i-Open Sets and Separating Axioms Spaces

Amir A. Mohammed **Sabih W. Askandar**
Department of Mathematics \ College of Education for Pure Sciences
University of Mosul
Mosul-Iraq

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المجاميع المفتوحة من النوع– i وفضاءات بديهيات الانفصال

أ.م.د. عامر عبد الإله محمد و م.م. صبيح وديع اسكندر
قسم الرياضيات/ كلية التربية للعلوم الصرفة/ جامعة الموصل
الموصل / العراق

الخلاصة:

الهدف من هذا البحث هو استخدام نوع من المجاميع المفتوحة المسماة بالمجاميع المفتوحة من النوع– i [9] لدراسة عدة أصناف من فضاءات بديهيات الانفصال للمجاميع المفتوحة، المفتوحة من النوع– α و شبه المفتوحة. فضلا عن ذلك، قمنا بدراسة العلاقة بينها.

الكلمات المفتاحية: $T_{\circ i}, T_{1i}$, T_{2i} , T_{3i} , $T_{(3\frac{1}{2})i}$, T_{4i} , T_{5i} .

Abstract:

The purpose of this paper is using a class of open sets called i-open sets [9] to study some classes of separating axioms spaces for open, α-open and semi-open sets. Further, we studied the relations between such spaces.

Keywords: $T_{\circ i}, T_{1i}$, T_{2i} , T_{3i} , $T_{(3\frac{1}{2})i}$, T_{4i} , T_{5i} .

Introduction:

Levine in 1963[5], introduced the concept of semi-open sets which improved many important basic theories of the general topology. Njastad in 1965[10], introduced the concept of α-open sets which is a subclass of generalized open sets. Also Levine in 1970[6] introduced the concept of generalized closed sets.. Mashhour A.S., Abd El-Monsef M.E. and El-Deeb, S.N., in 1982[8], introduced the concept of Pre-open sets. Dontchev and Maki, in 1999[3], introduced the concept of $\theta-$generalized closed sets. Devi, R., Selvakumar, A. and Parimala, M., in 2011[2], introduced the concept of $\alpha\psi-closed\ \ sets$ in topological spaces, which, it is complements were called $\alpha\psi-open\ sets$. Mohammed and Askandar In 2012 [9], introduced the concept of i-open sets which they could to entire them together with many other concepts of Generalized open sets mentioned above. In 2006 Fatima, M. Mohammad introduced Pre- Techonov and Pre-Hausdorff Separation Axioms in Intuitonistic Fuzzy special topological spaces [4] by using the concept of Pre-open sets [8]. In 2011 Y.K. Kim, R. Devi and A. Selvakumar used $\alpha\psi-Open\ sets$ [2] to introduce the concept of Weakly Ultra Separation Axioms [12]. In 2012 Al-Sheikhly, A.H. and Khudhair, H.K.[1] introduced another Type of Separation Axioms Depend on an $\theta g-open\ \ set$ [3]. The aim of this paper is to introduce another type of Separating Axioms spaces depend on i-open sets [9] for compare with the other separating axioms spaces. This work consists of two sections. In the first one, i-open sets[9] are defined and many related examples have been gave, the comparison between i-open sets, semi-open and α-open sets respectively are investigated, New class of mappings named, i-continuous [9] are introduced and comparison among i-continuity [9], continuity [11], semi-continuity [5] and α-continuity [13], are investigated (see Corollary 1.28). In the 2nd section, we study many types of separating axioms spaces as like as ($T_\circ, T_1$, T_2 , T_3 , $T_{(3\frac{1}{2})}$, T_4 and T_5) [11], ($T_{\circ\alpha}, T_{1\alpha}$, $T_{2\alpha}$, $T_{3\alpha}$, $T_{(3\frac{1}{2})\alpha}$, $T_{4\alpha}$ and $T_{5\alpha}$), ($T_{\circ s}, T_{1s}$, T_{2s} , T_{3s} , $T_{(3\frac{1}{2})s}$, T_{4s} and T_{5s}) and ($T_{\circ i}, T_{1i}$, T_{2i} , T_{3i} , $T_{(3\frac{1}{2})i}$, T_{4i} and T_{5i}) by using open, α-open[10], semi-open[5] and i-open sets[9] respectively. We give many examples to show that the converse may not be true. Also we discuss the relation among them. (See Corollary 2.5 and Corollaries 2.29). Throughout this work, (X,τ) and (Y,δ) are always topological spaces and *f* is always a mapping from (X,τ) into (Y,δ).

1. i-open sets

In this Section the concept of i-open sets [9] is defined and their position with the some other classes of generalized-open sets is determined. New class of mappings named i-continuous [9] is introduced and comparison between i-continuity [9], continuity [11], semi-continuity [5] and α-continuity [13], are investigated.

Definition1.1. [9] A subset A of (X, τ) is said to be an i-open if there exists an open set $G \neq \phi$, X such that $A \subseteq Cl(A \cap G)$ **.**The complement of an i-open set is called i-closed set.

Example1.2. Let $X= \{a, b, c\}$, $\tau= \{\phi, \{a\}, \{a, c\}, X\}$ by Definition 1.1, i-open sets are: $\phi, \{a\}, \{a, c\}, \{c\}, \{a, b\}, \{b, c\}, X$.

Example1.3. Let $X= \{d, e, f\}$, $\tau= \{\phi, \{d\}, \{e\}, \{d, e\}, X\}$. Therefore; i-open sets are: $\phi, \{d\}, \{e\}, \{d, e\}, \{d, f\}, \{e, f\}, X$.

Theorem1.4. [9] Every open set in a topological space is i-open, but the converse is not true.

Example1.5. Let $X= \{g, h, i\}$, $\tau= \{\phi, \{g\}, \{g, i\}, X\}$, $A= \{g, h\}$. $A= \{g, h\}$ is i-open set but it is not open.

Corollary1.6. [9] Every closed set in topological space is i-closed.

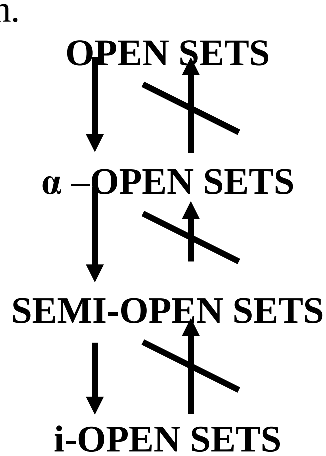
Theorem1.7. [9] Every semi-open set in a topological space is i-open.

