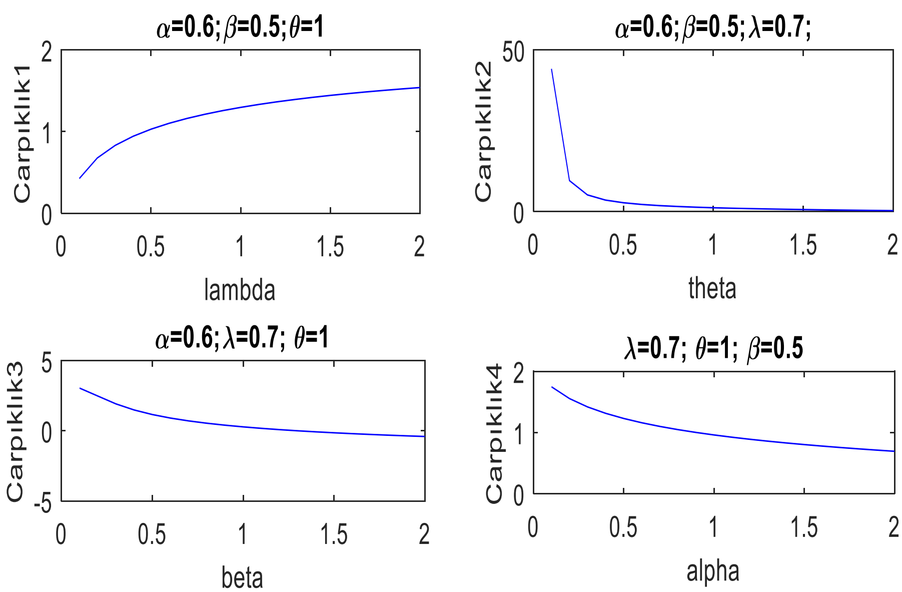


Figure 5. The plots of coefficient of Skewness for EPCh distribution

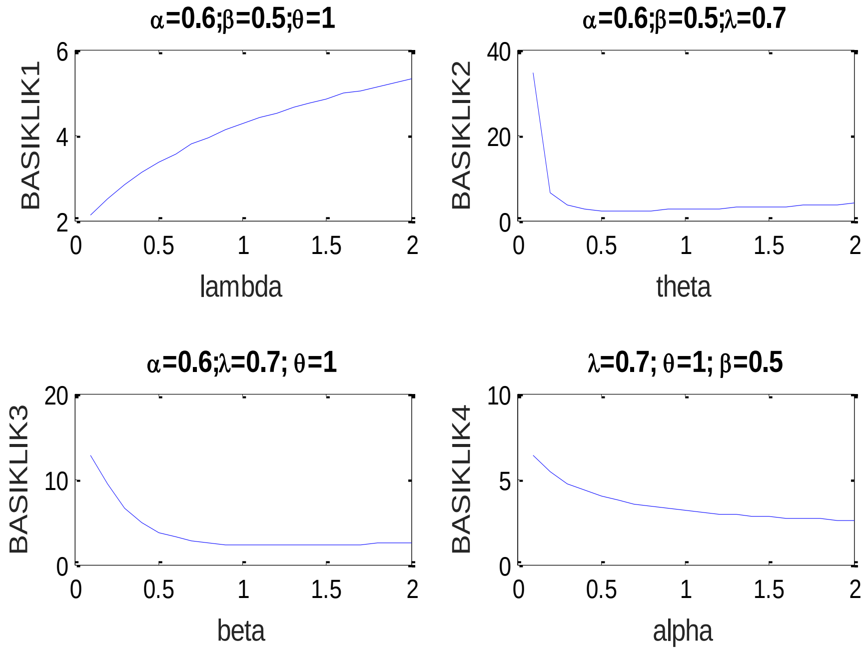


Figure 6. The plots of coefficient of Kurtosis for EPCh distribution

4.3.Moment Generating Function

The moment-generating function (mgf) of a random variable X having EP-Ch$(\lambda,\alpha,\beta,\theta)$ distribution, $M_x(t)$, is obtained as follows.

$$\begin{aligned} M_x(t) &= E\left(e^{tX}\right) \\ &= \int_0^\infty e^{tX} f(x)\,dx \\ &= \sum_{n=0}^{\infty}\frac{t^n}{n!}E\left(X^n\right) \qquad (20) \\ &= \sum_{n=0}^{\infty}\frac{t^n}{n!}\int_0^\infty\left(\ln\left(1+\frac{\alpha}{\lambda}u^{\frac{1}{\beta}}\right)\right)^{\frac{n}{\theta}} e^u e^{1-e^u}\,du \end{aligned}$$

4.4. Order Statistics for EP-Ch Distribution

Let $X_1, X_2 ..., X_n$ be a random sample taken from $EPCh(\lambda,\alpha,\beta,\theta)$ distribution. Let $X_{(1:n)} \le X_{(2:n)} ... \le X_{(n:n)}$ indicate the order statistics obtained from this sample. The pdf of the i^{th} order statistic for $i = 1,...,n$ is shown as $f_{i:n}(x)$ and it is given by;

$$f_{i:n}\left(x,\underline{\mu}\right)=\frac{1}{B(i,n-i+1)}f\left(x,\underline{\mu}\right)\left[F\left(x,\underline{\mu}\right)\right]^{i-1}\left[1-F\left(x,\underline{\mu}\right)\right]^{n-i}$$

$$=\frac{\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{\left(x^{\theta}\right)}-1\right)^{\beta-1}\beta\theta x^{\theta-1}e^{x^{\theta}}\exp\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{\left(x^{\theta}\right)}-1\right)^{\beta}\right)}{B\left(i,n-i+1\right)}\times \tag{21}$$

$$\times\exp\left(1-\exp\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{\left(x^{\theta}\right)}-1\right)^{\beta}\right)\right)^{n-i+1}\left[1-\exp\left(1-\exp\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{\left(x^{\theta}\right)}-1\right)^{\beta}\right)\right)\right]^{i-1}$$

Where $\underline{\mu}=\left(\lambda,\alpha,\beta,\theta\right)$, B(..) is the beta function. $F\left(x,\underline{\mu}\right)$ and $f\left(x,\underline{\mu}\right)$ are cdf and pdf of the EPCh distribution, respectively.

4.5. Mean Remaining Life

The mean remaining life function, $m\left(t\right)$, defined as the expected value of the remaining lifetime after a fixed time t for a continuous random variable T with a life function, $R\left(t\right)$, is stated as follows:

$$m\left(t\right)=E\left[T-t/T>t\right]=\frac{1}{R\left(t\right)}\int_{t}^{\infty}\left(x-t\right)f\left(x\right)dx=\frac{1}{R\left(t\right)}\int_{t}^{\infty}xf\left(x\right)dx-t. \tag{22}$$

Guess and Prosehan [11]. The other formula for $m\left(t\right)$ is obtained by the help of Tonelli's theorem [20,26] and is given as follows:

$$\begin{aligned}m\left(t\right)&=E\left[T-t/T>t\right]=\frac{1}{R\left(t\right)}\int_{t}^{\infty}\left(x-t\right)f\left(x\right)dx\\&=\frac{1}{R\left(t\right)}\int_{x=t}^{\infty}\left(\int_{u=t}^{x}du\right)f(x)dx\\&=\frac{1}{R\left(t\right)}\int_{u=t}^{\infty}\left(\int_{x=u}^{\infty}f(x)dx\right)du\\&=\frac{1}{R\left(t\right)}\int_{u=t}^{\infty}R(u)du.\end{aligned} \tag{23}$$

From equation (22), the mean remaining life function for the EP-Ch$\left(\alpha,\beta,\lambda,\theta\right)$distribution is given by

$$m(t)=\frac{1}{R\left(t\right)}\left(\int_{t}^{\infty}x\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^{\theta}}-1\right)^{\beta-1}\beta x^{\theta-1}\theta k(x,\alpha,\beta,\lambda,\theta)dx\right)-t \tag{24}$$

Where k can be written as follows:

$$k(x,\alpha,\beta,\lambda,\theta)=\exp\left(x^{\theta}+\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^{\theta}}-1\right)^{\beta}+1-\exp\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^{\theta}}-1\right)^{\beta}\right)\right)$$

As a result of applying the transformation $u=\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{\left(x^{\theta}\right)}-1\right)^{\beta}\right)$ in the Eq. (4.11), $m(t)$ is obtained as follows.

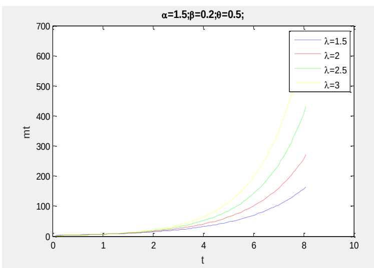
$$m(t)=\frac{1}{R(t)}\left(\int_{s}^{\infty}\left(\ln\left(1+\frac{\alpha}{\lambda}u^{\frac{1}{\beta}}\right)\right)^{\frac{1}{\theta}}e^{u}e^{1-e^{u}}du\right)-t \qquad (25)$$

where $s=\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{t^{\theta}}-1\right)^{\beta}$. According to Bryson and Siddique [5] and Ghitany et al.[10], if hazard function of a non-negative continuous random variable T is decreasing (increasing), then mean remaining life function of T is increasing (decreasing). The values of $m(t)$ for t = 1,2,3 and the different parameter values of EPCh distribution are given in Table 2 and its plot is given by Figure 7.

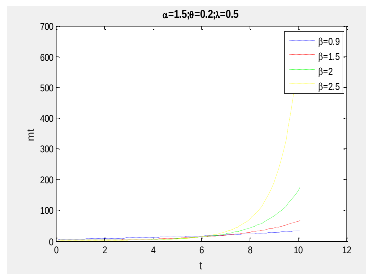
Table 2. $m(t)$ values for t = 1,2,3 and the different parameter values of EPCh distribution

$(\alpha,\beta,\theta,\lambda)$	$m(1)$	$m(2)$	$m(3)$
(1.5,0.2,0.5,1.5)	1.4529	3.5196	3.8974
(1.5,0.2,0.5,2)	1.0402	2.3229	3.7419
(1.5,0.2,0.5,2.5)	0.7466	3.1971	3.6839
(1.5,0.2,0.5,2.5)	0.5238	3.1419	3.6825
(1.5,0.9,0.2,0.5)	5.1339	6.0212	6.4235
(1.5,1.5,0.2,0.5)	2.4975	3.8020	4.0010
(1.5, 2,0.2,0.5)	2.9014	2.7051	2.4862
(1.5, 2,0.2,0.5)	2.7345	2.4192	2.1165
(0.2,0.5,0.3,0.01)	26.8366	27.9178	28.5566
(0.5,0.5,0.3,0.01)	56.9885	58.4539	59.342
(1.5,0.5,0.3,0.01)	123.8780	125.7015	126.8180
(2,0.5,0.3,0.01)	148.8599	150.7784	151.9081
(1.5,2,0.3,0.01)	165.5861	164.6076	163.6312
(1.5,2,0.5,0.01)	20.2103	19.2162	18.2262
(1.5,2,0.7,0.01)	7.8161	6.8215	5.8369
(1.5,2,0.9,0.01)	4.4221	3.4286	2.4650
(2,1.5,1.5,0.01)	1.8048	0.8286	0.0752
(1.5,0.6,0.5,0.01)	15.5495	15.2069	14.6775
(1.5,0.6,0.6,0.01)	9.1948	8.6193	8.0192
(1.5,0.6,0.7,0.01)	5.9921	5.5846	4.9763
(1.5,0.6,0.9,0.01)	3.1885	2.9301	2.4320
(0.2,0.5,0.2,0.5)	0.4386	0.5408	0.5999
(0.5,0.5,0.2,0.5)	2.0837	3.7894	4.3722