Example1.8. Let $X = \{j, k, l\}$, $\tau=\{ \phi, \{j, k\}, X\}$, $A=\{j, l\}$ is i-open set but is not semi-open in (X, τ) .

Corollary1.9. [9] Every α -open set in a topological space is i-open.

The converse of Corollary 1.9 is not true. Indeed, In Example 1.8 we see that $A = \{a, c\}$ is i-open set but is not α -open [$A \not\subseteq Int (Cl (Int (A)))$].

Corollary1.10. [9] By theorem (1.4), theorem (1.7) and corollary (1.9) we have the following Diagram.



Definition1.11. [9] the extension τ^i is the family of all i-open subsets of space X.

Definition1.12. Let (X, τ^i) be a topological space and let A be a subset of X then,
1. The intersection of all i-closed sets containing A is called i-closure of A [9], denoted by $Cl_i(A)$: $Cl_i(A)=\bigcap_{i\in\Lambda} F_i \,.\, A\subseteq F_i\ \forall i$ Where, F_i is i-closed set $\forall \boldsymbol{i}$ in (X,τ^i). $Cl_i(A)$ is the smallest i-closed set containing A.
2. The union of all i-open sets contained in A is called i-Interior of A [9], denoted by $Int_i(A)$. $Int_i(A)=\bigcup_{i\in\Lambda} I_i \ ,I_i\subseteq A\ \forall i$, where I_i is an i-open set $\forall \boldsymbol{i}$ in (X, τ^i). $Int_i(A)$ is the largest i-open set contained in A.

Definition1.13. A mapping $f: (X,\tau)\rightarrow(Y,\delta)$ is said to be i-continuous [9](respectively semi-continuous[5]) at the point $x_o\in X$ if and only if for each open set I^* in (Y,δ) containing $f(x_o)$ there exists an i-open set(respectively semi-open set[5]) I in (X,τ) containing $\boldsymbol{x}_\circ$ such that $f(I)\subseteq I^*$. f is i-continuous (respectively semi-continuous) map if it is i-continuous (respectively semi-continuous) at all points of X.

Theorem1.14. [9] A mapping $f: (X,\tau)\rightarrow(Y,\delta)$ is i-continuous if and only if,
1. $f^{-1}(I^*)$ is i-open set in (X,τ) for every open set I^* in (Y, δ).
2. $f^{-1}(I^*)$ is i-closed set in (X,τ) for every closed set I^* in (Y, δ).

Theorem1.15. [9] Every continuous mapping is i-continuous.

Theorem1.16. [9] Every semi-continuous mapping is i-continuous.

Definition1.17. [9] [13] A mapping $f: (X,\tau)\rightarrow(Y,\delta)$ is said to be α -continuous at the point $x_o\in X$ if and only if for each open set I^* in (Y,δ) containing $f(x_o)$ there exist an α -open set I in (X,τ) containing $\boldsymbol{x}_\circ$ such that $f(I)\subseteq I^*$. f is α -continuous map if it is α -continuous at all points of X.

Theorem1.18. [9] [13] A mapping f is α -continuous if and only if $f^{-1}(I^*)$ is α -open set in (X,τ) for every open set I^* in (Y, δ).

Theorem1.19. [9] Every α -continuous mapping is i-continuous.
Corollary1.20. [9] the following diagram is true:

Continuous Mappings

$\Downarrow \quad \not\Uparrow$

α- Continuous Mappings

$\Downarrow \quad \not\Uparrow$

Semi –Continuous Mappings

$\Downarrow \quad \not\Uparrow$

i- Continuous Mappings

2. i-Open Sets and Separating Axioms Spaces

In this section, we study new types of separating axioms spaces for i-open, semi-open and α-open sets for compare and find many relations among them.

Definition2.1. A topological space (X,τ) is said to be $T_{\circ}$ space [11] (respect. $T_{\circ\alpha}$, $T_{\circ s}$ [7]and $T_{\circ i}$ space) if it satisfies Klomogorov axiom[11] (respect. α-Klomogorov, s-Klomogorov [7] and i-Klomogorov axiom): [$T_{\circ}$ (respect. $T_{\circ\alpha}$, $T_{\circ s}$ and $T_{\circ i}$)]
$\forall x,y \in X\ (x \neq y)\ \exists I \in \tau\ (respect.\ \tau^{\alpha}, \tau^{s}\ and\ \tau^{i})$ s.t. $x \in I,\ y \notin I$.

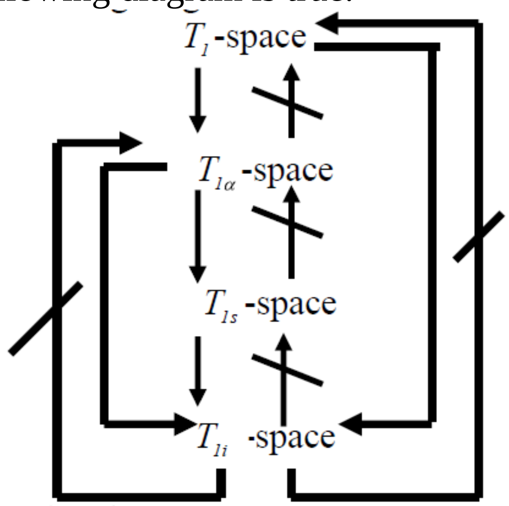
Example2.2. Let $X=\{a, b\}$, $\tau=\{\phi, \{a\}, X\}$, $\tau^{\alpha}=\tau^{s}=\tau^{i}=\tau$, (X,τ), (X,τ^{α}), (X,τ^{s}) and (X,τ^{i}) are topological spaces.
$a,b \in X\ (a \neq b)\ \exists\{a\} \in \tau (respect.\ \tau^{\alpha}, \tau^{s}\ and\ \tau^{i})$ s.t $a \in \{a\}, b \notin \{a\}$.
Therefore; (X,τ) is $T_{\circ}$, $T_{\circ\alpha}$, $T_{\circ s}$ and $T_{\circ i}$ space.