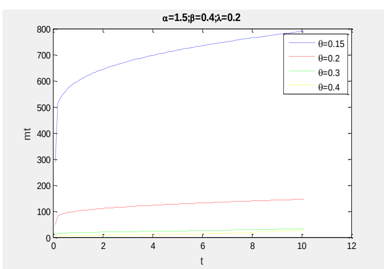
(1.5,0.5,0.2,0.5)	12.5983	14.3185	15.8991
(2,0.5,0.2,0.5)	19.8341	22.1319	23.7242
(1.5,0.4,0.15,0.2)	433.4017	463.4297	483.7846
(1.5,0.4,0.2,0.2)	72.1829	78.2397	82.4504
(1.5,0.4,0.3,0.2)	13.0904	14.1164	15.0439
(1.5,0.4,0.4,0.2)	5.61672	5.8228	6.33779
(2.5,0.5,0.2,0.1)	256.8916	266.7358	273.5476
(2.5,0.8,0.2,0.1)	207.7318	212.4193	215.5393
(2.5,0.9,0.2,0.1)	204.6623	208.2875	210.7467
(2.5,1.5,0.2,0.1)	214.6724	215.3445	215.6490

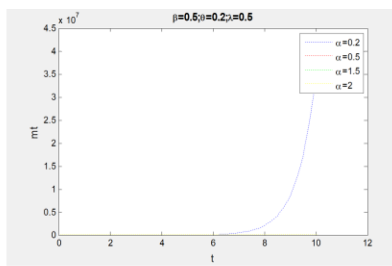


$\alpha = 1.5, \beta = 0.2, \theta = 0.5$ ve $\lambda = 1.5, 2, 2.5, 3$



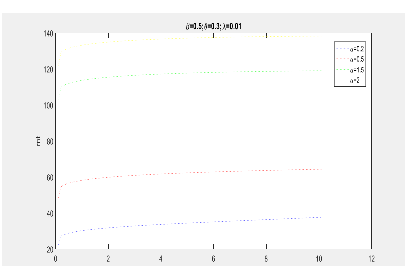
$\alpha = 1.5, \theta = 0.2, \lambda = 0.5$ ve $\beta = 0.9, 1.5, 2, 2.5$

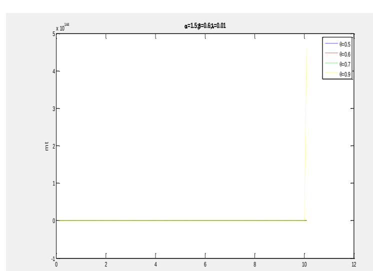




$\beta = 0.5, \theta = 0.2, \lambda = 0.5$ ve $\alpha = 0.2, 0.5, 1.5, 2$

$\alpha = 1.5, \beta = 0.4, \lambda = 0.2$ ve $\theta = 0.15, 0.2, 0.3, 0.4$





$\beta = 0.5, \theta = 0.3, \lambda = 0.01$ ve $\alpha = 0.2, 0.5, 1.5, 2$ $\alpha = 1.5, \beta = 0.6, \lambda = 0.01$ ve $\theta = 0.5, 0.6, 0.7, 0.9$

Figure 7. Plots of the mean remaining life function for EP-Ch $(\alpha, \beta, \lambda, \theta)$ distribution

4.5. Measures of Uncertainty for EPCh distribution

In this section, Renyi entropy (R'enyi [18]) and Shannon entropy (Shannon [20]) are presented for EPCh distribution. A larger entropy value indicates a higher level of uncertainty in the data.

4.5.1. R´enyi Entropy and Shannon Entropy

R´enyi entropy (R´enyi,[18]) is an extension of Shannon entropy and has been used in many fields such as physics, engineering, and economics. The Rényi entropy for any distribution is defined as follows:

$$H_\delta(X)=\frac{1}{1-\delta}\log\int_0^\infty f^{\delta}{}_{EP-Ch}(x;\alpha,\beta,\theta.\lambda)dx \qquad \delta\neq 1,\ \delta>0 \tag{26}$$

The Rényi entropy for EPCh $(\alpha,\beta,\lambda,\theta)$ distribution is given by

$$H_\delta(X)=\frac{1}{1-\delta}\log\int_0^\infty\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^\theta}-1\right)^{\beta-1}\beta x^{\theta-1}\theta k(x,\alpha,\beta,\lambda,\theta)\right)^{\delta}dx \tag{27}$$

Where k can be written as follows:

$$k(x,\alpha,\beta,\lambda,\theta)=\exp\left(x^\theta+\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^\theta}-1\right)^{\beta}+1-\exp\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^\theta}-1\right)^{\beta}\right)\right).$$

Renyi entropy values for various parameter values of $EPCh(\lambda,\alpha,\beta,\theta)$ distribution is given in Table 3.