Definition2.3. A topological space (X,τ) is said to be T_1 space [11] (respect. $T_{1\alpha}$, T_{1s} [7], T_{1i} space) if it satisfies Frechet axiom [11] (respect. α-Frechet, s- Frechet [7] and i-Frechet axiom) :[T_1(respect. $T_{1\alpha}, T_{1s}, T_{1i}$)]
$\forall x,y \in X\ (x \neq y)\ \exists I_1, I_2 \in \tau (respect.\ \tau^{\alpha}, \tau^{s}, \tau^{i})$ s.t. $x \in I_1,\ y \notin I_1 y \in I_2, x \notin I_2$.

Example2.4. Let $X=\{a, b, c\}$, $\tau=\{\phi, \{a\},\{b\},\{c\},\{a, b\},\{a, c\},\{b, c\},X\}$, $\tau^{\alpha}=\tau^{s}=\tau^{i}=\tau$, (X,τ), (X,τ^{α}), (X,τ^{s}) and (X,τ^{i}) are topological spaces.
$a,b \in X (a \neq b) \exists\{a\},\{b\} \in \tau, \tau^{\alpha}, \tau^{s}, \tau^{i}, s.t.\ a \in \{a\}, b \notin \{a\}\ b \in \{b\}, a \notin \{b\}$.

$a,c \in X(a \neq c)\ \exists\{a\},\{c\} \in \tau, \tau^{\alpha}, \tau^{s}, \tau^{i}$
$s.t.\ a \in \{a\}, c \notin \{a\}, c \in \{c\}, a \notin \{c\}$
$b,c \in X(b \neq c)\ \exists\{b\},\{c\} \in \tau, \tau^{\alpha}, \tau^{s}, \tau^{i}$
$s.t.\ b \in \{b\}, c \notin \{b\}, c \in \{c\}, b \notin \{c\}.$
Therefore; (X,τ) is $T_1, T_{1\alpha}, T_{1s}$, and T_{1i} -space.

Corollary2.5. The following diagram is true.



Proof: 1. Suppose that (X,τ) is T_1-space.
Then, $\forall x, y \in X\ (x \neq y)$ *there exists two open sets* I_1, I_2 s.t. $x \in I_1, y \notin I_1$, $y \in I_2, x \notin I_2$. Since every open set is $\alpha - open$ (corollary1.10).Then I_1 *and* I_2 are $\alpha - open$ sets. Therefore; (X,τ) is $T_{1\alpha}$-space (definition 2.3).
2. Similarly, by using corollary1.10 and definition 2.3, we can prove every $T_{1\alpha}$- space is T_{1s}- space.
3. Similarly, by using corollary1.10 and definition 2.3, we can prove every $\boldsymbol{T_{1s}}$ -space is $\boldsymbol{T_{1i}}$ -space.
4. From 1 and 2 we have, every $\boldsymbol{T_1}$-space is $\boldsymbol{T_{1s}}$ -space.
5. From 4 and 3 we have, every $\boldsymbol{T_1}$−space is $\boldsymbol{T_{1i}}$- space.
6. From 2 and 3 we have, every $\boldsymbol{T_{1\alpha}}$-space is $\boldsymbol{T_{1i}}$-space. ■

Example2.6. Let $X = \{1,2,3,4\}$, $\tau = \{\phi, \{1\},\{1,2\},\{1,2,3\},X\}$,
$\tau^{\alpha} = \tau^{s} = \{\phi,\{1\},\{1,2\},\{1,2,3\},\{1,3\},\{1,4\},\{1,3,4\},\{1,2,4\},X\}$,
$\tau^{i} = \{\phi,\{1\},\{1,2\},\{1,2,3\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},X\}$,
$(X,\tau), (X,\tau^{\alpha}), (X,\tau^{s})$ and (X,τ^{i}) are topological -spaces.
Take, $1 \neq 2$:

1. There is no exists two open sets G_1, G_2 s.t. $1 \in G_1, 2 \notin G_1, 2 \in G_2, 1 \notin G_2$, therefore; (X,τ) is not T_1-space.
2. There is no exists two α-open sets α_1, α_2 s.t. $1 \in \alpha_1, 2 \notin \alpha_1, 2 \in \alpha_2, 1 \notin \alpha_2$, therefore; (X,τ) is not $T_{1\alpha}$ -space.
3. There is no exists two semi-open sets S_1, S_2 s.t. $1 \in S_1, 2 \notin S_1$, $2 \in S_2, 1 \notin S_2$, therefore; (X,τ) is not T_{1s} −space.
4. $\forall x, y \in X \ (x \neq y) \ \exists I_1, I_2 \in \tau^i$ s.t. $x \in I_1, y \notin I_1, y \in I_2, x \notin I_2$, therefore; (X,τ) is T_{1i} −space.

Definition2.7. A topological space (X,τ) is said to be T_2-space [11] (respect. $T_{2\alpha}$, T_{2s} and T_{2i} −space) if it satisfies Hausdorff axiom [11](respect. α-Hausdorff, s-Hausdorff and i-Hausdorff axiom:[T_2 (respect. $T_{2\alpha}$, T_{2s} and T_{2i})]: $\forall x, y \in X \ (x \neq y) \ \exists I_1, I_2 \in \tau \ (respect. \tau^\alpha, \tau^s \ and \ \tau^i), \ I_1 \cap I_2 = \phi$ s.t. $x \in I_1, y \in I_2$.

Definition2.8. A topological space (X,τ) is said to be:
1. Regular space [11] (shortly R space) if it satisfies Vietoris axiom:[R] if F is a closed set in X and $x \in X, x \notin F$ $\exists S_1, S_2 \in \tau, S_1 \cap S_2 = \phi$ s.t. $F \subseteq S_1, \ x \in S_2$.
2. α-Regular space (shortly Rα- space) if it satisfies α-Vietoris axiom: [Rα] if F is an α-closed set in X and $x \in X, x \notin F$ $\exists S_1, S_2 \in \tau^\alpha, S_1 \cap S_2 = \phi$ s.t. $F \subseteq S_1, \ x \in S_2$.
3. s-Regular space (shortly R_s space) if it satisfies s-Vietoris axiom:[R_s] if F is a semi-closed set in X and $x \in X, x \notin F$ $\exists S_1, S_2 \in \tau^s, S_1 \cap S_2 = \phi$ s.t. $F \subseteq S_1, \ x \in S_2$.
4. i-Regular space (shortly R_i -space) if it satisfies i-Vietoris axiom:[R_i] if F is an i-closed set in X and $x \in X, x \notin F$ $\exists I_1, I_2 \in \tau^i, I_1 \cap I_2 = \phi$ s.t. $F \subseteq I_1, \ x \in I_2$.

Definition2.9. A T_1- space[11] (respect. $T_{1\alpha}$, T_{1s} and T_{1i} -space) is said to be $\boldsymbol{T_3}$ [11] (respect. $\boldsymbol{T_{3\alpha}}$, T_{3s} and T_{3i}) if it is Regular(respect. α-Regular, s-Regular and i-Regular).

Definition2.10. A topological space (X,τ) is said to be:
1. Normal space [11] (shortly N space) if it satisfies Urysohn axiom:

[N] $if\ F_1 \subseteq X,\ F_2 \subseteq X, F_1 \cap F_2 = \phi\ \exists S_1, S_2 \subseteq X\ s.t\ F_1 \subseteq S_1,\ F_2 \subseteq S_2$

$where\ S_1 \cap S_2 = \phi,\ F_1, F_2\ are\ closed\ sets,\ S_1, S_2\ are\ open\ sets.$

2. α-Normal space (shortly N_α- space) if it satisfies α-Urysohn axiom:
[N_α] $if\ F_1 \subseteq X,\ F_2 \subseteq X, F_1 \cap F_2 = \phi\ \exists S_1, S_2 \subseteq X\ s.t\ F_1 \subseteq S_1,\ F_2 \subseteq S_2$

$where\ S_1 \cap S_2 = \phi,\ F_1, F_2\ are\ \alpha-closed\ sets,\ S_1, S_2\ are\ \alpha-open\ sets.$

3. s-Normal space (shortly N_s space) if it satisfies s-Urysohn axiom:
[N_s] $if\ F_1 \subseteq X,\ F_2 \subseteq X, F_1 \cap F_2 = \phi\ \exists S_1, S_2 \subseteq X\ s.t\ F_1 \subseteq S_1,\ F_2 \subseteq S_2$

$where\ S_1 \cap S_2 = \phi,\ F_1, F_2\ are\ semi-closed\ sets,\ S_1, S_2\ are\ semi-open\ sets.$

4. i-Normal space (shortly N_i space) if it satisfies i-Urysohn axiom:
[N_i] $if\ F_1 \subseteq X,\ F_2 \subseteq X, F_1 \cap F_2 = \phi\ \exists I_1, I_2 \subseteq X\ s.t\ F_1 \subseteq I_1,\ F_2 \subseteq I_2$

$where\ I_1 \cap I_2 = \phi,\ F_1, F_2\ are\ i-closed\ sets,\ I_1, I_2\ are\ i-open\ sets.$

Definition2.11. A T_1 -space(respect. $T_{1\alpha}$, T_{1s} and T_{1i} −space) is said to be T_4 [11] (respect. $T_{4\alpha}$, T_{4s} and T_{4i} if it is Normal(respect. α- Normal, s- Normal and i- Normal).

Definition2.12. A topological space (X, τ) is said to be:
1. Completely regular space [11] (shortly CR space) if it satisfies the following axiom:[CR] if F is a closed set in X and $x \in X, x \notin F$ there exists a continuous mapping $f : X \to [0,1]$ s.t. $f(F) = 1,\ f(x) = 0$.
2. α-completely regular space (shortly CR_α space) if it satisfies the following axiom:[CR_α] if F is an α-closed set in X and $x \in X, x \notin F$ there exists an α-continuous mapping $f : X \to [0,1]$ s.t. $f(F) = 1,\ f(x) = 0$.
3. s-completely regular space (shortly CR_s space) if it satisfies the following axiom:[CR_s] if F is a semi-closed set in X and $x \in X, x \notin F$ there exists a semi-continuous mapping[5] $f : X \to [0,1]$ s.t. $f(F) = 1,\ f(x) = 0$.
4. i-completely regular space (shortly CR_i space) if it satisfies the following axiom: [CR_i] if F is an i-closed set in X and $x \in X, x \notin F$ there exist i-continuous mapping [9] $f : X \to [0,1]$ s.t. $f(F) = 1,\ f(x) = 0$.

Definition2.13. A T_1-space(respect. $T_{1\alpha}$, T_{1s} and T_{1i} −space) is said to be $T_{(3\frac{1}{2})}$ [11] (respect. $T_{(3\frac{1}{2})\alpha}$, $T_{(3\frac{1}{2})s}$ and $T_{(3\frac{1}{2})i}$) if it is completely Regular(respect. α- completely Regular, s- completely Regular and i-completely Regular).

Definition2.14. A topological space (X, τ) is said to be:

1. Completely Normal space [11] (shortly CN space) if it satisfies Tietze axiom:[CN] If,

$A_1 \subseteq X,\ A_2 \subseteq X, A_1 \cap A_2 = \phi\ \exists S_1, S_2 \subseteq X\ s.t\ A_1 \subseteq S_1, A_2 \subseteq S_2$

$where\ A_1, A_2\ are two\ separated sets, S_1 \cap S_2 = \phi, S_1, S_2\ are\ open sets.$

2. α-completely Normal space (shortly CN_α space) if it satisfies α- Tietze axiom:[CN_α] if,

$A_1 \subseteq X,\ A_2 \subseteq X, A_1 \cap A_2 = \phi\ , \exists S_1, S_2 \subseteq X\ s.t\ A_1 \subseteq S_1, A_2 \subseteq S_2$

$where\ A_1, A_2\ are two\ separated sets, S_1 \cap S_2 = \phi, S_1, S_2\ are\ \alpha - open sets.$

3. s-completely Normal space (shortly CN_s space) if it satisfies s- Tietze axiom: [CN_s] if,

$A_1 \subseteq X,\ A_2 \subseteq X, A_1 \cap A_2 = \phi\ \exists S_1, S_2 \subseteq X\ s.t\ A_1 \subseteq S_1,\ A_2 \subseteq S_2$

$where\ A_1, A_2\ are two\ separated sets, S_1 \cap S_2 = \phi, S_1, S_2\ are\ semi - open sets.$

4. i-completely Normal space (shortly CN_i space) if it satisfies i- Tietze axiom: [CN_i] if, $A_1 \subseteq X,\ A_2 \subseteq X, A_1 \cap A_2 = \phi\ \exists I_1, I_2 \subseteq X\ s.t\ A_1 \subseteq I_1,\ A_2 \subseteq I_2$

$where\ A_1, A_2\ are two\ separated sets, I_1 \cap I_2 = \phi, I_1, I_2\ are\ i - open sets.$

Definition2.15. A T_1- space (respect. $T_{1\alpha}$, T_{1s} and T_{1i}-space) is said to be T_5 [11](respect. $T_{5\alpha}$, T_{5s} and T_{5i} -space) if it is completely Normal(respect. α- completely Normal, s- completely Normal and i-completely Normal).