Table 3. R´enyi entropy for some selected parameter values of EPCh distribution

Parameters	Renyi entropy values		
$(\alpha,\beta,\theta,\lambda)$	$I_{(0.05)}$	$I_{(0.9999)}$	$I_{(2)}$
$\alpha=0.5,\beta=3,\theta=2,\lambda=0.1$	0.1726	-1.0210	-1.1790
$\alpha=1.5,\beta=3,\theta=2,\lambda=0.1$	0.3104	-0.6808	-1.2710
$\alpha=2,\beta=3,\theta=2,\lambda=0.1$	0.3356	-1.0690	-1.3010
$\alpha=1.5,\beta=0.5,\theta=2,\lambda=0.1$	0.7903	0.6042	0.5136
$\alpha=1.5,\beta=1.5,\theta=2,\lambda=0.1$	0.5325	-0.3378	-0.5624
$\alpha=1.5,\beta=2,\theta=2,\lambda=0.1$	0.4510	-0.6247	-0.8571
$\alpha=1.5,\beta=3,\theta=0.5,\lambda=0.1$	2.2360	1.7100	1.5470
$\alpha=1.5,\beta=3,\theta=1.5,\lambda=0.1$	0.5671	-0.5974	-0.8232
$\alpha=1.5,\beta=3,\theta=2,\lambda=0.1$	0.3104	-0.6808	-1.2710
$\alpha=1.5,\beta=3,\theta=2,\lambda=0.5$	0.8002	-0.9500	-1.6660
$\alpha=1.5,\beta=3,\theta=2,\lambda=1.5$	-0.1984	-0.9898	-1.2550
$\alpha=1.5,\beta=3,\theta=2,\lambda=2$	-0.2906	-1.1210	-1.3110

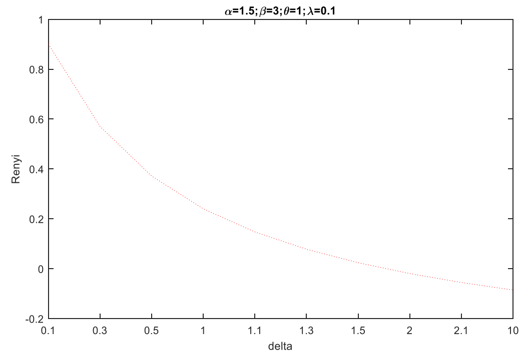
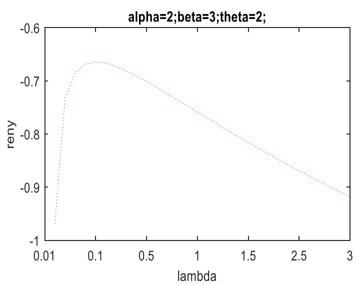
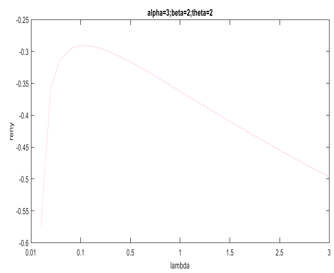
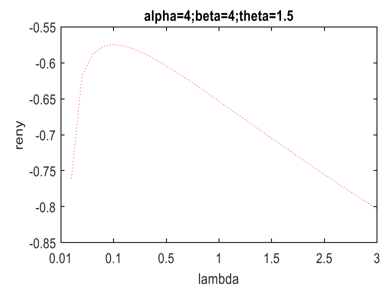


Figure 8. Plot of the R´enyi entropy is concave for different values of $\delta > 0$.

As seen from Figure7, Renyi entropy for EPCh distribution is a concave monotonically decreasing function. At the large δ values, the Rényi entropy is small.







$\alpha = 1.5, \beta = 0.2, \theta = 0.5$, $\lambda = 0.01:0.1:3$ $\alpha = 3, \beta = 2, \theta = 2$, $\lambda = 0.01:0.1:3$ $\alpha = 4, \beta = 4, \theta = 1.5$, $\lambda = 0.01:0.1:3$

R´enyi entropy tends to Shannon entropy for $\delta \to 1$[17]. The Shannon entropy is described as follows:

$$H\left(f_{EP-W}\right) = E\left(-\log\left(f\left(X;\lambda,\delta,\psi\right)\right)\right) \qquad (28)$$

The Shannon entropy for the EPCh distribution is obtained as follows.

$$H\left(x\right) = -\int_{-\infty}^{\infty} f\left(x\right)\log f\left(x\right)dx$$

$$H\left(x\right) = -\int_{0}^{\infty}\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^{\theta}}-1\right)^{\beta-1}\beta x^{\theta-1}\theta k(.)\log\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^{\theta}}-1\right)^{\beta-1}\beta x^{\theta-1}\theta k(.)\right)dx \qquad (29)$$

where $k(.) = k(x,\alpha,\beta,\lambda,\theta) = \exp\left(x^{\theta} + \left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^{\theta}}-1\right)^{\beta} + 1 - \exp\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{x^{\theta}}-1\right)^{\beta}\right)\right)$

the Shannon entropy values for different parameter values of the EP-Ch distribution are given in Table 4.

Table 4. Plots of Shannon entropy for different parameter values of EP-Ch distribution

$(\alpha,\beta,\theta,\lambda)$	Shannon Entropy
$\alpha=0.5,\beta=3,\theta=2,\lambda=0.1$	-1.021
$\alpha=1.5,\beta=3,\theta=2,\lambda=0.1$	-0.6806
$\alpha=2,\beta=3,\theta=2,\lambda=0.1$	-1.069
$\alpha=1.5,\beta=0.5,\theta=2,\lambda=0.1$	0.6042
$\alpha=1.5,\beta=1.5,\theta=2,\lambda=0.1$	-0.3378
$\alpha=1.5,\beta=2,\theta=2,\lambda=0.1$	-0.6247
$\alpha=1.5,\beta=3,\theta=0.5,\lambda=0.1$	1.71
$\alpha=1.5,\beta=3,\theta=1.5,\lambda=0.1$	-0.5974
$\alpha=1.5,\beta=3,\theta=2,\lambda=0.1$	-0.6808
$\alpha=1.5,\beta=3,\theta=2,\lambda=0.5$	-0.95
$\alpha=1.5,\beta=3,\theta=2,\lambda=1.5$	-0.9898
$\alpha=1.5,\beta=3,\theta=2,\lambda=2$	-1.121