Example2.16. Let $X = \{a, b\}$, $\tau = \{\phi, \{a\}, \{b\}, X\}$, $\tau^\alpha = \tau^s = \tau^i = \tau$

$(X, \tau), (X, \tau^\alpha), (X, \tau^s)$ and (X, τ^i) are topological spaces.

Open, $\alpha - open, s - open\ and\ i - open$ sets are: ϕ, $\{a\}, \{b\}, X$.

Closed, $\alpha - closed\ , s - closed\ and\ i - closed$ sets are: ϕ, $\{a\}, \{b\}, X$

$1.\ a, b \in X (a \neq b)\ \exists \{a\}, \{b\} \in \tau\ (respect.\ \tau^\alpha, \tau^s\ and\ \tau^i)$

$s.t.\ a \in \{a\}, b \in \{b\}$. Therefore; (X, τ) is T_1, $T_{1\alpha}$, T_{1s} and T_{1i}-space.

$2.\ a, b \in X (a \neq b) \exists \{a\}, \{b\} \in \tau (respect.\ \tau^\alpha, \tau^s\ and\ \tau^i)\ s.t.\ a \in \{a\}, b \in \{b\},$

$\{a\} \cap \{b\} = \phi$. Therefore; (X, τ) is T_2, $T_{2\alpha}$, T_{2s} and T_{2i}-space.

$3.\ i.\ \{b\}\ is\ a\ closed\ \ set\ \ and\ a \notin \{b\}\ there\ are\ two\ open\ sets\ \{a\}, \{b\}$

$s.t.\ \ a \in \{a\}, \{b\} \subseteq \{b\}$. Therefore; (X, τ) is Regular space.

$ii.\ \{b\}\ is\ \ \alpha - closed\ \ set\ \ and\ a \notin \{b\}\ there\ are\ two\ \alpha - open\ sets\ \{a\}, \{b\}$

$s.t.\ \ a \in \{a\}, \{b\} \subseteq \{b\}$. Therefore; (X, τ) is α-Regular space.

$iii.\ \{b\}\ is\ a\ semi - closed\ \ set\ \ and\ a \notin \{b\}\ there\ are\ two\ semi - open\ sets\ \{a\}, \{b\}$

$s.t.\ \ a \in \{a\}, \{b\} \subseteq \{b\}$. Therefore; (X, τ) is s-Regular space.

$iv.\ \{b\}$ *is an* $i-closed$ *set and* $a \notin \{b\}$ *there is two* $i-open$ *sets* $\{a\},\{b\}$
$s.t.\ \ a \in \{a\}, \{b\} \subseteq \{b\}$. Therefore; (X,τ) is i-Regular space.

4. By (1) and (3) (i)(respect. (ii), (iii) and (iv)) we have: (X,τ) is T_3-space (respect. $T_{3\alpha}$, T_{3s} and T_{3i}-space).

5. $i.\ \{a\},\{b\}$ *are closed sets, there are two open sets* $\{a\},\{b\}$
$s.t.\ \ \{a\} \subseteq \{a\}, \{b\} \subseteq \{b\}, \{a\} \cap \{b\} = \phi$. Therefore; (X,τ) is Normal space.

$ii.\ \{a\},\{b\}$ *are* $\alpha-closed$ *sets, there are two* $\alpha-open$ *sets* $\{a\},\{b\}$
$s.t.\ \ \{a\} \subseteq \{a\}, \{b\} \subseteq \{b\}, \{a\} \cap \{b\} = \phi$. Therefore; (X,τ) is α-Normal space.

$iii.\ \{a\},\{b\}$ *are* $semi-closed$ *sets, there are two* $semi-open$ *sets* $\{a\},\{b\}$
$s.t.\ \ \{a\} \subseteq \{a\}, \{b\} \subseteq \{b\}, \{a\} \cap \{b\} = \phi$. Therefore; (X,τ) is s-Normal space.

$iv.\ \{a\},\{b\}$ *are* $i-closed$ *sets there are two* $i-open$ *sets* $\{a\},\{b\}$
$s.t.\ \ \{a\} \subseteq \{a\}, \{b\} \subseteq \{b\}, \{a\} \cap \{b\} = \phi$. Therefore; (X,τ) is i-Normal space.

6. By (1) and (5) (i)(respect. (ii), (iii) and (iv)) we have: (X,τ) is T_4-space (respect. $T_{4\alpha}$, T_{4s} and T_{4i}-space).

7. $i.$ *let* $f : X \to [0,1]$ *be a continuous mapping and* $\{b\}$ *is a closed set and* $a \notin \{b\}$ $s.t.\ \ f(a) = 0, f(\{b\}) = 1.$
$s.t.\ \ f(a) = 0, f(\{b\}) = 1.$ Therefore; (X,τ) is Completely Regular space.

$ii.$ *let* $f : X \to [0,1]$ *be an* $\alpha-continuous$ *mapping and* $\{b\}$ *is* $\alpha-closed$ *set and* $a \notin \{b\}$ $s.t.\ \ f(a) = 0, f(\{b\}) = 1.$
Therefore; (X,τ) is α-Completely Regular space.

$iii.$ *let* $f : X \to [0,1]$ *be a* $semi-continuous$ *mapping and* $\{b\}$ *is a* $semi-closed$ *set and* $a \notin \{b\}$ $s.t.\ \ f(a) = 0, f(\{b\}) = 1.$ Therefore; (X,τ) is s-Completely Regular space.

$iv.$ *Let* $f : X \to [0,1]$ *be an* $i-continuous$ *mapping and* $\{b\}$ *is an* $i-closed$ *set and* $a \notin \{b\}$ $s.t.\ \ f(a) = 0, f(\{b\}) = 1.$ Therefore; (X,τ) is i-Completely Regular space.

8. By (1) and (7) (i)(respect. (ii), (iii) and (iv)) we have: (X,τ) is $T_{(3\frac{1}{2})}$ -space (respect. $T_{(3\frac{1}{2})\alpha}$, $T_{(3\frac{1}{2})s}$ and $T_{(3\frac{1}{2})i}$ -space).

9. $i.\ \{a\},\{b\} \subseteq X$, *there are two open sets* $\{a\},\{b\}$
$s.t.\ \ \{a\} \subseteq \{a\}, \{b\} \subseteq \{b\}$ *where* $\{a\} \cap \{b\} = \phi.$

Therefore; (X,τ) is Completely Normal space.

$ii. \{a\},\{b\} \subseteq X$, there are two $\alpha-open$ sets $\{a\},\{b\}$

s.t. $\{a\} \subseteq \{a\}, \{b\} \subseteq \{b\}$ where $\{a\} \cap \{b\} = \phi$.

Therefore; (X,τ) is α-Completely Normal space.

$iii. \{a\},\{b\} \subseteq X$, there are two $semi-open$ sets $\{a\},\{b\}$

s.t. $\{a\} \subseteq \{a\}, \{b\} \subseteq \{b\}$ where $\{a\} \cap \{b\} = \phi$.

Therefore; (X,τ) is s-Completely Normal space.

$iv. \{a\},\{b\} \subseteq X$, there are two $i-open$ sets $\{a\},\{b\}$

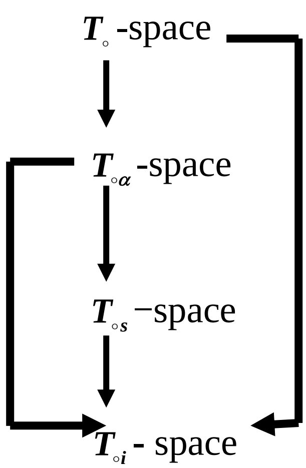
s.t. $\{a\} \subseteq \{a\}, \{b\} \subseteq \{b\}$ where $\{a\} \cap \{b\} = \phi$.

Therefore; (X,τ) is i-Completely Normal space.

10. By (1) and (9)(i)(respect. (ii), (iii) and (iv)) we have: (X,τ) is T_5-space (respect. $T_{5\alpha}$, T_{5s} and $T_{5i}-$ space).

Corollaries2.17. The following diagrams are true.

i.



Proof: 1. Suppose that (X,τ) is $T_\circ$ space.

Then $\forall x,y \in X\ (x \neq y)$ there exists open set I s.t. $x \in I,\ y \notin I$. Since every open set is $\alpha-open$ (corollary1.10).Then I is $\alpha-open$ set. Therefore; (X,τ) is $T_{\circ\alpha}$-space (definition 2.1).

2. Similarly, by using (corollary1.10) and (definition 2.1), we can prove every $T_{\circ\alpha}$ -space is $T_{\circ s}$- space.

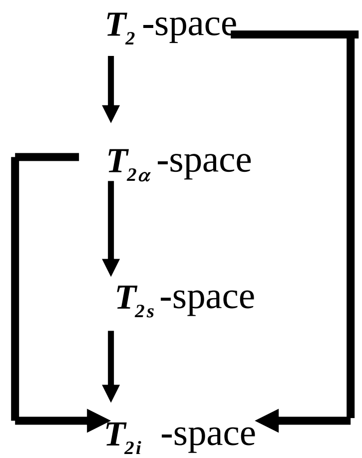
3. Similarly, by using corollary1.10 and definition 2.1, we can prove every $T_{\circ s}$-space is $T_{\circ i}$ space.

4. From 1 and 2 we have, every $T_\circ$-space is $T_{\circ s}$-space.

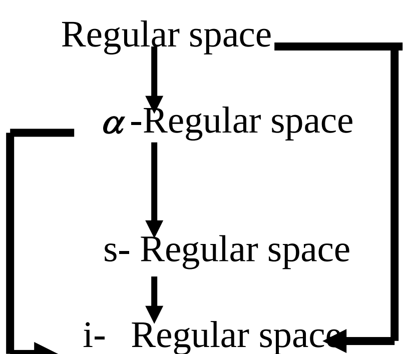
5. From 4 and 3 we have, every $T_\circ-$space is $T_{\circ i}$-space.

6. From 2 and 3 we have, every $T_{\circ\alpha}$ -space is $T_{\circ i}$ -space. ■

ii.



Proof: 1. Suppose that (X,τ) is T_2 -space.
Then $\forall x,y \in X\ (x \neq y)$ *there exists two open sets* $I_1, I_2, I_1 \cap I_2 = \phi$ s.t. $x \in I_1$, $y \in I_2$. Since every open set is $\alpha - open$ (corollary1.10).Then I_1 *and* I_2 are $\alpha - open$ sets. Therefore; (X,τ) is $T_{2\alpha}$ space (definition 2.7).
2. Similarly, by using (corollary1.10) and (definition 2.7), we can prove every $T_{2\alpha}$ -space is T_{2s} - space.
3. Similarly, by using corollary1.10 and (definition 2.7), we can prove every T_{2s} -space is T_{2i} - space.
4. From 1 and 2 we have, every T_2 space is T_{2s} space.
5. From 4 and 3 we have, every T_2 space is T_{2i} space.
6. From 2 and 3 we have, every $T_{2\alpha}$ space is T_{2i} space. ■
iii.

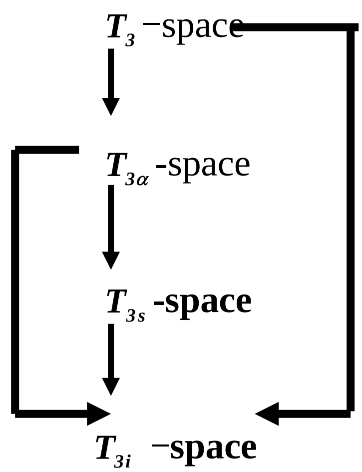


Proof: 1. Suppose that (X,τ) is a regular space. Then for every closed set F in X with $x \in X, x \notin F$ *there exists two open sets* $S_1, S_2, S_1 \cap S_2 = \phi$ s.t. $F \subseteq S_1$, $x \in S_2$. Since every open (closed) set is $\alpha - open(\alpha - closed)$

(corollary1.10).Then S_1 *and* S_2 are $\alpha-open$ sets and F is $\alpha-closed$ set. Therefore; (X,τ) is α-regular space (definition 2.8(2)).