6. Maximum Likelihood Estimation

Let $X_{1,}X_2,....,X_n$ be a random sample with size n taken from $EPCh(\lambda,\alpha,\beta,\theta)$ distribution. The log-likelihood function is given as follows.

$$\begin{aligned} l(\underline{\mu}\,|\,\underline{x}) &= n\,\beta\log\left(\frac{\lambda}{\alpha}\right)+(\beta-1)\sum_{i=1}^{n}\log\left(e^{\left(x^{\theta}{}_{i}\right)}-1\right)+n\log\beta+n\log(\theta) \\ &+(\theta-1)\sum_{i=1}^{n}\log(x_i)+n \\ &+\left(\sum_{i=1}^{n}\left(-\exp\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{\left(x^{\theta}{}_{i}\right)}-1\right)^{\beta}\right)+\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{\left(x^{\theta}{}_{i}\right)}-1\right)^{\beta}+x^{\theta}{}_{i}\right)\right) \end{aligned} \quad (30)$$

where $\underline{\mu}=(\lambda,\alpha,\beta,\theta)$. Derivatives according to unknown parameters of the log-likelihood function are as follows:

$$\begin{aligned} \frac{\partial\log L}{\partial\alpha} &= -\beta\alpha^{-1}\left(n+\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(\sum_{i=1}^{n}\left(e^{\left(x_i^{\theta}\right)}-1\right)^{\beta}\right)\right) \\ &-\alpha^{-1}\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(\sum_{i=1}^{n}\left(e^{\left(x_i^{\theta}\right)}-1\right)^{\beta}\exp\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{\left(x_i^{\theta}\right)}-1\right)^{\beta}\right)\right) \end{aligned} \quad (31)$$

$$\begin{aligned}\frac{\partial \log L}{\partial \beta} &= n\log(\lambda) - n\log(\alpha) + \left(\sum_{i=1}^{n}\log\left(e^{(x_i^{\theta})}-1\right)\right) + \frac{n}{\beta} \\ &+\left(\frac{\lambda}{\alpha}\right)^{\beta}\log\left(\frac{\lambda}{\alpha}\right)\left(\sum_{i=1}^{n}\left(e^{(x_i^{\theta})}-1\right)^{\beta}\right)+\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(\sum_{i=1}^{n}\left(e^{(x_i^{\theta})}-1\right)^{\beta}\log\left(e^{(x_i^{\theta})}-1\right)\right) \\ &-\sum_{i=1}^{n}\exp\left(\left(\frac{\lambda}{\alpha}\right)\left(e^{(x_i^{\theta})}-1\right)^{\beta}\right) \\ &\quad\times\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\log\left(\frac{\lambda}{\alpha}\right)\left(e^{(x_i^{\theta})}-1\right)^{\beta}+\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{(x_i^{\theta})}-1\right)^{\beta}\log\left(e^{(x_i^{\theta})}-1\right)\right)\end{aligned} \quad (32)$$

$$\begin{aligned}\frac{\partial \log L}{\partial \theta} &= \left(\sum_{i=1}^{n}\frac{x_i^{\theta}\log(x_i)e^{(x_i^{\theta})}}{e^{(x_i^{\theta})}-1}\right)\theta\beta-\left(\sum_{i=1}^{n}\frac{x_i^{\theta}\log(x_i)e^{(x_i^{\theta})}}{e^{(x_i^{\theta})}-1}\right)\theta+n \\ &+\left(\sum_{i=1}^{n}\log(x_i)\right)\theta+\left(x_i^{\theta}\log(x_i)\right)\theta \\ &+\left(\frac{\lambda}{\alpha}\right)^{\beta}\beta\left(\sum_{i=1}^{n}\left(e^{(x_i^{\theta})}-1\right)^{\beta-1}x_i^{\theta}\log(x_i)e^{(x_i^{\theta})}\right)\theta \\ &-\left(\frac{\lambda}{\alpha}\right)^{\beta}\beta\left(\sum_{i=1}^{n}\left(e^{(x_i^{\theta})}-1\right)^{\beta-1}x_i^{\theta}\log(x_i)\exp\left(x_i^{\theta}+\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{(x_i^{\theta})}-1\right)^{\beta}\right)\theta\right)/\theta\end{aligned} \quad (33)$$

$$\begin{aligned}\frac{\partial \log L}{\partial \lambda} &= n\frac{\beta}{\lambda}+\left(\frac{\lambda}{\alpha}\right)^{\beta}\frac{\beta}{\lambda}\sum_{i=1}^{n}\left(e^{(x_i^{\theta})}-1\right)^{\beta} \\ &-\left(\frac{\lambda}{\alpha}\right)^{\beta}\frac{\beta}{\lambda}\sum_{i=1}^{n}\left(e^{(x_i^{\theta})}-1\right)^{\beta}\exp\left(\left(\frac{\lambda}{\alpha}\right)^{\beta}\left(e^{(x_i^{\theta})}-1\right)^{\beta}\right)\end{aligned} \quad (34)$$

MLEs of λ,α,β and θ parameters are obtained by the simultaneous solutions of the equations (31) - (34). These nonlinear equations can be solved using iterative methods.

6. Simulation Study

In this section, a simulation study based on 5000 replications to investigate the performances of MLEs of the unknown parameters in terms of bias and mean squared error (MSE) for $EPCh(\lambda,\alpha,\beta,\theta)$ distribution for different sample sizes n =100,150,200,300,500 and for different parameter values such as (0.5,1.4,0.2,0.5), (0.2,0.8,0.6,0.5), (0.3,0.9,0.6,0.4), (0.2,1.5,0.4,0.2) and (0.3, 2,0.5,0.9) is performed. The simulation results are given in Table 5.