2. Similarly, by using corollary 1.10 and definitions 2.8(2), 2.8(3), we can prove every α-regular space is s-regular space
3. Similarly, by using corollary 1.10 and (definitions 2.8(3), 2.8(4), we can prove every s-regular space is i-regular space.
4. From 1 and 2 we have, every regular space is s-regular space.
5. From 4 and 3 we have, every regular space is i-regular space.
6. From 2 and 3 we have, every α-regular space is i-regular space. ■

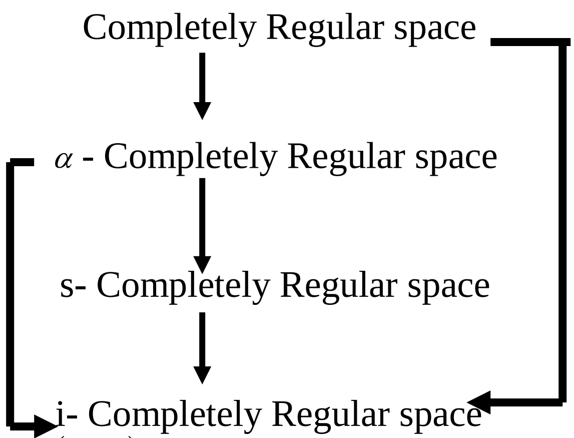
iv.



Proof: 1. Suppose that (X,τ) is T_3-space. Then (X,τ) is T_1 and regular space (definition 2.9). Since every T_1 space is $T_{1\alpha}$ (corollary 2.5(1) and since every Regular space is α-Regular(corollaries 2.17(iii)(1)), we have (X,τ) is $T_{3\alpha}$-space.

2. Similarly, by using (definition 2.8(4)), (corollary 2.5(2)) and corollaries 2.29(iii)(2), we can prove every $T_{3\alpha}$ space is T_{3s} space.
3. Similarly, by using (definition 2.9), (corollary 2.5(3)) and corollaries 2.17(iii)(3), we can prove every T_{3s} space is T_{3i} space.
4. From 1 and 2 we have, every T_3 space is T_{3s} space.
5. From 4 and 3 we have, every T_3 space is T_{3i} space.
6. From 2 and 3 we have, every $T_{3\alpha}$ space is T_{3i} space. ■

v.



Proof: 1. Suppose that (X,τ) is a completely regular space. Then for every closed set F in X with $x \in X, x \notin F$ there exists a continuous mapping $f : X \to [0,1]$ s.t. $f(F)=1,\ f(x)=0$. Since every open (closed) set is $\alpha - open(\alpha - closed)$ (corollary1.10) and since every continuous mapping is $\alpha -$continuous(corollary 1.20) .Then F is $\alpha - closed$ set and $f : X \to [0,1]$ is $\alpha -$continuous. Therefore; (X,τ) is α- completely regular space (definition 2.18(2)).

2. Similarly, by using (corollary 1.10), (corollary 1.20) and (definition 2.12(3)), we can prove every α- completely regular space is s- completely regular space.

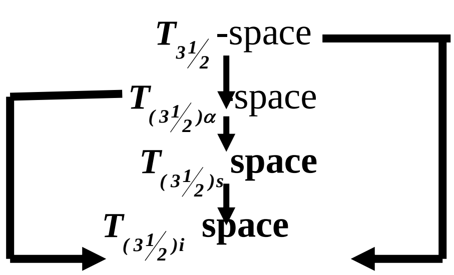
3. Similarly, by using (corollary 1.10), (corollary 1.20) and (definition 2.12(4)), we can prove every s- completely regular space is i- completely regular space.

4. From 1 and 2 we have, every completely regular space is s- completely regular space.

5. From 4 and 3 we have, every completely regular space is i- completely regular space.

6. From 2 and 3 we have, every α- completely regular space is i- completely regular space.■

vi.



Proof: 1. Suppose that (X,τ) is $T_{3\frac{1}{2}}$ space. Then (X,τ) is T_1 and completely-regular space (definition 2.13). Since every T_1 space is $T_{1\alpha}$ (corollary 2.5(1) and since every completely regular space is α-completely regular (corollaries 2.17(v)(1)), we have (X,τ) is $T_{(3\frac{1}{2})\alpha}$ space.

2. Similarly, by using (corollary 2.5(2)) and corollaries 2.17(v)(2), we can prove every $T_{(3\frac{1}{2})\alpha}$ space is $T_{(3\frac{1}{2})s}$ space.

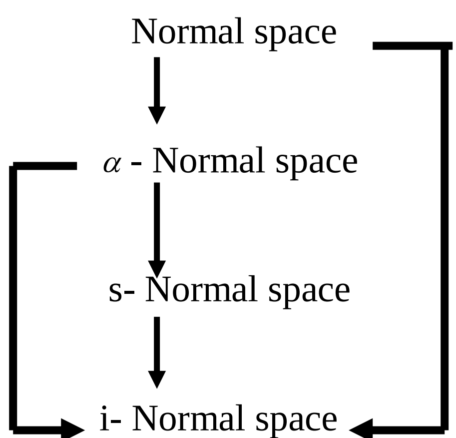
3. Similarly, by using (corollary 2.5(3)) and corollaries 2.17(v)(3), we can prove every $T_{(3\frac{1}{2})s}$ space is $T_{(3\frac{1}{2})i}$ space.

4. From 1 and 2 we have, every $T_{3\frac{1}{2}}$ space is $T_{(3\frac{1}{2})s}$ space.

5. From 4 and 3 we have, every $T_{3\frac{1}{2}}$ space is $T_{(3\frac{1}{2})i}$ space.

6. From 2 and 3 we have, every $T_{(3\frac{1}{2})\alpha}$ space is $T_{(3\frac{1}{2})i}$ space. ■

vii.



Proof: 1. Suppose that (X,τ) is a normal space. Then $for\ every\ F_1 \subseteq X,\ F_2 \subseteq X, F_1 \cap F_2 = \phi\ \exists S_1, S_2 \subseteq X\ s.t\ F_1 \subseteq S_1,\ F_2 \subseteq S_2$ $where\ S_1 \cap S_2 = \phi,\ F_1, F_2\ are\ closed\ sets,\ S_1, S_2\ are\ open\ sets.$ Since every open (closed) set is $\alpha - open\,(\alpha - closed)$(corollary1.10).Then $S_1\ and\ S_2$ are $\alpha - open$ sets and F_1, F_2 are $\alpha - closed$ sets. Therefore; (X,τ) is α-normal space (definition2.10 (2)).

2. Similarly, by using (corollary 1.10) and (definition 2.10(3)), we can prove every α - normal space is s- normal space.

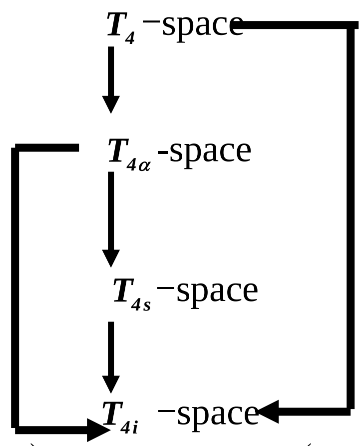
3. Similarly, by using (corollary 1.10) and (definition 2.10(4)), we can prove every s- normal space is i- normal space.

4. From 1 and 2 we have, every normal space is s- normal space.

5. From 4 and 3 we have, every normal space is i- normal space.

6. From 2 and 3 we have, every α - normal space is i- normal space.■

viii.



Proof: 1. Suppose that (X,τ) is T_4 space. Then (X,τ) is T_1 and normal space (definition 2.11). Since every T_1 space is $T_{1\alpha}$ (corollary 2.5(1) and since every normal space is α-normal(corollaries 2.17(vii)(1)), we have (X,τ) is $T_{4\alpha}-$ space.

2. Similarly, by using (definition 2.11), (corollary 2.5(2)) and corollaries 2.17(vii)(2), we can prove every $T_{4\alpha}$ -space is $T_{4s}-$ space.

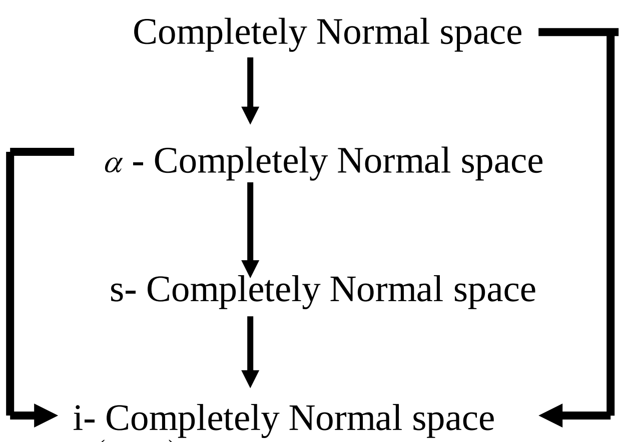
3. Similarly, by using (definition 2.11), (corollary 2.5(٣)) and corollaries 2.17(vii)(٣), we can prove every $T_{4s}-$ space is T_{4i} -space.

4. From 1 and 2 we have, every T_4−space is T_{4s}- space.

5. From 4 and 3 we have, every T_4-space is T_{4i} -space.

6. From 2 and 3 we have, every $T_{4\alpha}$- is T_{4i} -space. ■

ix.



Proof: 1. Suppose that (X,τ) is a completely normal space.
Then for every two separated subsets of X, $A_1, A_2, A_1 \cap A_2 = \phi$, there exists two open sets, $S_1, S_2 \subseteq X$ $s.t$ $A_1 \subseteq S_1$, $A_2 \subseteq S_2$, $S_1 \cap S_2 = \phi$. Since every

open (closed) set is $\alpha-open$ ($\alpha-closed$) (corollary1.10).Then S_1,S_2 are $\alpha-closed$ sets. Therefore; (X,τ) is α - completely normal space (definition 2.14(2)).

2. Similarly, by using (corollary 1.10) and (definition 2.14(3)), we can prove every α - completely normal space is s- completely normal space.

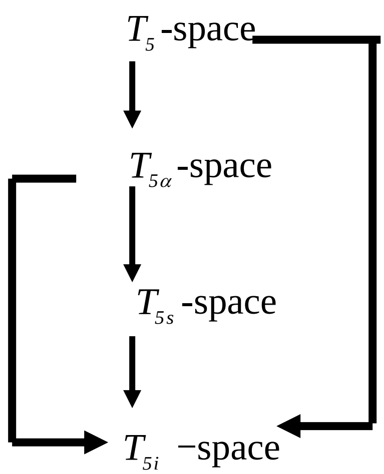
3. Similarly, by using (corollary 1.10) and (definition 2.14(4)), we can prove every s- completely normal space is i- completely normal space.

4. From 1 and 2 we have, every completely normal space is s- completely normal space.

5. From 4 and 3 we have, every completely normal space is i- completely normal space.

6. From 2 and 3 we have, every α - completely normal space is i- completely normal space. ■

x.



Proof: 1. Suppose that (X,τ) is $\boldsymbol{T_5}$ space. Then (X,τ) is T_1 and completely normal space (definition 2.15). Since every T_1 space is $T_{1\alpha}$ (corollary 2.5(1) and since every completely normal space is α - completely normal(corollaries 2.17(ix)(1)), we have (X,τ) is $T_{5\alpha}$ space.

2. Similarly, by using (definition 2.15), (corollary 2.5(2)) and corollaries 2.17(ix)(2), we can prove every $T_{5\alpha}$ space is T_{5s} space.

3. Similarly, by using (definition 2.15), (corollary 2.5(٣)) and corollaries 2.17(ix)(٣), we can prove every T_{5s} space is T_{5i} space.

4. From 1 and 2 we have, every T_5 space is T_{5s} space.

5. From 4 and 3 we have, every T_5 space is T_{5i} space.

6. From 2 and 3 we have, every $T_{5\alpha}$ space is T_{5i} space. ■

From above we have the converses of corollaries 2.17 are not necessary to be true.

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