Table 5. Bias and MSE for various values of λ,α,β and θ parameters

Parameters		$\hat{\lambda}$		$\hat{\alpha}$		$\hat{\beta}$		$\hat{\theta}$	
$(\lambda,\alpha,\beta,\theta)$	n	bias	Mse	bias	mse	Bias	mse	bias	Mse
(0.5,1.4,0.2,0.5)	100	-0.0222	0.0438	0.0134	0.1736	0.0063	0.0139	0.0329	0.0169
	150	-0.0147	0.0132	0.0103	0.0725	0.0032	0.0034	0.0209	0.0106
	200	-0.0077	0.0061	0.0086	0.0432	0.0025	0.0019	0.0139	0.0073
	300	-0.0019	0.0037	0.0069	0.0268	0.0026	0.0011	0.0077	0.0044
	500	0.0009	0.0018	0.0096	0.0150	0.0019	0.0005	0.0043	0.0022
(0.2,0.8,0.6,0.5)	100	-0.0080	0.0543	0.0261	0.1965	0.1574	2.0267	0.0320	0.0374

	150	-0.0108	0.0030	0.0221	0.0386	0.0491	0.2309	0.0277	0.0235
	200	-0.0061	0.0027	0.0164	0.0260	0.0388	0.0817	0.0171	0.0179
	300	-0.0033	0.0009	0.0111	0.0148	0.0226	0.0312	0.0099	0.0114
	500	-0.0013	0.0006	0.0135	0.0093	0.0102	0.0153	0.0078	0.0069
(0.3,0.9,0.6,0.4)	100	0.0100	0.7033	0.0799	2.1006	0.2131	4.6283	0.0540	0.0378
	150	-0.0067	0.1823	0.0269	0.5555	0.0844	0.6790	0.0320	0.0237
	200	-0.0074	0.0518	0.0198	0.1855	0.0462	0.1454	0.0239	0.0174
	300	-0.0061	0.0024	0.0108	0.0182	0.0286	0.0450	0.0131	0.0110
	500	-0.0023	0.0011	0.0123	0.0094	0.0126	0.0198	0.0095	0.0064
(0.2,1.5,0.4,0.2)	100	-0.0120	0.0082	0.0798	0.7168	0.0289	0.1880	0.0076	0.0024
	150	-0.0087	0.0021	0.0611	0.1454	0.0116	0.0141	0.0055	0.0016
	200	-0.0055	0.0015	0.0442	0.1099	0.0099	0.0098	0.0033	0.0012
	300	-0.0025	0.0010	0.0380	0.0721	0.0055	0.0056	0.0025	0.0007
	500	-0.0009	0.0006	0.0375	0.0466	0.0017	0.0031	0.0020	0.0004
(0.3, 2,0.5,0.9)	100	-0.0213	0.0100	0.0666	0.3942	0.0515	0.1520	0.0317	0.0644
	150	-0.0127	0.0048	0.0597	0.2636	0.0250	0.0355	0.0215	0.0400
	200	-0.0096	0.0033	0.0563	0.2045	0.0175	0.0215	0.0173	0.0303
	300	-0.0041	0.0022	0.0480	0.1337	0.0105	0.0114	0.0105	0.0189
	500	-0.0009	0.0013	0.0467	0.0812	0.0050	0.0062	0.0083	0.0113

7. Real Data Analysis

In this section, two real data analysis are considered to illustrate that the EPCh distribution can be better than known distributions such as Exponentiated exponential, Weibull and Chen distribution. For this aim, EPCh distribution are compared with above distributions using goodness of fit measures such as the Akaike's Information Criterion (AIC), corrected Akaike's Information Criterion (AICc), the Bayesian Information Criterion (BIC) and -2×log-likelihood value. These measures are given by

$$AIC = -2l + 2k \tag{35}$$

$$AICc = AIC + \left(\frac{2k(k+1)}{n-k-1}\right) \tag{36}$$

$$BIC = -2l + k\log(n) \tag{37}$$

where k is a number of parameters, n is sample size and l is the value of log–likelihood function. The first data set which shows failure times of components is the real data set taken from book of Murthy et al [14] are given in Table 6.

Table6. Real Data set based on failure times (Data Set 1)

0.0014 0.0623 1.3826 2.0130 2.5274 2.8221 3.1544 4.9835 5.5462 5.8196 5.8714 7.4710 7.5080 7.6667 8.6122 9.0442 9.1153 9.6477 10.1547 10.7582

The second data set which states graft survival times in months of 148 renal transplant patients was obtained by Henderson and Milner [13] and was included in the book of Hand et al. [12].

Table 7. Real Data set based on surviving times (Data Set 2)

0.0035, 0.0068, 0.01, 0.0101, 0.0167, 0.0168, 0.0197, 0.0213, 0.0233, 0.0234, 0.0508, 0.0508, 0.0533, 0.0633, 0.0767, 0.0768, 0.077, 0.1066, 0.1267, 0.13, 0.1639, 0.1803, 0.1867, 0.218, 0.2967, 0.3328, 0.37, 0.3803, 0.4867, 0.6233, 0.6367, 0.66, 0.66, 0.718, 0.78, 0.7933, 0.7967, 0.8016, 0.83, 0.841, 0.91, 0.9233, 1.0541, 1.0607, 1.0633, 1.0667, 1.1067, 1.2213, 1.2508, 1.2533, 1.38, 1.4267, 1.4475, 1.45, 1.5213, 1.5333, 1.5525, 1.5533, 1.5541, 1.5934, 1.62, 1.63, 1.6344, 1.66, 1.7033, 1.7067, 1.7475, 1.7667, 1.77, 1.7967, 1.8115, 1.8115, 1.8933, 1.8934, 1.9508, 1.9733, 2.018, 2.09, 2.1167, 2.1233, 2.21, 2.2148, 2.2267, 2.25, 2.2533, 2.3738, 2.4082, 2.418, 2.4705, 2.5213, 2.5705, 3.1934, 3.218, 3.2367, 3.2705, 3.3148, 3.3567, 3.4836, 3.4869, 3.6213, 3.941, 3.9433, 4.0001, 4.1733, 4.1734, 4.2311, 4.2869, 4.3279, 4.3902, 4.4267.

The MLE(s) and their standard errors for the unknown parameters of above the distributions are given in Table 8 for data set 1 and Table 10 for data set 2. Goodness of fit measures for these data sets are shown in Table 9 for data set 1 and Table 11 for data set 2. Plots of empirical and theoritical distribution functions of random variables having compared distributions are given by Figure 8 for data set 1 and Figure 9 for data set 2.

Table 8. Parameter estimates (standard errors) for Data set 1

Distribution	**MLE**
EP-Ch	$\hat{\lambda}=0.0338(1.6891)$, $\hat{\alpha}=0.1254(2.8571)$ $\hat{\beta}=2.2694(0.4971)$, $\hat{\theta}=0.5092(0.0642)$
Exponentiated Exponential	$\hat{\theta}=0.8377(0.2300)$, $\hat{\alpha}=0.1570(0.0467)$
Weibull	$\hat{\mu}=1.0893(0.2210)$, $\hat{\sigma}=5.8164\ (1.2216)$
Exponential power	$\hat{\alpha}=8.6650(1.2778)$, $\hat{\beta}=0.9446(0.1958)$

Table 9. Selective criteria statistics for Real data set 1

Dağılım	**-2LogL**	**AIC**	**BIC**	**K-S**	**p-value**
EP-Ch	**93.6475**	**101.6475**	**105.6304**	**0.1383**	**0.8359**
Exponentiated Exponential	109.2411	113.2411	115.2325	0.2493	0.1663
Weibull	109.5036	133.5036	115.4950	0.2205	0.2853
Exponential power	102.9713	106.9713	108.9628	0.2049	0.3706

Table 4.10. Parameter estimates (standard errors) for Data set 2

Distribution	**MLE**
EP-Ch	$\hat{\lambda}=0.0124(32.7709)$, $\hat{\alpha}=0.0854\ (225.346)$ $\hat{\beta}=0.6267(0.2233)$, $\hat{\theta}=0.6821(0.1897)$
Exponential power	$\hat{\alpha}=2.6841(0.2115)$, $\hat{\beta}=0.7464(0.0629)$
Chen	$\hat{\lambda}=0.3212(0.0000)$, $\hat{\theta}=0.6060(6.000)$

Table 11. Selective criteria statistics for Real data set 2

Dağılım	**-2LogL**	**AIC**	**AICc**	**K-S**	**p-value**
EP-Ch	**290.2455**	**298.2455**	**298.6265**	**0.0743**	**0.5776**

Exponential power	300.3445	304.3445	304.4566	0.1158	0.1044
Chen	295.3769	299.3769	299.4891	0.1158	0.1044

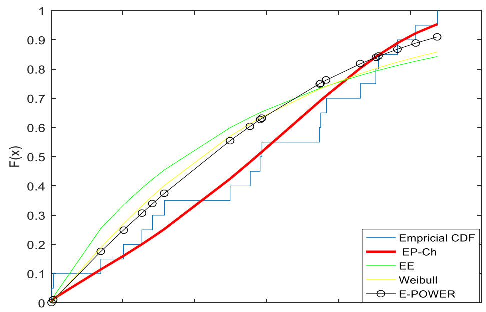


Figure 8. Goodness of fit plots for data set 1

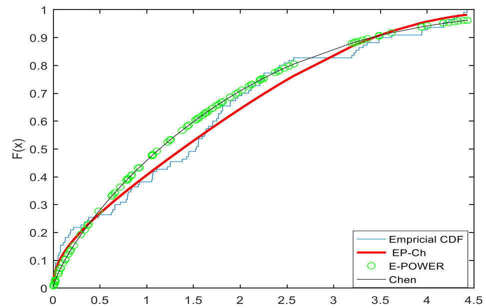


Figure 9. Goodnes of fit plots for data set 2

3. CONCLUSION

In this paper, we have introduced a new lifetime distribution named as Exponential Power Chen (EPCh) distribution by using method of Alzaatreh et al. [2]. Some statistical properties of EPCh distribution such as moments, moment generating function, order statistics**,** mean remaining life, Renyi and Shannon entropies are obtained. Furthermore, shapes of pdf, cdf, hf and ihf for this distribution are examined. From these shapes, it is concluded that EPCh distribution can be used to model the data having increasing, decreasing and bathtube shaped hazard rates. Further, the maximum likelihood estimators (MLE) for unknown parameters of EPCh distribution are derived. An Monte-Cario simulation study has been carried out to examine the performance of this estimators in terms of mean square error and bias. To illustrate the applicability of this new distribution, EPCh distribution for two real data sets are compared with some known distributions such as Exponentiated exponential, Weibull and Chen distribution using some goodness of fit measures. According to real data analysis results obtained from both data sets, EPCh distribution has the best fitting among compared distributions. This demonstrates the applicability of the EP-Ch distribution in real life.

REFERENCES

1. Akinsete, A., Famoye, F., Lee, C. (2008). The beta-Pareto distribution. *Statistics*, 42(6), 547-563.
2. Alzaatreh, A., Lee, C., Famoye, F. (2013). A new method for generating families of continuous distributions. *Metron*, *71*(1), 63-79.
3. Alzaatreh, A., Famoye, F., Lee, C. (2013). Weibull-Pareto distribution and its applications. *Communications in Statistics-Theory and Methods*, 42(9), 1673-1691.
4. Alzaghal, A., Famoye, F., Lee, C. (2013). Exponentiated T - X family of distributions with some applications. *International Journal of Statistics and Probability*, *2*(3), 31.
5. Bryson, M.C. and Siddiqui, M.M. (1969). Some criteria for aging. Journal of the American Statistical Association 64; 1472-14838
6. Cordeiro, G. M., de Castro, M. (2011). A new family of generalized distributions. *Journal of* statistical computation and simulation, 81(7), 883-898.
7. Chen, Z. (2000). A new two-parameter lifetime distribution with bathtub shape or increasing failure rate function. Statistics & Probability Letters, 49(2), 155-161.
8. Çelik, N., Guloksuz, C. T. (2017). A New Lifetime Distribution, Nowy Rozkład Cyklu Życıa. *Eksploatacja I Nıezawodnosc*, *19*(4), 634.

9. Eugene, N., Lee, C., Famoye, F. (2002). Beta-normal distribution and its applications, *Communications in Statistics-Theory and methods*, *31*(4), 497-512.
10. Ghitany, M. E., Alqallaf, F., Al-Mutairi, D. K., & Husain, H. A. (2011). A two-parameter weighted Lindley distribution and its applications to survival data. *Mathematics and Computers in simulation*, *81*(6), 1190-1201.
11. Guess,F., Prosehan.F.(1985). Mean Residual Life: Theory and Applications, The Florida State University ,Department of Statistics , Afosr Technical Report No. 85-178 .
12. Hand, D. J. , Daly, F., Lunn, A. D., McConway , K. J. , Ostrowski, E. (1994). A Handbook of Small Data Sets, London: Chapman & Hall, 255.
13. Henderson, R. and Milner, A. (1991) Aalenplots under proportional hazards. *Applied Statistics*, 40, 401-409.
14. Murthy D, Xie M, Jiang R. (2004). Weibull models. John Wiley & Sons, Inc.
15. Nadarajah, S., Kotz, S. (2004). The beta Gumbel distribution. *Mathematical Problems in engineering*, (4), 323-332.
16. Nadarajah, S., Kotz, S. (2006). The beta exponential distribution. *Reliability engineering & system safety*, 91(6), 689-697.
17. Nielsen, F. and Nock, R. (2011). On R´enyi and Tsallis entropies and divergences for exponential families, arXiv preprint arXiv:1105.3259 .
18. R´enyi, A. (1961). On measures of entropy and information, *Proceedings of the 4th Berkeley Symposium on Mathematical Statistics and Probability*, Berkeley (CA):University of California Press, I, 547 - 561.
19. Rudin. W. Principles of Mathematical Analysis. McGraw-Hill, New York, 3rd edition, 1976
20. Shannon, E.A. (1948). A Mathematical Theory of Communication, *The Bell System Technical Journal*, 27(10), 623 - 656.
21. Smith, R. M., Bain, L. J. (1975). An exponential power life-testing distribution. *Communications in Statistics-Theory and Methods*, 4(5), 469-481.
22. Tahir, M. H., Cordeiro, G. M., Alizadeh, M., Mansoor, M., Zubair, M., Hamedani, G. G. (2015). The odd generalized exponential family of distributions with applications. *Journal of Statistical Distributions and Applications*, *2*(1), 1.
23. Tahir, M. H., Cordeiro, G. M., Alzaatreh, A., Mansoor, M., & Zubair, M. (2016a). The Logistic-X family of distributions and its applications. *Communications in Statistics-Theory and Methods*, *45*(24), 7326-7349.
24. Tahir, M. H., Zubair, M., Mansoor, M., Cordeiro, G. M., Alizadeh, M., & Hamedani, G. G. (2016b). A new Weibull-G family of distributions. *Hacet. J. Math. Stat*, *45*, 629-647.
25. Yeh, J. (2006). Real Analysis: Theory of Measure and Integration Second Edition, World Scientific Publishing Company.

القوة الأسية–توزيع تشن وبعض خصائصه

نورسل احمد زين العابدين[1] و بوكرا سراكوكلو[2]

[1]جامعة سلجوق ، كلية الدراسات العليا للعلوم الطبيعية والتطبيقية ، قسم الإحصاء ، طالب دكتوراه ، قونية ، تركيا. [2]قسم الاحصاء، كلية العلوم ، جامعة سيلوك، كونيا، تركيا

الخلاصة: يهدف البحث إلى إدخال توزيع إحصائي جديد يسمى القوة الأسية–تشن باستخدام الطريقة التي اقترحها الزعترة وآخرون. ((2013). يتم الحصول على بعض الخصائص الإحصائية مثل العزوم ، معاملات الانحراف والتفرطح ، مولد رقم عشوائي لتوزيع الطاقة الأسية تشن (الجيش الشعبي–الفصل). وعلاوة على ذلك ، تم اشتقاق مقدرات الاحتمالية القصوى (ملس) لمعلمات غير معروفة لتوزيع إب–تش ودراسة محاكاة مونت كارلو على أساس متوسط أخطاء مربع والتحيزات من هذه المقدرات لمختلف أحجام العينات قد أجريت. أخيرا ، تم تقديم تطبيق يستخدم مجموعة بيانات حقيقية لهذا التوزيع الجديد.

الكلمات المفتاحية: القوة الأسية–عائلة إكس للتوزيعات ، توزيع القوة الأسية تشن ، تقدير الاحتمالية القصوى ، محاكاة مونت كارلو.